Electronic Supplementary Material (ESI) for Physical Chemistry Chemical Physics.
This journal is © the Owner Societies 2015

Exceptional $\mathrm{H}_{2}$ Sorption Characteristics in a $\mathrm{Mg}^{2+}$-Based Metal-Organic Framework with Small Pores: Insights From Experimental and Theoretical Studies

Tony Pham, ${ }^{\dagger}$ Katherine A. Forrest, ${ }^{\dagger}$ Eduardo H. L. Falcão, ${ }^{\ddagger}$ Juergen Eckert,,${ }^{*} \dagger$ and Brian Space ${ }^{*}, \dagger$
${ }^{\dagger}$ Department of Chemistry, University of South Florida,
4202 East Fowler Avenue, CHE205, Tampa, FL 33620-5250, United States
${ }^{\ddagger}$ Departamento de Química Fundamental, Universidade Federal de Pernambuco,
Av. Jorn. Anibal Fernandes, $s / n$, Cidade Universitária,
Recife, PE, 50740-560, Brazil
*juergen@usf.edu; brian.b.space@gmail.com

MOF System Cell


Figure S1. (a) The orthographic $a$-axis and (b) $b$-axis views of the $2 \times 2 \times 2$ system cell of $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$. $\mathrm{The}^{2} \mathrm{Mg}^{2+}$ ions are color-coded to highlight the four chemically distinct $\mathrm{Mg}^{2+}$ ions. Atoms colors: $\mathrm{C}=$ cyan, $\mathrm{H}=$ white, $\mathrm{O}=\mathrm{red}, \mathrm{Mg} 1=$ magenta, Mg 2 $=$ lime green, $\mathrm{Mg} 3=$ violet, $\mathrm{Mg} 4=$ blue.


Figure S2. (a) The orthographic $a$-axis and (b) $b$-axis views of the $2 \times 2 \times 2$ system cell of $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$. The formate ligands (L) are color-coded to highlight the six chemically distinct linkers. Atoms colors: $\mathrm{Mg}=$ gray, $\mathrm{L} 1=$ red $, \mathrm{L} 2=\mathrm{blue}, \mathrm{L} 3=$ green, $\mathrm{L} 4=$ yellow, $\mathrm{L} 5=$ orange, $\mathrm{L} 6=$ cyan.

## MOF Partial Charges



Figure S3. The numbering of the chemically distinct atoms in $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$ as referred to in Tables S 1 and S2: (a) view showing the Mg 1 ion; (b) view showing the Mg 2 and Mg 3 ions; (c) view showing the Mg 4 ion. Atom colors: $\mathrm{C}=$ cyan, $\mathrm{H}=$ white, $\mathrm{O}=$ red, $\mathrm{Mg}=$ gray.

Table S1. The calculated partial charges $\left(e^{-}\right)$for the chemically distinct atoms in $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$. Label of atoms correspond to Figure S3. Component types correspond to Figure S1 for the $\mathrm{Mg}^{2+}$ ions and Figure S2 for the formate ligands.

| Atom | Label | Component | $q\left(e^{-}\right)$ |
| :---: | :---: | :---: | :---: |
| Mg | 1 | Mg 1 | 1.83260 |
| Mg | 2 | Mg 2 | 1.78830 |
| Mg | 3 | Mg 3 | 1.72240 |
| Mg | 4 | Mg 4 | 1.86400 |
| O | 5 | L 1 | -0.97390 |
| O | 6 | L 1 | -0.92820 |
| O | 7 | L 2 | -0.98040 |
| O | 8 | L 2 | -0.69340 |
| O | 9 | L 3 | -0.82760 |
| O | 10 | L 3 | -0.91300 |
| O | 11 | L 4 | -0.93010 |
| O | 12 | L 4 | -0.84180 |
| O | 13 | L 5 | -1.00110 |
| O | 14 | L 5 | -0.98990 |
| O | 15 | L 6 | -0.79080 |
| O | 16 | L 6 | -0.73780 |
| C | 17 | L 1 | 0.82200 |
| C | 18 | L 2 | 0.96540 |
| C | 19 | L 3 | 0.85510 |
| C | 20 | L 4 | 0.92050 |
| C | 21 | L 5 | 0.75950 |
| C | 22 | L6 | 0.94070 |
| H | 23 | L1 | 0.02510 |
| H | 24 | L2 | -0.06720 |
| H | 25 | L3 | 0.15290 |
| H | 26 | L4 | -0.02510 |
| H | 27 | L5 | 0.08220 |
| H | 28 | L6 | -0.18210 |

Table S2. The crystallographic distances (in $\AA$ ) between various atoms in $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$. Label of atoms correspond to Figure S3.

