Appendix 4. Deriving the $\sqrt{N_s+N_b}$ rule from the $\sqrt{2N_b}$ rule:

We assume that we know N_b as an estimate of the mean background activity; the critical level is given as follows:

$$L_c = (N_s - N_b)_c = k_\alpha\sqrt{2N_b}$$

Consequently, a gross signal activity amounting to $N_b + k_\alpha\sqrt{2N_b}$ is not yet detected (although any gross signal activity larger then this is detected). Now, we have two background estimates: N_b and $N_b + k_\alpha\sqrt{2N_b}$.

Accordingly, the mean background value is estimated as $\dfrac{N_b + N_b + k_\alpha\sqrt{2N_b}}{2} = N_b + \dfrac{k_\alpha}{2}\sqrt{2N_b}$

For the critical level, we now obtain: $L_c = (N_s - N_b)_c = k_\alpha\sqrt{2\left(N_b + \dfrac{k_\alpha}{2}\sqrt{2N_b}\right)} = k_\alpha\sqrt{2N_b + k_\alpha\sqrt{2N_b}}$

Since now, the problem allows for a convergent iterative solution. We have the following background estimates: N_b and $N_b + k_\alpha\sqrt{2N_b + k_\alpha\sqrt{2N_b}}$. The mean background value is estimated as

$$\frac{N_b + N_b + k_\alpha\sqrt{2N_b + k_\alpha\sqrt{2N_b}}}{2} = N_b + \frac{k_\alpha}{2}\sqrt{2N_b + k_\alpha\sqrt{2N_b}}$$

For the critical value, we obtain:

$$L_c = (N_s - N_b)_c = k_\alpha\sqrt{2\left(N_b + \frac{k_\alpha}{2}\sqrt{2N_b + k_\alpha\sqrt{2N_b}}\right)} = k_\alpha\sqrt{2N_b + k_\alpha\sqrt{2N_b + k_\alpha\sqrt{2N_b}}}$$

etc.

In the end, we have to calculate the critical level from a infinite radical:

$$L_c = k_\alpha\sqrt{2N_b + k_\alpha\sqrt{2N_b + k_\alpha\sqrt{2N_b + k_\alpha\sqrt{2N_b + k_\alpha\ldots}}}}$$

Finding the limit of this radical is a simple arithmetic exercise:

$$L_c^2 = k_\alpha^2\left[2N_b + k_\alpha\sqrt{2N_b + k_\alpha\sqrt{2N_b + k_\alpha\sqrt{2N_b + k_\alpha\ldots}}}\right] = 2k_\alpha^2 N_b + k_\alpha^2 L_c$$

$$L_c^2 - k_\alpha^2 L_c - 2k_\alpha^2 N_b = 0 \;\Rightarrow\; L_c = \frac{k_\alpha^2 + \sqrt{k_\alpha^4 + 8k_\alpha^2 N_b}}{2} = \frac{k_\alpha^2}{2} + k_\alpha\sqrt{\frac{k_\alpha^2}{4} + 2N_b}$$