Supplementary information

1. Detailed deduction of exchange energy E_{ex}

The exchange energy E_{ex} is denoted as the summation of magnetostatic energy between all magnetic moments. In the model of chain of ellipsoid-rings, we only consider the dominant energy of magnetic interactions, namely, magnetic interaction energy within each ring $E_{ex(rings)}$ and within each chain $E_{ex(chains)}$ of ellipsoids. Thus, the exchange energy E_{ex} can be described as

$$E_{ex} = E_{ex(rings)} + E_{ex(chains)} \ . \tag{S1}$$

In the following, we shall deduce the mathematical form of the exchange energy E_{ex} in detail.

The exchange energy E_{ex} stems from the magnetic interaction between magnetic moments. The energy between two magnetic moments is

$$E_{ij} = \frac{1}{r_{ij}^{\ 3}}[(\boldsymbol{\mu}_i\boldsymbol{\mu}_j) - \frac{3}{r_{ij}^{\ 2}}(\boldsymbol{\mu}_i\boldsymbol{r}_{ij})(\boldsymbol{\mu}_j\boldsymbol{r}_{ij})] , \tag{S2}$$

where $\boldsymbol{\mu}_i$ and $\boldsymbol{\mu}_j$ are two dipoles of magnetic moments and $\boldsymbol{r}_{ij}$ is vector between $\boldsymbol{\mu}_i$ and $\boldsymbol{\mu}_j$. Thus, both $E_{ex(rings)}$ and $E_{ex(chains)}$ can be deduced from the Eq.(S2).

First, starting from the Eq.(S2), the magnetic interaction energy within each ring $E_{ex(rings)}$ can be calculated as follows:

$$\begin{aligned} E_{ex(rings)} &= \frac{N_r}{2}\sum_{j=0}^{N_e-1}\sum_{i=1}^{N_e-1} E_{ij} \\ &= \frac{N_r}{2}\sum_{j=0}^{N_e-1}\sum_{i=1}^{N_e-1} \frac{1}{r_{ij}^{\ 3}}[(\boldsymbol{\mu}_i\boldsymbol{\mu}_j) - \frac{3}{r_{ij}^{\ 2}}(\boldsymbol{\mu}_i\boldsymbol{r}_{ij})(\boldsymbol{\mu}_j\boldsymbol{r}_{ij})] , \\ &= \frac{N_r}{2}\sum_{j=0}^{N_e-1}\sum_{i=1}^{N_e-1} \frac{1}{r_{ij}^{\ 3}}[\mu_e^{\ 2} - \frac{3}{r_{ij}^{\ 2}}(\boldsymbol{\mu}_e\boldsymbol{r}_{ij})^2] \end{aligned} \tag{S3}$$

where each ellipsoid is treated as a dipole of magnetic moment $\boldsymbol{\mu}_e$ and the N_r and N_e represent the number of rings in a nanotube and number of ellipsoids in a ring, respectively. As shown in the Eq.(S3), there are two items of $\boldsymbol{r}_{ij}$ and $\boldsymbol{\mu}_e\boldsymbol{r}_{ij}$ to be solved before the final mathematical form of $E_{ex(rings)}$ is obtained. To solve $\boldsymbol{r}_{ij}$ and $\boldsymbol{\mu}_e\boldsymbol{r}_{ij}$, we schematically illustrate the relation between the $\boldsymbol{\mu}_{e(i)}$ and $\boldsymbol{\mu}_{e(j)}$ in a ring in the Fig. S1 according to the coordinate described in Fig. 1(c).

(1) Calculation of $\boldsymbol{r}_{ij}$ item. As shown in Fig. S1, the azimuthal angle of $\boldsymbol{\mu}_{e(i)}$ and $\boldsymbol{\mu}_{e(j)}$ in ring plane are φ_i and φ_j, and there are i-1 ellipsoids between $\boldsymbol{\mu}_{e(i)}$ and $\boldsymbol{\mu}_{e(j)}$. Thus, the distance r_{ij} between $\boldsymbol{\mu}_{e(i)}$ and $\boldsymbol{\mu}_{e(j)}$ can be calculated as

$$r_{ij}{=}2(R-r_e)^2\text{-}2(R-r_e)^2\cos(\varphi_i\text{-}\varphi_j){=}\{2(R-r_e)^2[1-\cos(2\pi i/N_e)]\}^{\frac{1}{2}} . \qquad \text{(S4)}$$

