

**Determination of As in particulate matter using Se as an  
internal standard by multi-element electrothermal atomic  
absorption spectrometry.**

**Supplementary Material**

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<i>Quantity</i>	<i>Units</i>	<i>Definition</i>
alpha	$\mu\text{L}/(\mu\text{L } ^\circ\text{C})$	Thermal expansion coefficient
reps		Replicates
temperature	$^\circ\text{C}$	Temperature
Vpipet	$\mu\text{L}$	Volume of the bulk solution pipetted for preparing the stock solution
Vpipet2	$\mu\text{L}$	Volume of the stock solution pipetted for preparing the first dilution solution
Vfirstdil	$\mu\text{L}$	Volume of the first dilution solution pipetted for preparing the second dilution solution
Vsecdil	$\mu\text{L}$	Volume of the second dilution pipetted for preparing the working solution
Vsampling	$\mu\text{L}$	Sampling volume
Vafterdigest	mL	Final volume of the sample after digestion
Fsample		Dilution factor of the sample
Cbulk	$\mu\text{g}/\text{L}$	Arsenic concentration of the bulk solution
Cstock	$\mu\text{g}/\text{L}$	Arsenic concentration of the stock solution
Cworksolution	$\mu\text{g}/\text{L}$	Arsenic concentration of the working solution
Cstd	$\mu\text{g}/\text{L}$	Mean concentration of all calibration standards
A <sub>ij</sub>	AU	Integrated absorbance of the j measurement of the calibration standard i
Astd	AU	Mean absorbance of all calibration standards
$\beta_1$	$\text{AU}/(\mu\text{g}/\text{L})$	Calculated best fit slope (gradient) of the calibration curve
$\beta_0$	AU	Calculated best fit intercept of the calibration curve
AR <sub>ij</sub>	AU	Integrated absorbance of the j measurement of the i fortification level solution
Cobsi	$\mu\text{g}/\text{L}$	Mean arsenic concentration of the i fortification level solution
dbiasi		Bias component of the uncertainty of Cobsi
drepi		Reproducibility component of the uncertainty of Cobsi
C <sub>i</sub>	$\mu\text{g}/\text{L}$	Final arsenic concentration of the i fortification level solution (all components of uncertainty included)

**Suppl. Table 1:** Terms and definitions of the Mathematical code used in MCM when Se was not used as internal standard.



**Mathematica program code used for the uncertainty estimation based on MCM  
without the use of an internal standard**

```
reps=1000000;  
alpha=0.00021;  
meansN={meanVpipet,meanVpipet2,meanVfirstdil,meanVsecdil,meanVsampling};  
sdN={sdVpipet,sdVpipet2,sdVfirstdil,sdVsecdil,sdVsampling};  
meanU={meanVafterdigest,meanT ,meanCbulk};  
sdU={sdVafterdigest,sdT ,sdCbulk};  
Temperature=22;  
meanA1={A11,A12,A13};  
meanA2={A21,A22,A23};  
meanA3={A31,A32,A33};  
meanA4={A41,A42,A43};  
meanA5={A51,A52,A53};  
meanAR1={AR11,AR12,AR13};  
meanAR2={AR21,AR22,AR23};  
meanAR3={AR31,AR32,AR33};  
meanAR4={AR41,AR42,AR43};  
sdAi=0.001897;  
a=(2meanU-sdU Sqrt[12])/2;  
b=(2meanU+sdU Sqrt[12])/2;  
Vpipet=Table[RandomReal[NormalDistribution[meansN[[1]],sdN[[1]]],{reps}];  
Vpipet2=Table[RandomReal[NormalDistribution[meansN[[2]],sdN[[2]]],{reps}];  
Vfirstdil=Table[RandomReal[NormalDistribution[meansN[[3]],sdN[[3]]],{reps}];  
Vsecdil=Table[RandomReal[NormalDistribution[meansN[[4]],sdN[[4]]],{reps}];  
Vsampling=Table[RandomReal[NormalDistribution[meansN[[5]],sdN[[5]]],{reps}];
```

