# Supplemental Information for

# Protein-protein interactions generate hidden feedback and feed-forward loops to trigger bistable switches, oscillations and biphasic dose-responses

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\* *Corresponding authors:* <u>lan.nguyen@ucd.ie</u> (LKN), <u>boris.kholodenko@ucd.ie</u> (BK), In this supplementary information (SI), we present in detail the reactions, parameter values, and ordinary differential equations of the various models presented and discussed in the main text. In all the following tables, the concentrations and the Michaelis-Menten constants ( $K_ms$ ) are given in nM. First- and second-order rate constants are expressed in s<sup>-1</sup> and nM<sup>-1</sup> s<sup>-1</sup>. Maximum rates Vs of the enzymes catalysed reactions are expressed in nM s<sup>-1</sup>. In addition, the supplementary figures are shown in section 4.

S1. Single isolated PPI event



Table S1. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter
number			values
1	$A + B \leftarrow \rightarrow AB$	$v_1 = k_{1f} \cdot [A][B] - k_{1r} \cdot [AB]$	$k_{1f} = 0.00001,$
			$k_{1r} = 0.0005$

 Table S1. Ordinary differential equations of the kinetic model. The reaction rates are given in Table S1.

Left-hand	<b>Right-hand Sides</b>	Initial
Sides		Concentrations
		( <b>nM</b> )
d[A]/dt	$-v_1$	150
d[B]/dt	$-v_1$	0
d[AB]/dt	<i>v</i> <sub>1</sub>	150

S2. Single PPI network exhibiting bistability



Table S2. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter
number			values
1	$A + B \leftarrow \rightarrow AB$	$v_1 = k_{1f} \cdot [A][B] - k_{1r} \cdot [AB]$	$k_{1f} = 0.444116,$
			$k_{1r} = 0.0138931$
2	$B \rightarrow B^*$	$n_{12} = \frac{kc_2 \cdot [A] \cdot [B]}{kc_2 \cdot [A] \cdot [B]}$	$kc_2 = 2.17889,$
		$v_2 = Km_2 + [B]$	$Km_2 = 48.8039$
3	$B^* \rightarrow B$	$V_{3}.[B *]$	$V_3 = 0.0149644,$
		$v_3 - Km_3 + [B*]$	Km <sub>3</sub> = 3.09516

**Table S2. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S2.

Left-hand	<b>Right-hand Sides</b>	Initial Concentrations
Sides		( <b>nM</b> )
d[A]/dt	$-v_1$	15
d[B]/dt	$v_3 - v_1 - v_2$	100
d[AB]/dt	<i>v</i> <sub>1</sub>	0
d[B*]/dt	$v_2 - v_3$	0

S3. Single PPI network exhibiting Sustained Oscillation



Table S3. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter
number			values
1	$A + B \leftarrow \rightarrow AB$	$v_1 = k_{1f} \cdot [A][B] - k_{1r} \cdot [AB]$	$k_{1f} = 0.00143087,$
			$k_{1r} = 0.000653442$
2	$C^* \rightarrow C$	$w_{2} = \frac{kc_{2} \cdot [A] \cdot [C *]}{kc_{2} \cdot [A] \cdot [C *]}$	$kc_2 = 24.0512,$
		$V_2 = Km_2 + [C *]$	Km <sub>2</sub> = 151.308
3	$C \rightarrow C^*$	$v_{2} = \frac{V_{3}.[C]}{V_{3}.[C]}$	V <sub>3</sub> = 2.69421,
		$V_3 = Km_3 + [C]$	Km <sub>3</sub> = 1.93629
4	$B^* \rightarrow B$	$w_{4} = \frac{kc_{4} \cdot [C] \cdot [B *]}{kc_{4} \cdot [C] \cdot [B *]}$	$kc_4 = 0.723006,$
		$V_4 = Km_4 + [B *]$	$Km_4 = 48.6098$
5	$B \rightarrow B^*$	$v_{-} = \frac{V_{5} \cdot [B]}{V_{5} \cdot [B]}$	$V_5 = 11.8224,$
		$v_5 - Km_5 + [B]$	$Km_5 = 2.2509$