(2) Calculation of $\boldsymbol{\mu}_e\boldsymbol{r}_{ij}$ item. According to the mathematical definition, $\boldsymbol{\mu}_e\boldsymbol{r}_{ij}{=}\mu_e r_{ij}\cos(<\boldsymbol{\mu}_e,\boldsymbol{r}_{ij}>)$. As shown in Fig. S1, the projection of all magnetic moments in the ring plane (the dotted arrows in the Fig. S1) is parallel to the X-Z plane (the dotted line in the Fig. S1); thus, the angle between projection of $\boldsymbol{\mu}_{e(i)}$ and $\boldsymbol{r}_{ij}$ becomes $\frac{\pi}{2}+\left(\frac{\varphi_i+\varphi_j}{2}\right)=\frac{\pi}{2}+\varphi_j+\frac{\pi}{N_e}i$. At the same time, the external field $\boldsymbol{H}$ respectively makes the angles $(\alpha+\theta_0)/(-\alpha+\theta_0)$ and θ_0 with respect to the magnetic moment $\boldsymbol{\mu}_e$ and tube axis direction according to the coordinate described in Fig.1(c), and we correspondingly show the relation among $(\alpha+\theta_0)/(-\alpha+\theta_0)$, θ_0 and $\boldsymbol{H}$ in the Figure S2. Thus, the angle between the magnetic moment $\boldsymbol{\mu}_e$ and the ring plane becomes $\frac{\pi}{2}-\alpha$ and the cosine of the angle of $\boldsymbol{\mu}_{e(i)}$ to $\boldsymbol{r}_{ij}$ is calculated as $\cos(<\mu_e,r_{ij}>)=\cos\left[\frac{\pi}{2}-\alpha\right]\cos\left[\frac{\pi}{2}+\varphi_j+\frac{\pi}{N_e}i\right]=-\sin\alpha\sin(\varphi_j+\frac{\pi}{N_e}i)$.

Therefore, the $\boldsymbol{\mu}_e\boldsymbol{r}_{ij}$ can be written as:

$$\boldsymbol{\mu}_e\boldsymbol{r}_{ij}{=}\mu_e r_{ij}\cos(<\boldsymbol{\mu}_e,\boldsymbol{r}_{ij}>){=}{-}\mu_e r_{ij}\sin\alpha\sin(\varphi_j+\frac{\pi}{N_e}i) . \qquad \text{(S5)}$$

Substituting Eq.(S4) and Eq.(S5) for Eq.(S3), the magnetic interaction energy within each ring $E_{ex(rings)}$ can be written as

$$E_{ex(rings)} = \frac{N_r}{2}\sum_{j=0}^{N_e-1}\sum_{i=1}^{N_e-1}\frac{1}{r_{ij}^{3}}[\mu_e^{2} - \frac{3}{r_{ij}^{2}}(\boldsymbol{\mu_e r_{ij}})^2]$$

$$= \frac{N_r\mu_e^{2}}{2}\sum_{j=0}^{N_e-1}\sum_{i=1}^{N_e-1}\frac{1-3\left(1-\cos^2\alpha\right)\sin^2(\varphi_j+\frac{\pi i}{N_e})}{\{2(R-r_e)^2[1-\cos(2\pi i/N_e)]\}^{3/2}} \quad \text{(S6)}$$