| Atom Pair | Distance $(\AA)$ |
| :---: | :---: |
| $1-5$ | 2.06287 |
| $1-7$ | 2.03841 |
| $1-9$ | 2.10633 |
| $2-6$ | 2.09236 |
| $2-8$ | 2.06497 |
| $2-9$ | 2.07079 |
| $2-12$ | 2.10005 |
| $2-14$ | 2.10018 |
| $2-16$ | 2.08418 |
| $3-6$ | 2.11379 |
| $3-10$ | 2.01986 |
| $3-11$ | 2.09291 |
| $3-12$ | 2.03424 |
| $3-14$ | 2.09006 |
| $3-16$ | 2.05853 |
| $4-8$ | 2.11284 |
| $4-13$ | 2.05076 |
| $4-15$ | 2.06418 |

## Simulated $\mathbf{H}_{2}$ Sorption Results



Figure S4. (a) Absolute $\mathrm{H}_{2}$ sorption isotherms at 77 K (solid) and 87 K (dashed) and (b) isosteric heats of adsorption $\left(Q_{s t}\right)$ for $\mathrm{H}_{2}$ in $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$ for experiment (black) and simulation (red). The simulations were performed using the nonpolarizable potential by Belof et al. ${ }^{1}$


Figure S5. Molecular illustration of the sorbed $\mathrm{H}_{2}$ molecules within the pore of $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$ as determined from simulation: (a) $b$-axis view and (b) $a$-axis view. The $\mathrm{H}_{2}$ molecules and $\mathrm{Mg}^{2+}$ ions are color-coded to highlight the different $\mathrm{H}_{2}$ sorption sites and chemically distinct $\mathrm{Mg}^{2+}$ ions, respectively. $\mathrm{H}_{2}$ molecule colors: site $1 \mathrm{~A}=$ green, site $1 \mathrm{~B}=$ yellow, site $2 \mathrm{~A}=$ orange, site $2 \mathrm{~B}=$ gray. Atoms colors: $\mathrm{C}=$ cyan, $\mathrm{H}=$ white, $\mathrm{O}=$ red, $\mathrm{Mg} 1=$ magenta, $\mathrm{Mg} 2=$ lime green, $\mathrm{Mg} 3=$ violet, $\mathrm{Mg} 4=\mathrm{blue}$. Note, sites 1 B and 2B are reflected on the opposite side of the pore from sites 1 A and 2 A , respectively, for illustrative purposes.


Figure S6. Molecular illustration of the sorbed $\mathrm{H}_{2}$ molecule within the pore of $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$ as determined from simulation. The $\mathrm{H}_{2}$ molecule (orange) shown is sorbed at site 1 C . The $\mathrm{Mg}^{2+}$ ions are color-coded to highlight the four chemically distinct $\mathrm{Mg}^{2+}$ ions. Atoms colors: $\mathrm{C}=$ cyan, $\mathrm{H}=$ white, $\mathrm{O}=\mathrm{red}, \mathrm{Mg} 1=$ magenta, $\mathrm{Mg} 2=$ lime green, $\mathrm{Mg} 3=$ violet, $\mathrm{Mg} 4=$ blue.


Figure S7. Molecular illustration of the sorbed $\mathrm{H}_{2}$ molecules within the pore of $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$ as determined from simulation. The $\mathrm{H}_{2}$ molecules (orange) shown are sorbed at: (a) site 3 A , (b) site 3 B , and (c) site 3 C . The $\mathrm{Mg}^{2+}$ ions are color-coded to highlight the four chemically distinct $\mathrm{Mg}^{2+}$ ions. Atoms colors: $\mathrm{C}=$ cyan, $\mathrm{H}=$ white, $\mathrm{O}=$ red, $\mathrm{Mg} 1=$ magenta, $\mathrm{Mg} 2=\operatorname{lime}$ green, $\mathrm{Mg} 3=$ violet, $\mathrm{Mg} 4=$ blue.


Figure S8. Molecular illustration of a $\mathrm{H}_{2}$ molecule (orange) sorbed at (a) site 1 A , (b) site 1 B , and (c) site 1 C in $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$ as determined from simulation showing the distances between the H atoms of the $\mathrm{H}_{2}$ molecule and the nearest O atoms of the framework.


Figure S9. Molecular illustration of a $\mathrm{H}_{2}$ molecule (orange) sorbed at (a) site 2A, (b) site 2 B in $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$ as determined from simulation showing the distances between the H atoms of the $\mathrm{H}_{2}$ molecule and the nearest O atoms of the framework.


Figure S10. Molecular illustration of a $\mathrm{H}_{2}$ molecule (orange) sorbed at (a) site 3 A , (b) site 3 B , and (c) site 3 C in $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$ as determined from simulation showing the distances between the H atoms of the $\mathrm{H}_{2}$ molecule and the nearest O atoms of the framework.