**Table S3. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S3.

Left-hand	<b>Right-hand Sides</b>	Initial Concentrations (nM)
Sides		
d[A]/dt	$-v_1$	39
d[B]/dt	$v_4 - v_1 - v_5$	455
d[AB]/dt	<i>v</i> <sub>1</sub>	0
d[B*]/dt	$v_5 - v_4$	0
d[C]/dt	$v_2 - v_3$	122

d[C*]/dt	<i>v</i> <sub>3</sub> - <i>v</i> <sub>2</sub>	0
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S4. Single PPI network exhibiting Damped Oscillation



Table S4. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter
number			values
1	$A + B \leftarrow \rightarrow AB$	$v_1 = k_{1f} \cdot [A][B] - k_{1r} \cdot [AB]$	k <sub>1f</sub> =0.000021458,
			$k_{1r} = 0.000227335$
2	$B^* \rightarrow B$	$n_{12} = \frac{kc_2 \cdot [A] \cdot [B *]}{kc_2 \cdot [A] \cdot [B *]}$	kc <sub>2</sub> =6.63297,
		$V_2 = Km_2 + [B *]$	$Km_2 = 0.0555913$
3	$B \rightarrow B^*$	$V_{3}.[B]$	V <sub>3</sub> =13.9797,
		$\nu_3 = \frac{1}{Km_3 + [B]}$	Km <sub>3</sub> = 0.0102194

**Table S4. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S4.

Left-hand	Right-hand Sides	Initial Concentrations
Sides		( <b>nM</b> )
d[A]/dt	$-v_1$	25
d[B]/dt	$v_2 - v_1 - v_3$	0
d[AB]/dt	v <sub>1</sub>	0
d[B*]/dt	$v_3 - v_2$	290

S5. Single PPI network exhibiting Biphasic Response



Table S5. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter
number			values
1	$A + B \leftarrow \rightarrow AB$	$v_1 = k_{1f} \cdot [A][B] - k_{1r} \cdot [AB]$	k <sub>1f</sub> =0.000662316,
			$k_{1r} = 0.000701878$
2	$B^* \rightarrow B$	$n_{2} = \frac{kc_{2} \cdot [A] \cdot [B *]}{kc_{2} \cdot [A] \cdot [B *]}$	kc <sub>2</sub> =2.98182,
		$V_2 = Km_2 + [B *]$	$Km_2 = 11.0657$
3	$B \rightarrow B^*$	$v_{2} = \frac{V_{3} [B]}{V_{3} [B]}$	V <sub>3</sub> =53.6473,
		$V_3 = Km_3 + [B]$	Km <sub>3</sub> = 1880.36

**Table S5. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S5.

Left-hand	<b>Right-hand Sides</b>	Initial
Sides		Concentrations (nM)
d[A]/dt	$-v_1$	50
d[B]/dt	$v_2 - v_1 - v_3$	0
d[AB]/dt	<i>v</i> <sub>1</sub>	0
d[B*]/dt	<i>v</i> <sub>3</sub> - <i>v</i> <sub>2</sub>	134

S6. Single PPI network exhibiting Coherent Feedforward Regulation



Table S6. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter
number			values
1	$A + B \leftarrow \rightarrow AB$	$v_1 = k_{1f} \cdot [A][B] - k_{1r} \cdot [AB]$	$k_{1f} = 0.00907778,$
			$k_{1r} = 0.0295832$
2	$B \rightarrow B^*$	$n_{1} = \frac{kc_{2} \cdot [A] \cdot [B]}{kc_{2} \cdot [A] \cdot [B]}$	$kc_2 = 0.0208362,$
		$v_2 = \frac{1}{Km_2 + [B]}$	Km <sub>2</sub> = 79.0018
3	$B^* \rightarrow B$	$V_{3} = \frac{V_{3} \cdot [B *]}{V_{3} \cdot [B *]}$	$V_3 = 2.37913,$
		$\nu_3 = \frac{1}{Km_3 + [B *]}$	Km <sub>3</sub> = 24.4436