Second, the magnetic interaction energy within a chain $E_{ex(chains)}$ can also be calculated from the Eq.(S2). Considering the exchange energy $E_{ex(chain)}$ in a single chain, it is denoted as the sum of all exchange energy between the magnetic moments in a chain, namely, from the nearest-neighbor exchange energy to the furthest-neighbor exchange energy. Here we set the number of rings is even. Thus, the total energy in a single chain can be calculated as:

$$E_{ex(chain)} = \sum E_{ex(nearest)} + \sum E_{ex(next-nearest)} + \ldots E_{ex(furthest)}$$

$$= \sum_{neastest}\frac{1}{r_{ij}^{3}}[\mu_e^{2} - \frac{3}{r_{ij}^{2}}(\boldsymbol{\mu_e r_{ij}})^2] + \sum_{next-neastest}\frac{1}{r_{ij}^{3}}[\mu_e^{2} - \frac{3}{r_{ij}^{2}}(\boldsymbol{\mu_e r_{ij}})^2] + \ldots\frac{1}{r_{ij}^{3}}[\mu_e^{2} - \frac{3}{r_{ij}^{2}}(\boldsymbol{\mu_e r_{ij}})^2]$$

$$= \frac{(N_r-1)\mu_e^{2}}{(1\times 2kr_e)^3}\left(\cos 2\alpha - 3\cos^2\alpha\right) + \frac{(N_r-2)\mu_e^{2}}{(2\times 2kr_e)^3}\left(1-3\cos^2\alpha\right) \quad \text{, (S7)}$$

$$+\ldots\frac{\mu_e^{2}}{[(N_r-1)\times 2kr_e]^3}\left(\cos 2\alpha - 3\cos^2\alpha\right)$$

$$= \frac{N_r K_{N_r}\mu_e^{2}}{(2kr_e)^3}\left[L_{N_r}\left(\cos 2\alpha - 3\cos^2\alpha\right) + M_{N_r}\left(1-3\cos^2\alpha\right)\right]$$

where

$$L_{N_r} = \sum_{i=1}^{\frac{1}{2}(N_r-1)<i\le\frac{1}{2}(N_r+1)}\frac{N_r - (2i-1)}{N_r(2i-1)^3},$$

$$M_{N_r} = \sum_{i=1}^{\frac{1}{2}(N_r-2)<i\le\frac{1}{2}N_r}\frac{N_r - 2i}{N_r(2i)^3},$$

$$K_{N_r} = M_{N_r} + L_{N_r} = \sum_{i=1}^{N_r}\frac{N_r - i}{N_r i^3}$$

Finally, by substituting Eq.(S6) and Eq.(S7) into Eq.(S1), the total exchange energy E_{ex} can be described in the following mathematical form:

$$
\begin{aligned}
E_{ex} = & \frac{N_r \mu_e^2}{2} \sum_{j=0}^{N_e-1} \sum_{i=1}^{N_e-1} \frac{1-3\left(1-\cos^2\alpha\right)\sin^2\left(\phi_j + \frac{\pi i}{N_e}\right)}{\left\{2\left(R-r_e\right)^2\left[1-\cos\left(2\pi i / N_e\right)\right]\right\}^{3/2}} \\
& + \frac{N_r N_e K_{N_r} \mu_e^2}{\left(2kr_e\right)^3} \left[\begin{array}{l} L_{N_r}\left(\cos 2\alpha - 3\cos^2\alpha\right) \\ + M_{N_r}\left(1-3\cos^2\alpha\right) \end{array} \right]
\end{aligned} . \qquad \text{(S8)}
$$

Figure. S1

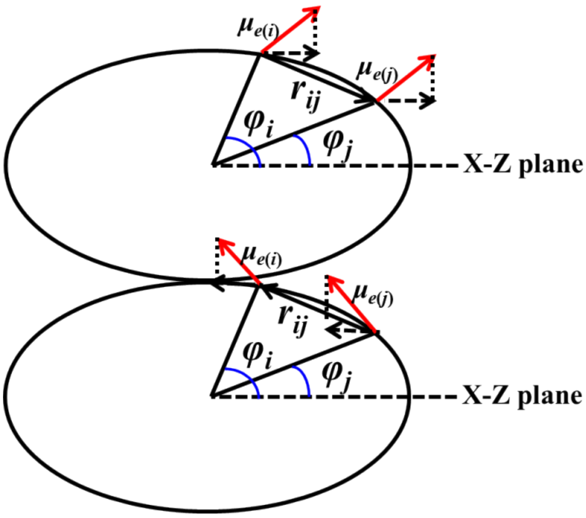

FIG. S1 Schematic diagram of geometric relation between $\boldsymbol{\mu}_{e(i)}$ and $\boldsymbol{\mu}_{e(j)}$.