Vafterdigest=Table[RandomReal[{a[[1]],b[[1]]}],{reps}];

T=Table[RandomReal[{a[[2]],b[[2]]}],{reps}];

Cbulk=Table[RandomReal[{a[[3]],b[[3]]}],{reps}];

Vpipet=Vpipet(1+alpha(T-Temperature));

Vpipet2=Vpipet2(1+alpha(T-Temperature));

Vfirstdil=Vfirstdil(1+alpha(T-Temperature));

Vsecdil=Vsecdil(1+alpha(T-Temperature));

Vsampling=Vsampling(1+alpha(T-Temperature));

Vafterdigest=Vafterdigest(1+alpha(T-Temperature));

Fsample=Vafterdigest/Vsampling;

Cstock=(Cbulk\*Vpipet)/Vfirstdil;

Cworksolution=(Cstock\*Vpipet2)/Vsecdil;

Cstd=(6Cworksolution)/25;

aA1=(2meanA1-sdAi Sqrt[12])/2;bA1=(2meanA1+sdAi Sqrt[12])/2;aA2=(2meanA2-sdAi Sqrt[12])/2;bA2=(2meanA2+sdAi Sqrt[12])/2;

aA3=(2meanA3-sdAi Sqrt[12])/2;bA3=(2meanA3+sdAi Sqrt[12])/2;aA4=(2meanA4-sdAi Sqrt[12])/2;bA4=(2meanA4+sdAi Sqrt[12])/2;

aA5=(2meanA5-sdAi Sqrt[12])/2;bA5=(2meanA5+sdAi Sqrt[12])/2;

A11=RandomReal[{aA1[[1]],bA1[[1]]},reps];A12=RandomReal[{aA1[[2]],bA1[[2]]},reps];

A13=RandomReal[{aA1[[3]],bA1[[3]]},reps];A21=RandomReal[{aA2[[1]],bA2[[1]]},reps];

A22=RandomReal[{aA2[[2]],bA2[[2]]},reps];A23=RandomReal[{aA2[[3]],bA2[[3]]},reps];

A31=RandomReal[{aA3[[1]],bA3[[1]]},reps];A32=RandomReal[{aA3[[2]],bA3[[2]]},reps];

A33=RandomReal[{aA3[[3]],bA3[[3]]},reps];A41=RandomReal[{aA4[[1]],bA4[[1]]},reps];

A42=RandomReal[{aA4[[2]],bA4[[2]]},reps];A43=RandomReal[{aA4[[3]],bA4[[3]]},reps];

A51=RandomReal[{aA5[[1]],bA5[[1]]},reps];A52=RandomReal[{aA5[[2]],bA5[[2]]},reps];

A53=RandomReal[{aA5[[3]],bA5[[3]]},reps];

Astd1=A11+A12+A13;

Astd2=A21+A22+A23;

Astd3=A31+A32+A33;

Astd4=A41+A42+A43;

Astd5=A51+A52+A53;

Astd=(Astd1+Astd2+Astd3+Astd4+Astd5)/15;

$\sqrt{\text{Beta}}_1 = (5\text{Total}[\text{Astd} * \text{Cstd}] - \text{Total}[\text{Cstd}] * \text{Total}[\text{Astd}]) / (5\text{Total}[\text{Cstd}^2] - \text{Total}[\text{Cstd}]^2)$ ;

$\sqrt{\text{Beta}}_0 = \text{Mean}[\text{Astd}] - \sqrt{\text{Beta}}_1 * \text{Mean}[\text{Cstd}]$ ;

aAR1=(2meanAR1-sdAi Sqrt[12])/2;bAR1=(2meanAR1+sdAi Sqrt[12])/2;

aAR2=(2meanAR2-sdAi Sqrt[12])/2;bAR2=(2meanAR2+sdAi Sqrt[12])/2;

aAR3=(2meanAR3-sdAi Sqrt[12])/2;bAR3=(2meanAR3+sdAi Sqrt[12])/2;

aAR4=(2meanAR4-sdAi Sqrt[12])/2;bAR4=(2meanAR4+sdAi Sqrt[12])/2;

