Exercises for Chapter 3

HOWARD A. STONE AND CAMILLE DUPRAT

Exercise 1. Substitute eqn (3.3) into the right-hand side of eqn (3.2) and show that the magnitude of the net forcing is $2\pi\mu V_0/\left(\ln\epsilon^{-1}\right)^2$. Then, make the equation nondimensional. For example, how should you scale z and u_s? How do the typical fiber deformations vary with the fiber radius, ℓ, $\ln\epsilon^{-1}$, B, and μ? *Answer:* See eqns (3.4) and (3.5).

Exercise 2. Verify that the rescalings shown in eqn (3.10) reduce eqn (3.9) to eqn (3.11).

Exercise 3. Verify that the variable η in eqn (3.29) is dimensionless, *i.e.* $\left(B/\zeta_\perp\right)^{1/4} t^{1/4}$ has dimensions of length. Also, verify that the similarity solution in eqn (3.29) reduces the original fourth-order PDE to the fourth-order ODE (3.30).

Exercise 4.

(i) Show that the problem in §3.3.2 has a similarity solution of the form

$$u(z,t) = v_0 t U(\eta), \quad \text{where} \quad \eta = \frac{z}{\left(B/\zeta_\perp\right) t^{1/4}}. \tag{1}$$

(ii) Explain why the similarity variable has a dependence on $t^{1/4}$.

(iii) Find the ODE satisfied by $U(\eta)$ and give sufficient boundary conditions for its solution.

Exercise 5. Carry out all of the steps to verify the solution given in §3.3.3 for a finite-length elastica.

Supplementary information for Fluid-Structure Interactions in Low-Reynolds-Number Flows

Exercise 6.

(i) Explain why nondimensionalization of the PDE and boundary conditions for the problem in §3.3.5 leads naturally to the definitions

$$T = \omega t, \qquad Z = \frac{z}{\ell_B}, \quad \text{where} \quad \ell_B = \left(\frac{B}{\omega \zeta_\perp}\right)^{1/4}, \qquad U = \frac{u}{M_0 \ell_B^2 / B}. \tag{2}$$

(ii) Conclude that the corresponding nondimensional problem statement is

$$\frac{\partial^4 U}{\partial Z^4} = -\frac{\partial U}{\partial T}, \quad \text{with} \quad U(0,T) = 0, \quad \frac{\partial^2 U}{\partial Z^2}(0,T) = \cos T, \quad U(\infty, T) \to 0. \tag{3}$$

(iii) Show that the time-periodic solution can be written

$$U(Z,T) = \Re\left\{e^{iT} W(Z)\right\}, \quad \text{where} \quad W'''' = -iW, \tag{4}$$

and solve for $W(Z)$ using steps and algebra similar to those in the problem discussed in §3.3.4.