

Figure S1. Plots of the ensemble average  $D_p$  versus  $q$  for the four concentrations of  $\text{SrI}_2$  in  $\text{D}_2\text{O}$ . These are the same values as shown in Figure 3, but with the abscissa as  $q = 2\pi/d$ . Error bars ( $\pm\sigma$ ) are those of the statistics only.

## The CQENS equations

The incoming neutron beam can be modeled as a plane wave, and the scattered neutron's field as spherical waves originating from the scattering nuclei. The magnitude of the scattered waves have amplitudes given by the scattering length,  $b_i$ , of each scattering atom. The value of  $b_i$  depends on the neutron wavelength, the isotope identity, and somewhat on the chemical environment of the scattering isotope nucleus. This treatment of neutron scattering can be used because the nuclear scattering process occurs over a very short range compared with the neutron wavelength. The short range can be modeled as a delta function in distance, so the nucleus-neutron interaction can be written as occurring through a Fermi pseudo-potential:

$$V(\mathbf{r}) = \frac{2\pi\hbar^2}{m_n} b_i \delta(\mathbf{r} - \mathbf{r}_i) \quad (\text{S1})$$

Here,  $m_n$  is the neutron mass and  $\mathbf{r}_i$  is the position of the nucleus  $i$ .

The properties of the coherent quasielastic scattering is found, first, by summing the time and distance dependence of the interference between all the sets of two scattering centers—identified generally as  $\alpha$  and  $\beta$  at positions  $\mathbf{R}_\alpha$  and  $\mathbf{R}_\beta$ —but not yet accounting for their scattering lengths. The letter  $N$  labels the total numbers of the specific isotopes. The equation for this sum is,

$$I^{\alpha\beta}(\mathbf{q}, t) = \frac{1}{\sqrt{N_\alpha N_\beta}} \sum_{i_\alpha=1}^{N_\alpha} \sum_{i_\beta=1}^{N_\beta} \left\langle \exp \left\{ i \left[ \mathbf{q} \cdot \mathbf{R}_{i_\alpha}(t) - \mathbf{q} \cdot \mathbf{R}_{i_\beta}(0) \right] \right\} \right\rangle \quad (\text{S2})$$

where the  $\mathbf{k}$ -values are the incoming/initial and scattered/final neutron wavevectors and  $\mathbf{q} = \mathbf{k}_i - \mathbf{k}_f$ .

The time-Fourier transform of Equation S1 yields the equation for the pairs that describes the  $q$  dependence of the experimentally measured scattering;  $S^{\alpha\beta}(\mathbf{q}, E)$  has the form

$$S^{\alpha\beta}(\mathbf{q}, E) = \frac{1}{2\pi\sqrt{N_\alpha N_\beta}} \sum_{i_\alpha=1}^{N_\alpha} \sum_{i_\beta=1}^{N_\beta} \int_{-\infty}^{\infty} \left\langle \exp \left\{ i \left[ \mathbf{q} \cdot \mathbf{R}_{i_\alpha}(t) - \mathbf{q} \cdot \mathbf{R}_{i_\beta}(0) \right] \right\} \right\rangle \exp(-iEt) dt \quad (\text{S3})$$

where  $E = E_i - E_f$ , the incoming/initial and scattered/final neutron kinetic energies.

The magnitude of each of the pairs' contributions to the scattering must be weighted by the coherent scattering lengths to reflect the experimentally obtained data. The function of Eq. S4 also can be normalized to the total number of atoms rather than to the number of pairs. The resulting function,  $S_{coh}^n(\mathbf{q}, E)$ , is called the coherent dynamic structure factor for neutron scattering, and is given by

$$S_{coh}^n(\mathbf{q}, E) = \frac{1}{N} \sum_{\alpha=1}^n \sum_{\beta=1}^n b_\alpha^{coh} b_\beta^{coh} \sqrt{N_\alpha N_\beta} S^{\alpha\beta}(\mathbf{q}, E) \quad (\text{S4})$$

The experimental measurement is comprised of the scattered neutron flux  $\sigma$  over some solid angle between  $\Omega$  and  $\Omega+d\Omega$  and over some energy range  $E$  and  $E+dE$ . This measurement described by Equation S5 is, then, related through the above functions to the coherent dynamics expressed in Equation S2 after the incoherent scattering has been subtracted.

$$\frac{\partial^2 \sigma_{coh}}{\partial \Omega \partial E} = \frac{k_f}{k_i} S_{coh}^n(\mathbf{q}, E) \quad (S5)$$

The factor  $k_f/k_i$  accounts for the flux of the neutrons, which is dependent on the velocity of the neutrons.

The relationship between the diffusion rate and the scattering is found by realizing that the distribution of the relative positions of pairs of particles changing in time by translational motion in three dimensions  $G(r, t)$  is given by

$$G(r, t) = (4\pi D_p t)^{-3/2} \exp\left(\frac{-r^2}{4D_p t}\right) \quad (S6)$$

The double Fourier transform can be performed in closed form to give

$$S(q, E) = \frac{1}{\pi} \frac{D_p q^2}{(D_p q^2)^2 + E^2} \quad (S7)$$

which is a Lorentzian function that has a full width at half maximum of  $2D_p q^2$ , and the left hand side of this equation is the experimental function from Equation S5.