Figure. S2

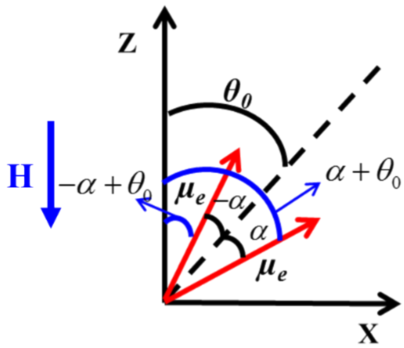

FIG. S2 Angles between magnetic moment $\boldsymbol{\mu}_e$, axis direction of nanotube, and external field ***H***.

2. Detailed calculation of coercivity in the fanning rotation and coherent rotation

According to the Eq. (11) in our previous work (see the ref. [24] of this article) and the Eq. (9) in this article, we can easily obtain the coercivity $\boldsymbol{H_c}$ of coherent rotation and fanning rotation. The equations are as follows:

For the coherent rotation,

$$H_c = \begin{cases} -\dfrac{2a}{b}(cos^{\frac{2}{3}}\theta_0 + sin^{\frac{2}{3}}\theta_0)^{-\frac{3}{2}}, & 0^{\circ} < \theta_0 \le 45^{\circ} \\ -\dfrac{asin2\theta_0}{b}, & 45^{\circ} < \theta_0 \le 90^{\circ} \end{cases}, \quad \text{(S9)}$$

where

$$a = \frac{N_r\mu_e{}^2}{2}\sum_{j=0}^{N_e-1}\sum_{i=1}^{N_e-1}\frac{3sin^2(\varphi_j + \dfrac{\pi i}{N_e})}{\{2(R-r_e)^2[1-cos(2\pi i/N_e)]\}^{3/2}} - 3\frac{N_rN_eK_{N_r}\mu_e{}^2}{(2kr_e)^3}$$
$$-\frac{1}{2}N_rN_eI_s\mu_e(N_t - N_o),$$

$$b = N_rN_e\mu_e.$$

For the fanning rotation,

$$H_c = \begin{cases} -\dfrac{2d}{e}(\cos^{\frac{2}{3}}\theta_0 + \sin^{\frac{2}{3}}\theta_0)^{-\frac{3}{2}}, (0^{\circ} < \theta_0 \le 45^{\circ}) \\ -\dfrac{d}{e}\sin 2\theta_0, (45^{\circ} < \theta_0 \le 90^{\circ}) \end{cases}, \quad \text{(S10)}$$

where,

$$d = \frac{N_r\mu_e^2}{2}\sum_{j=0}^{N_e-1}\sum_{i=1}^{N_e-1}\frac{3sin^2\left(\varphi_j + \dfrac{\pi i}{N_e}\right)}{\left\{2\left(R-r_e\right)^2\left[1-\cos\left(2\pi i/N_e\right)\right]\right\}^{3/2}}$$
$$-\frac{N_rN_eK_{N_r}\mu_e^2L_{N_r}}{\left(2kr_e\right)^3} - \frac{3N_rN_eK_{N_r}\mu_e^2M_{N_r}}{\left(2kr_e\right)^3}$$
$$+\frac{1}{2}N_rN_eI_s\mu_e\left(N_t - N_o\right);$$

$$e = \frac{N_r}{2}N_e\mu_e.$$

It is clearly seen that there are four coefficients of a, b, d and e in the Eq. (S9) and Eq. (S10); all these coefficients are function of the geometric parameters of nanotubes, including axis ratio k, the length of nanotube N_r, the number of ellipsoid in a rings N_e and the thickness of nanotube, $2r_e$. To simply calculate the coercivity, we set N_e is equal to 50 and then study the influence of the geometric parameters of nanotube on the magnetic properties, especially for the coercivity $\boldsymbol{H_c}$. All the calculation results are demonstrated in the Fig. 3, Fig. 4 and Fig.5. In addition, in order to compare the coercivity of fanning rotation with that of coherent rotation, we set the same values of k, N_r, and $2r_e$ to calculate magnetic properties, as shown in the Fig. 5.