**Table S6. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S6.

Left-hand	<b>Right-hand Sides</b>	Initial
Sides		Concentrations
		( <b>nM</b> )
d[A]/dt	$-v_1$	200
d[B]/dt	$v_3 - v_1 - v_2$	40
d[AB]/dt	<i>v</i> <sub>1</sub>	0
d[B*]/dt	$v_2 - v_3$	0

#### S7. In vivo PPI examples



S8. Coupled isolated PPI event



Table S8. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter
number			values
1	$A + i \leftrightarrow Ai$	$v_1 = k_{1f} \cdot [A][i] - k_{1r} \cdot [Ai]$	$k_{1f}$ = 0.00005, $k_{1r}$ =
			0.002
2	B+i ←→ Bi	$v_2 = k_{2f} \cdot [B][i] - k_{2r} \cdot [Bi]$	$k_{2f}=0.00005, k_{2r}=$
			0.002

**Table S8. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S8.

Left-hand	Right-hand Sides	Initial Concentrations
Sides		( <b>nM</b> )
d[A]/dt	$-v_1$	200
d[i]/dt	$-v_1 - v_2$	300
d[B]/dt	$-v_{2}$	200
d[Ai]/dt	<i>v</i> <sub>1</sub>	0
d[Bi]/dt	<i>v</i> <sub>2</sub>	0

S9. Coupled PPI network exhibiting Sustained Oscillation



Table S9. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter values
number			
1	$A + i \leftrightarrow Ai$	$v_1 = k_{1f} \cdot [A][i] - k_{1r} \cdot [Ai]$	$k_{1f} = 0.000156153,$
			$k_{1r} = 0.0000766173$
2	B+i ←→ Bi	$v_2 = k_{2f} \cdot [B][i] - k_{2r} \cdot [Bi]$	$k_{2f} = 0.000125176,$
			$k_{2r} = 0.00208669$
3	$B \rightarrow B^*$	$v_2 = \frac{kc_3.[A].[B]}{kc_3.[A].[B]}$	$kc_3 = 4.95119,$
		$Km_3 + [B]$	$Km_3 = 0.103278$
4	$B^* \rightarrow B$	$v_4 = \frac{V_4 \cdot [B *]}{V_4 \cdot [B *]}$	$V_4 = 8.30799,$
		$Km_4 + [B *]$	$Km_4 = 21.4724$

**Table S9. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S9.

Left-hand	Right-hand Sides	Initial Concentrations (nM)
Sides		
d[A]/dt	$-v_1$	25
d[i]/dt	$-v_1 - v_2$	74
d[B]/dt	$v_4 - v_3 - v_2$	672
d[Ai]/dt	v <sub>1</sub>	0
d[Bi]/dt	<i>v</i> <sub>2</sub>	0
d[B*]/dt	<i>v</i> <sub>3</sub> - <i>v</i> <sub>4</sub>	0

S10. Coupled PPI network exhibiting Biphasic Response



Table S10. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter
number			values
1	$A + i \leftrightarrow Ai$	$v_1 = k_{1f} \cdot [A][i] - k_{1r} \cdot [Ai]$	$k_{1f} = 0.0147496,$
			$k_{1r} = 0.000192892$
2	B+i ←→ Bi	$v_2 = k_{2f} \cdot [B][i] - k_{2r} \cdot [Bi]$	$k_{2f} = 0.175098,$
			$k_{2r} = 0.127203$
3	$B \rightarrow B^*$	$v_2 = \frac{kc_3.[A].[B]}{kc_3.[A].[B]}$	$kc_3 = 0.136761,$
		$Km_3 + [B]$	$Km_3 = 10.9548$
4	$B^* \rightarrow B$	$v_4 = \frac{V_4 \cdot [B *]}{V_4 \cdot [B *]}$	$V_4 = 5.2157,$
		$Km_4 + [B *]$	Km <sub>4</sub> = 195.135

Table S10. Or	rdinary	differential	equations	of the	kinetic	model.	The	reaction	rates	are
given in Table	S10.									