### Abbreviations

$k_f$	Incoming neutron wavevector
$k_i$	Scattered neutron wavevector
$\alpha, \beta$	Isotope type labels
$N$	Total number of specific isotope
$S_{coh}^{\alpha\beta}(\mathbf{q}, E)$	Coherent dynamic neutron structure factor
$b_{\alpha}^{coh}$	Coherent scattering length
$\sigma$	Flux of scattered neutrons
$q$	Momentum transfer
$E$	Energy

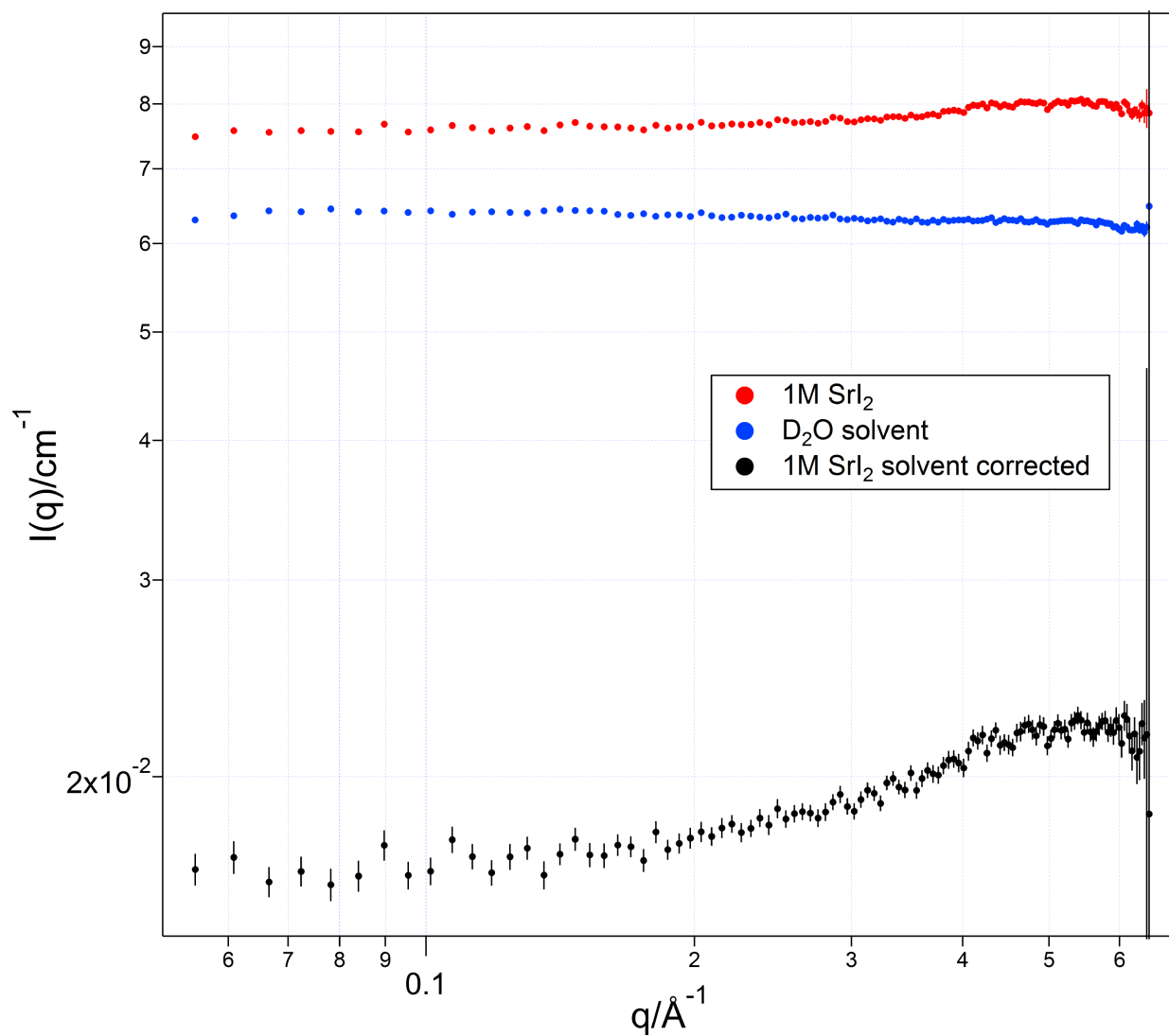


Figure S2. Example of the SANS data solvent subtraction. Here, the volume fraction of the  $\text{D}_2\text{O}$  signal was subtracted, *i.e.*, the black curve is results from

$$1 \text{ M SrI}_2 (\text{Red}) - 0.925 \times \text{D}_2\text{O} (\text{Blue})$$

The final S/N of the difference curve indicates the quality of the data for curve fitting.