AR11=RandomReal[{aAR1[[1]],bAR1[[1]]},reps];AR12=RandomReal[{aAR1[[2]],bAR1[[2]]},reps];

AR13=RandomReal[{aAR1[[3]],bAR1[[3]]},reps];AR21=RandomReal[{aAR2[[1]],bAR2[[1]]},reps];

AR22=RandomReal[{aAR2[[2]],bAR2[[2]]},reps];AR23=RandomReal[{aAR2[[3]],bAR2[[3]]},reps];

AR31=RandomReal[{aAR3[[1]],bAR3[[1]]},reps];AR32=RandomReal[{aAR3[[2]],bAR3[[2]]},reps];

AR33=RandomReal[{aAR3[[3]],bAR3[[3]]},reps];AR41=RandomReal[{aAR4[[1]],bAR4[[1]]},reps];

AR42=RandomReal[{aAR4[[2]],bAR4[[2]]},reps];AR43=RandomReal[{aAR4[[3]],bAR4[[3]]},reps];

AR1mean=(AR11+AR12+AR13)/3;AR2mean=(AR21+AR22+AR23)/3;AR3mean=(AR31+AR32+AR33)/3;AR4mean=(AR41+AR42+AR43)/3;

Cobs1=(AR1mean-\[Beta]0)^\[Beta]1;Cobs2=(AR2mean-\[Beta]0)^\[Beta]1;Cobs3=(AR3mean-\[Beta]0)^\[Beta]1;Cobs4=(AR4mean-\[Beta]0)^\[Beta]1;

dbias1=Table[RandomReal[NormalDistribution[0,0.094]],{reps}];

dbias2=Table[RandomReal[NormalDistribution[0,0.081]],{reps}];

dbias3=Table[RandomReal[NormalDistribution[0,0.029]],{reps}];

dbias4=Table[RandomReal[NormalDistribution[0,0.018]],{reps}];

drep1=Table[RandomReal[NormalDistribution[0,2.41]],{reps}];

drep2=Table[RandomReal[NormalDistribution[0,14.7]],{reps}];

drep3=Table[RandomReal[NormalDistribution[0,10.5]],{reps}];

drep4=Table[RandomReal[NormalDistribution[0,7.69]],{reps}];

C1=Cobs1\*Fsample+dbias1+drep1;C2=Cobs2\*Fsample+dbias2+drep2;

C3=Cobs3\*Fsample+dbias3+drep3;C4=Cobs4\*Fsample+dbias4+drep4;

Mean[C1]

Mean[C2]

Mean[C3]

Mean[C4]

**Suppl. Table 2:** Terms and definitions of the Mathematica code used in MCM when Se was used as internal standard.

<i>Quantity</i>	<i>Units</i>	<i>Definition</i>
alpha	$\mu\text{L}/(\text{uL } ^\circ\text{C})$	Thermal expansion coefficient
reps		Replicates
temperature	$^\circ\text{C}$	Temperature
Vpipet	$\mu\text{L}$	Volume of the bulk arsenic solution pipetted for preparing the stock solution
Vpipet2	$\mu\text{L}$	Volume of the stock arsenic solution pipetted for preparing the first dilution solution
Vfirstdil	$\mu\text{L}$	Volume of the first dilution of the arsenic solution pipetted for preparing the second dilution solution
Vsecdil	$\mu\text{L}$	Volume of the second dilution of the arsenic solution pipetted for preparing the working solution
Vsampling	$\mu\text{L}$	Sampling volume
Vafterdigest	mL	Final volume of the sample after digestion
Fsample		Dilution factor of the sample
Cbulk	$\mu\text{g/L}$	Arsenic concentration of the bulk solution
Cstock	$\mu\text{g/L}$	Arsenic concentration of the stock solution
Cworksolution	$\mu\text{g/L}$	Arsenic concentration of the working solution
Cstd	$\mu\text{g/L}$	Mean concentration of all calibration standards
Aij	AU	Integrated absorbance of the j measurement of the calibration standard i
Astd	AU	Mean absorbance of all calibration standards
$\beta_1$	$\text{AU}/(\mu\text{g/L})$	Calculated best fit slope (gradient) of the calibration curve
$\beta_0$	AU	Calculated best fit intercept of the calibration curve
ARij	AU	Integrated absorbance of the j measurement of the i fortification level solution
Cobsi	$\mu\text{g/L}$	Mean arsenic concentration of the i fortification level solution
dbiasi		Bias component of the uncertainty of Cobsi
drepi		Reproducibility component of the uncertainty of Cobsi
Ci	$\mu\text{g/L}$	Final arsenic concentration of the i fortification level solution (all components of uncertainty included)



