Appendix 3. A proof of the unity standard deviation for the $\dfrac{N_s - N_b}{\sqrt{2N_b}}$ statistic under the null hypothesis $\overline{N_s} = \overline{N_b}$ ($\overline{N_s - N_b} = 0$)

Considering the above statistic as a function of two variables, $N_s - N_b$ and N_b, and propagating its uncertainty in the vicinity of point $(\overline{N_s - N_b}, \overline{N_b})$, we obtain:

$$Var\left(\frac{N_s - N_b}{\sqrt{2N_b}}\right)\Bigg|_{\substack{\overline{N_s - N_b}\\ \overline{N_b}}} = \frac{1}{2N_b}Var(N_s - N_b) + \frac{(N_s - N_b)^2}{4N_b^2}Var(\sqrt{2N_b}) - 2\frac{N_s - N_b}{(2N_b)^{3/2}}Cov(N_s - N_b, \sqrt{2N_b})\Bigg|_{\substack{\overline{N_s - N_b}\\ \overline{N_b}}}$$

At $\overline{N_s} = \overline{N_b}$, or $\overline{N_s - N_b} = 0$, the last two terms in the right-hand part of this equation can be omitted. Hence,

$$Var\left(\frac{N_s - N_b}{\sqrt{2N_b}}\right)\Bigg|_{\substack{\overline{N_s - N_b}\\ \overline{N_b}}} = \frac{1}{2N_b}Var(N_s - N_b)\Bigg|_{\substack{\overline{N_s - N_b}\\ \overline{N_b}}} = \frac{\overline{N_s + N_b}}{2\overline{N_b}} = 1\ \Big|_{\overline{N_s} = \overline{N_b}}$$