## Quantum Rotation Calculations

The two-dimensional quantum rotational levels for a $\mathrm{H}_{2}$ molecule sorbed about the eight considered sites in $\alpha-$ $\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$ were calculated by diagonalizing the rotor Hamiltonian in the spherical harmonic basis, $Y_{j m}$, which is the following:

$$
\begin{equation*}
\hat{H}=B \mathbf{j}^{2}+V(\theta, \phi) \tag{1}
\end{equation*}
$$

where $B$ is the rotational constant for molecular $\mathrm{H}_{2}(7.35 \mathrm{meV}),{ }^{2} \mathbf{j}^{2}$ is the angular momentum operator, and $V(\theta, \phi)$ is the potential energy surface for the rotation of the $\mathrm{H}_{2}$ molecule with its center-of-mass held fixed within the MOF- $\mathrm{H}_{2}$ system. Each matrix element, $\left\langle Y_{j m}\right| V(\theta, \phi)\left|Y_{j m}\right\rangle$, was constructed using Gauss-Legendre quadrature ${ }^{3}$ with a basis set consisting of $\pm m$ functions. ${ }^{4}$ The kinetic energy term, $j(j+1)$, was added to all of the diagonal elements. The matrix was diagonalized using the LAPACK linear algebra package, ${ }^{5}$ yielding the rotational eigenvalues and the eigenvector coefficients. All twodimensional rotational levels were calculated with $j=7$, leading to 64 basis functions. The calculated rotational levels for a $\mathrm{H}_{2}$ molecule sorbed about the eight considered sites in $\alpha-\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$ are provided in Table S3.

The rotational barrier for a $\mathrm{H}_{2}$ molecules sorbed about site 1 A was determined by calculating the potential energy of the MOF $-\mathrm{H}_{2}$ system as the $\mathrm{H}_{2}$ molecule was rotated at various angles of $\theta\left(0\right.$ to $\left.180^{\circ}\right)$ and $\phi\left(0\right.$ to $\left.360^{\circ}\right)$. Single point energies were obtained on a sphere ( 4,096 points based on $64 \times 64$ Gaussian quadrature integration) and the rotational barrier was estimated by taking the difference between the high and low values. All calculations were performed using the Massively Parallel Monte Carlo (MPMC) code, which is currently available for download on GitHub. ${ }^{6}$

Table S3. The calculated two-dimensional quantum rotational levels for a $\mathrm{H}_{2}$ molecule sorbed at the eight considered sites in $\alpha-$ $\left[\mathrm{Mg}_{3}\left(\mathrm{O}_{2} \mathrm{CH}\right)_{6}\right]$. Sites $1 \mathrm{~A}, 1 \mathrm{~B}, 2 \mathrm{~A}$, and 2 B are depicted in Figure S 5 , while site 1 C is depicted in Figure S 6 , and sites $3 \mathrm{~A}, 3 \mathrm{~B}$, and 3 C are depicted in Figure S7. Relative energies are given in meV.

| $n$ | $j$ | Site $1 \mathrm{~A} \Delta E(\mathrm{meV})$ | Site 1B $\Delta E(\mathrm{meV})$ | Site $1 \mathrm{C} \Delta E(\mathrm{meV})$ | Site 2A $\Delta E(\mathrm{meV})$ | Site 2B $\Delta E(\mathrm{meV})$ | Site 3A $\Delta E(\mathrm{meV})$ | Site 3B $\Delta E(\mathrm{meV})$ | Site 3C $\Delta E(\mathrm{meV})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 |  | 2.57 | 4.19 | 4.54 | 6.48 | 7.07 | 8.09 | 8.80 | 9.18 |
| 3 | 1 | 31.54 | 20.77 | 24.39 | 19.20 | 20.45 | 14.75 | 19.40 | 18.22 |
| 4 |  | 34.47 | 36.67 | 29.12 | 26.33 | 22.02 | 27.79 | 23.19 | 19.16 |
| 5 |  | 45.54 | 39.58 | 42.59 | 40.86 | 42.01 | 38.86 | 40.62 | 41.67 |
| 6 |  | 47.71 | 42.73 | 45.13 | 41.71 | 42.34 | 39.44 | 43.52 | 42.49 |
| 7 | 2 | 51.48 | 50.68 | 45.94 | 45.88 | 43.59 | 48.45 | 46.79 | 43.20 |
| 8 |  | 69.44 | 63.32 | 60.59 | 54.93 | 53.09 | 53.08 | 51.80 | 49.64 |
| 9 |  | 70.07 | 70.38 | 61.60 | 57.16 | 52.22 | 57.88 | 54.10 | 49.71 |

${ }^{1}$ J. L. Belof, A. C. Stern and B. Space, J. Chem. Theory Comput., 2008, 4, 1332-1337.
${ }^{2}$ J. Bigeleisen and M. G. Mayer, J. Chem. Phys., 1947, 15, 261-267.
${ }^{3}$ M. Abramowitz, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, Dover Publications, Inc., Mineola, NY, 1965.
${ }^{4}$ J. L. Belof, Ph.D. thesis, University of South Florida, 2009.
${ }^{5}$ E. Anderson, Z. Bai, J. Dongarra, A. Greenbaum, A. McKenney, J. Du Croz, S. Hammerling, J. Demmel, C. Bischof and D. Sorensen, Proceedings of the 1990 ACM/IEEE conference on Supercomputing, 1990, pp. 2-11.
${ }^{6}$ J. L. Belof and B. Space, Massively Parallel Monte Carlo (MPMC), Available on GitHub, 2012, https://github.com/mpmccode/mpmc.