VpipetSe	$\mu\text{L}$	Volume of the bulk selenium solution pipetted for preparing the stock solution
VpipetSe2	$\mu\text{L}$	Volume of the stock selenium solution pipetted for preparing the first dilution solution
VfirstdilSe	$\mu\text{L}$	Volume of the first dilution of the selenium solution pipetted for preparing the second dilution solution
VsecdilSe	$\mu\text{L}$	Volume of the second dilution of the selenium solution pipetted for preparing the working solution
CbulkSe	$\mu\text{g/L}$	Selenium concentration of the bulk solution
CstockSe	$\mu\text{g/L}$	Selenium concentration of the stock solution
CworksolutionSe	$\mu\text{g/L}$	Selenium concentration of the work solution
ARISij	AU	Integrated absorbance of the j measurement of the fortification level solution of the analyte to internal standard
AntoISij	AU	Integrated absorbance of the j measurement of the calibration standard i (arsenic to internal standard selenium)

**Mathematica code used for the uncertainty estimation based on MCM when Se  
was used as an internal standard**

```
alpha=0.00021;
reps=1000000;
meansN={meanVpipet,meanVpipet2,meanVfirstdil,meanVsecdil,meanVsampling,me
anVpipetSe,meanVpipetSe2,meanVfirstdilSe,meanVsecdilSe};
sdN={sdVpipet,sdVpipet2,sdVfirstdil,sdVsecdil,sdVsampling,sdVpipetSe,sdVpipetSe
2,sdVfirstdilSe,sdVsecdilSe};
meanU={meanVafterdigest,meanT ,meanCbulk,meanCbulkSe};
sdU={sdVafterdigest,sdT ,sdCbulk,sdCbulkSe};
Temperature=22;
meanAntoIS1={AntoIS11,AntoIS12,AntoIS13};
meanAntoIS2={AntoIS21,AntoIS22,AntoIS23};
meanAntoIS3={AntoIS31,AntoIS32,AntoIS33};
meanAntoIS4={AntoIS41,AntoIS42,AntoIS43};
meanAntoIS5={AntoIS51,AntoIS52,AntoIS53};
meanARIS1={ARIS11,ARIS12,ARIS13};
meanARIS2={ARIS21,ARIS22,ARIS23};
meanARIS3={ARIS31,ARIS32,ARIS33};
meanARIS4={ARIS41,ARIS42,ARIS43};
sdAi=0.63797521;
a=(2meanU-sdU Sqrt[12])/2;
b=(2meanU+sdU Sqrt[12])/2;
Vpipet=Table[RandomReal[NormalDistribution[meansN[[1]],sdN[[1]]],{reps}];
Vpipet2=Table[RandomReal[NormalDistribution[meansN[[2]],sdN[[2]]],{reps}];
VpipetSe=Table[RandomReal[NormalDistribution[meansN[[6]],sdN[[6]]],{reps}];
VpipetSe2=Table[RandomReal[NormalDistribution[meansN[[7]],sdN[[7]]],{reps}];
Vfirstdil=Table[RandomReal[NormalDistribution[meansN[[3]],sdN[[3]]],{reps}];
Vsecdil=Table[RandomReal[NormalDistribution[meansN[[4]],sdN[[4]]],{reps}];
VfirstdilSe=Table[RandomReal[NormalDistribution[meansN[[3]],sdN[[3]]],{reps}];
VsecdilSe=Table[RandomReal[NormalDistribution[meansN[[8]],sdN[[8]]],{reps}];
Vsampling=Table[RandomReal[NormalDistribution[meansN[[9]],sdN[[9]]],{reps}];
Vafterdigest=Table[RandomReal[{a[[1]],b[[1]]}],{reps}];
T=Table[RandomReal[{a[[2]],b[[2]]}],{reps}];
Cbulk=Table[RandomReal[{a[[3]],b[[3]]}],{reps}];
CbulkSe=Table[RandomReal[{a[[4]],b[[4]]}],{reps}];
Vpipet=Vpipet(1+alpha(T-Temperature));
Vpipet2=Vpipet2(1+alpha(T-Temperature));
VpipetSe=VpipetSe(1+alpha(T-Temperature));
VpipetSe2=VpipetSe2(1+alpha(T-Temperature));
```

