Frequency modulated microrheology - Supporting Information

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Collagen rheology

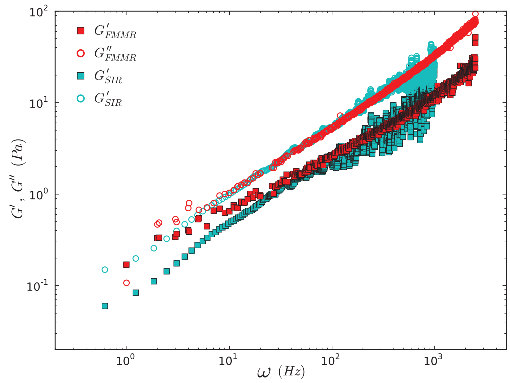

Figure 1: Recombinant collagen complex modulus measured by FMMR and SIR. Measurements were performed 45 hours after incubating with a bifunctional cross-linker.

FMMR measurement range

The FM input can be decomposed into a linear superposition of sinusoids, each with amplitude A. Neglecting inertial effects, the response of a particle trapped in a viscoelastic medium to each component of the driving force has the form

$$w(t) = D(\omega)\cos(\omega t + \delta(\omega)) \tag{1}$$

The amplitude and phase-lag of the time-dependent displacement from the optical trap are

$$D(\omega) = A\left(1 - \frac{\xi}{\sqrt{(\xi + {G'}/{3})^2 + G''^2}}\right) \tag{2}$$

and

$$\tan(\delta) = \frac{G''}{\xi + {G'}/{3}} \tag{3}$$

G' and G'' are the storage and loss moduli of the medium, respectively. The optical trap's characteristic elasticity, $\xi = \frac{\kappa}{6\pi a}$ is a function of trap stiffness, κ, and probe radius, a.

The work required to drive the probe through the fluid is balanced by the optical potential created by the particle's displacement from the trap. The trapping force is conservative, yielding a potential with the form $\frac{\kappa\langle r'^2\rangle}{2}$, where the effective mean-square displacement (msd) $\langle r'^2\rangle$, contains contributions from both the deterministic sinusoidal driving force ($\frac{D^2}{2}$) and the stochastic thermal force ($\langle r^2\rangle$).

$$\frac{\kappa}{2}\left(\frac{D^2}{2} + \langle r^2\rangle\right) = Ga(A - D)^2 \tag{4}$$

The smallest detectable displacement is set by the detector resolution, s. The minimum measurable shear modulus is therefore

$$G = \frac{1}{2(A-s)^2}\left(3\pi\xi s^2 + \frac{k_B T}{a}\right) \tag{5}$$

Alternatively, the largest modulus ($D \gg \langle r^2 \rangle^{\frac{1}{2}}$) that can be measured is

$$G = \frac{3\pi\xi}{2}\left(\frac{A}{s} - 1\right)^2 \tag{6}$$