$V_{\text{firstdil}}=V_{\text{firstdil}}(1+\alpha(T-\text{Temperature}));$   
 $V_{\text{secdil}}=V_{\text{secdil}}(1+\alpha(T-\text{Temperature}));$   
 $V_{\text{sampling}}=V_{\text{sampling}}(1+\alpha(T-\text{Temperature}));$   
 $V_{\text{afterdigest}}=V_{\text{afterdigest}}(1+\alpha(T-\text{Temperature}));$   
 $F_{\text{sample}}=V_{\text{afterdigest}}/V_{\text{sampling}};$   
 $C_{\text{stock}}=(C_{\text{bulk}}*V_{\text{pipet}})/V_{\text{firstdil}};$   
 $C_{\text{stockSe}}=(C_{\text{bulkSe}}*V_{\text{pipetSe}})/V_{\text{firstdilSe}};$   
 $C_{\text{worksolution}}=(C_{\text{stock}}*V_{\text{pipet2}})/V_{\text{secdil}};$   
 $C_{\text{worksolutionSe}}=(C_{\text{stockSe}}*V_{\text{pipetSe2}})/V_{\text{secdilSe}};$   
 $C_{\text{std}}=(6C_{\text{worksolution}})/25;$

$a_{\text{AntoIS1}}=(2\text{meanAntoIS1}-s_{\text{dAi}} \sqrt{12})/2; b_{\text{AntoIS1}}=(2\text{meanAntoIS1}+s_{\text{dAi}} \sqrt{12})/2;$   
 $a_{\text{AntoIS2}}=(2\text{meanAntoIS2}-s_{\text{dAi}} \sqrt{12})/2; b_{\text{AntoIS2}}=(2\text{meanAntoIS2}+s_{\text{dAi}} \sqrt{12})/2;$   
 $a_{\text{AntoIS3}}=(2\text{meanAntoIS3}-s_{\text{dAi}} \sqrt{12})/2; b_{\text{AntoIS3}}=(2\text{meanAntoIS3}+s_{\text{dAi}} \sqrt{12})/2;$   
 $a_{\text{AntoIS4}}=(2\text{meanAntoIS4}-s_{\text{dAi}} \sqrt{12})/2; b_{\text{AntoIS4}}=(2\text{meanAntoIS4}+s_{\text{dAi}} \sqrt{12})/2;$   
 $a_{\text{AntoIS5}}=(2\text{meanAntoIS5}-s_{\text{dAi}} \sqrt{12})/2; b_{\text{AntoIS5}}=(2\text{meanAntoIS5}+s_{\text{dAi}} \sqrt{12})/2;$   
 $\text{AntoIS11}=\text{RandomReal}\{a_{\text{AntoIS1}}[[1]],b_{\text{AntoIS1}}[[1]]\},\text{reps}; \text{AntoIS12}=\text{RandomReal}\{a_{\text{AntoIS1}}[[2]],b_{\text{AntoIS1}}[[2]]\},\text{reps};$   
 $\text{AntoIS13}=\text{RandomReal}\{a_{\text{AntoIS1}}[[3]],b_{\text{AntoIS1}}[[3]]\},\text{reps}; \text{AntoIS21}=\text{RandomReal}\{a_{\text{AntoIS2}}[[1]],b_{\text{AntoIS2}}[[1]]\},\text{reps};$   
 $\text{AntoIS22}=\text{RandomReal}\{a_{\text{AntoIS2}}[[2]],b_{\text{AntoIS2}}[[2]]\},\text{reps}; \text{AntoIS23}=\text{RandomReal}\{a_{\text{AntoIS2}}[[3]],b_{\text{AntoIS2}}[[3]]\},\text{reps};$   
 $\text{AntoIS31}=\text{RandomReal}\{a_{\text{AntoIS3}}[[1]],b_{\text{AntoIS3}}[[1]]\},\text{reps}; \text{AntoIS32}=\text{RandomReal}\{a_{\text{AntoIS3}}[[2]],b_{\text{AntoIS3}}[[2]]\},\text{reps};$   
 $\text{AntoIS33}=\text{RandomReal}\{a_{\text{AntoIS3}}[[3]],b_{\text{AntoIS3}}[[3]]\},\text{reps}; \text{AntoIS41}=\text{RandomReal}\{a_{\text{AntoIS4}}[[1]],b_{\text{AntoIS4}}[[1]]\},\text{reps};$   
 $\text{AntoIS42}=\text{RandomReal}\{a_{\text{AntoIS4}}[[2]],b_{\text{AntoIS4}}[[2]]\},\text{reps}; \text{AntoIS43}=\text{RandomReal}\{a_{\text{AntoIS4}}[[3]],b_{\text{AntoIS4}}[[3]]\},\text{reps};$   
 $\text{AntoIS51}=\text{RandomReal}\{a_{\text{AntoIS5}}[[1]],b_{\text{AntoIS5}}[[1]]\},\text{reps}; \text{AntoIS52}=\text{RandomReal}\{a_{\text{AntoIS5}}[[2]],b_{\text{AntoIS5}}[[2]]\},\text{reps};$   
 $\text{AntoIS53}=\text{RandomReal}\{a_{\text{AntoIS5}}[[3]],b_{\text{AntoIS5}}[[3]]\},\text{reps};$   
 $\text{AntoISstd1}=\text{AntoIS11}+\text{AntoIS12}+\text{AntoIS13};$   
 $\text{AntoISstd2}=\text{AntoIS21}+\text{AntoIS22}+\text{AntoIS23};$   
 $\text{AntoISstd3}=\text{AntoIS31}+\text{AntoIS32}+\text{AntoIS33};$   
 $\text{AntoISstd4}=\text{AntoIS41}+\text{AntoIS42}+\text{AntoIS43};$   
 $\text{AntoISstd5}=\text{AntoIS51}+\text{AntoIS52}+\text{AntoIS53};$   
 $\text{AntoISstd}=(\text{AntoISstd1}+\text{AntoISstd2}+\text{AntoISstd3}+\text{AntoISstd4}+\text{AntoISstd5})/15;$

$\sqrt{[\text{Beta}]1}=(5\text{Total}[\text{AntoISstd}*C_{\text{std}}]-\text{Total}[C_{\text{std}}]*\text{Total}[\text{AntoISstd}])/(5\text{Total}[C_{\text{std}}^2]-\text{Total}[C_{\text{std}}]^2);$   
 $\sqrt{[\text{Beta}]0}=\text{Mean}[\text{AntoISstd}]-\sqrt{[\text{Beta}]1}*\text{Mean}[C_{\text{std}}];$

$aARIS1=(2\text{mean}ARIS1-sdAi \sqrt{12})/2$ ;  $bARIS1=(2\text{mean}ARIS1+sdAi \sqrt{12})/2$ ;  
 $aARIS2=(2\text{mean}ARIS2-sdAi \sqrt{12})/2$ ;  $bARIS2=(2\text{mean}ARIS2+sdAi \sqrt{12})/2$ ;  
 $aARIS3=(2\text{mean}ARIS3-sdAi \sqrt{12})/2$ ;  $bARIS3=(2\text{mean}ARIS3+sdAi \sqrt{12})/2$ ;  
 $aARIS4=(2\text{mean}ARIS4-sdAi \sqrt{12})/2$ ;  $bARIS4=(2\text{mean}ARIS4+sdAi \sqrt{12})/2$ ;  
 $ARIS11=RandomReal[\{aARIS1[[1]],bARIS1[[1]]\},reps]$ ;  $ARIS12=RandomReal[\{aA$   
 $RIS1[[2]],bARIS1[[2]]\},reps]$ ;  
 $ARIS13=RandomReal[\{aARIS1[[3]],bARIS1[[3]]\},reps]$ ;  $ARIS21=RandomReal[\{aA$   
 $RIS2[[1]],bARIS2[[1]]\},reps]$ ;  
 $ARIS22=RandomReal[\{aARIS2[[2]],bARIS2[[2]]\},reps]$ ;  $ARIS23=RandomReal[\{aA$   
 $RIS2[[3]],bARIS2[[3]]\},reps]$ ;  
 $ARIS31=RandomReal[\{aARIS3[[1]],bARIS3[[1]]\},reps]$ ;  $ARIS32=RandomReal[\{aA$   
 $RIS3[[2]],bARIS3[[2]]\},reps]$ ;  
 $ARIS33=RandomReal[\{aARIS3[[3]],bARIS3[[3]]\},reps]$ ;  $ARIS41=RandomReal[\{aA$   
 $RIS4[[1]],bARIS4[[1]]\},reps]$ ;  
 $ARIS42=RandomReal[\{aARIS4[[2]],bARIS4[[2]]\},reps]$ ;  $ARIS43=RandomReal[\{aA$   
 $RIS4[[3]],bARIS4[[3]]\},reps]$ ;  
 $ARIS1\text{mean}=(ARIS11+ARIS12+ARIS13)/3$ ;  $ARIS2\text{mean}=(ARIS21+ARIS22+ARIS2$   
 $3)/3$ ;  $ARIS3\text{mean}=(ARIS31+ARIS32+ARIS33)/3$ ;  $ARIS4\text{mean}=(ARIS41+ARIS42+A$   
 $RIS43)/3$ ;  
 $Cobs1=(ARIS1\text{mean}-\sqrt{[Beta]0})^{\sqrt{[Beta]1}}$ ;  $Cobs2=(ARIS2\text{mean}-$   
 $\sqrt{[Beta]0})^{\sqrt{[Beta]1}}$ ;  $Cobs3=(ARIS3\text{mean}-\sqrt{[Beta]0})^{\sqrt{[Beta]1}}$ ;  $Cobs4=(ARIS4\text{mean}-$   
 $\sqrt{[Beta]0})^{\sqrt{[Beta]1}}$ ;

$dbias1=Table[RandomReal[NormalDistribution[0,0.048]],\{reps\}]$ ;  
 $dbias2=Table[RandomReal[NormalDistribution[0,0.063]],\{reps\}]$ ;  
 $dbias3=Table[RandomReal[NormalDistribution[0,0.017]],\{reps\}]$ ;  
 $dbias4=Table[RandomReal[NormalDistribution[0,0.008]],\{reps\}]$ ;  
 $drep1=Table[RandomReal[NormalDistribution[0,0.042]],\{reps\}]$ ;  
 $drep2=Table[RandomReal[NormalDistribution[0,0.048]],\{reps\}]$ ;  
 $drep3=Table[RandomReal[NormalDistribution[0,0.015]],\{reps\}]$ ;  
 $drep4=Table[RandomReal[NormalDistribution[0,0.083]],\{reps\}]$ ;  
 $C1=Cobs1*F\text{sample}+dbias1+drep1$ ;  $C2=Cobs2*F\text{sample}+dbias2+drep2$ ;  
 $C3=Cobs3*F\text{sample}+dbias3+drep3$ ;  $C4=Cobs4*F\text{sample}+dbias4+drep4$ ;  
 $Mean[C1]$   
 $Mean[C2]$   
 $Mean[C3]$   
 $Mean[C4]$