Left-hand	Right-hand Sides	Initial Concentrations
Sides		( <b>nM</b> )
d[A]/dt	$-v_1$	100
d[i]/dt	$-v_1 - v_2$	100
d[B]/dt	$v_4 - v_3 - v_2$	100
d[Ai]/dt	<i>v</i> <sub>1</sub>	0
d[Bi]/dt	<i>v</i> <sub>2</sub>	0
d[B*]/dt	<i>v</i> <sub>3</sub> - <i>v</i> <sub>4</sub>	0

S11. Coupled PPI network exhibiting Bistability



Table S11. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter
number			values
1	$A + i \leftarrow \rightarrow Ai$	$v_1 = k_{1f} \cdot [A][i] - k_{1r} \cdot [Ai]$	$k_{1f} = 1.30884,$
			$k_{1r} = 1.22917$
2	B + i ← → Bi	$v_2 = k_{2f} \cdot [B][i] - k_{2r} \cdot [Bi]$	$k_{2f} = 0.0154033,$
			$k_{2r} = 0.0533712$
3	$B^* \rightarrow B$	$v_2 = \frac{kc_3. [A]. [B *]}{m}$	$kc_3 = 0.0103728,$
		$Km_3 + [B *]$	$Km_3 = 1.42502$
4	$B \rightarrow B^*$	$v_{4} = \frac{V_{4} \cdot [B]}{V_{4} \cdot [B]}$	$V_4 = 0.120356,$
		$Km_4 + [B]$	$Km_4 = 3.22874$

**Table S11. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S11.

Left-hand	<b>Right-hand Sides</b>	Initial Concentrations
Sides		( <b>nM</b> )
d[A]/dt	$-v_l$	70
d[i]/dt	$-v_1 - v_2$	100
d[B]/dt	$v_3 - v_4 - v_2$	100
d[Ai]/dt	<i>v</i> <sub>1</sub>	0
d[Bi]/dt	<i>v</i> <sub>2</sub>	0
d[B*]/dt	$v_4 - v_3$	0

S12. The MST2-Raf-1 signaling network

## a. Facilitates Sustained Oscillation



Table S12.a. Reactions	and rates of	the kinetic model.
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Reaction	Reactions	Reaction rates	Parameter values
number			
1	pMST2 + pRaf1	$v_1$	k <sub>1f</sub> =0.185164,
	$\leftrightarrow$ pMST2-pRaf1	$= k_{1f} \cdot [pMST2][pRaf1]$	k <sub>1r</sub> =0.0985695
		$- k_{1r}.[pMST2.pRaf1]$	
2	$pMST2 \rightarrow MST2^*$	$v_2 = \frac{V_2 \cdot [MST2]}{V_2 \cdot [MST2]}$	V <sub>2</sub> =0.0454014,
		$v_2 = Km_2 + [MST2 *]$	Km <sub>2</sub> =330.376
3	$MST2^* \rightarrow pMST2$	$V_3.[MST2*]$	V <sub>3</sub> =0.0525323,
		$V_3 = Km_3 + [pMST2]$	Km <sub>3</sub> =99.4278
4	$LATS1 \rightarrow LATS1^*$	$m_{12} = \frac{kc_4.[MST2*].[LATS1]}{kc_4.[MST2*].[LATS1]}$	kc <sub>4</sub> =1.367,
		$Km_4 + [LATS1]$	Km <sub>4</sub> =41.4402
5	$LATS1^* \rightarrow LATS1$	$w_{-} = \frac{V_5. [LATS1 *]}{V_5 V_5 V_5 V_5 V_5 V_5 V_5 V_5 V_5 V_5 $	V <sub>5</sub> =0.757472,
		$Km_5 = Km_5 + [LATS1 *]$	Km5=0.581622
6	$Raf1^* \rightarrow pRaf1$	$v_6$	kc <sub>6</sub> =0.00229352,
		_ kc <sub>6</sub> . [LATS1 *]. [Raf1 *]	Km <sub>6</sub> =0.151116
		$-\frac{1}{Km_6 + [Raf1*]}$	
7	$pRaf1 \rightarrow Raf1^*$	$V_7.[pRaf1]$	V <sub>7</sub> =0.0387405,
		$Km_7 + [pRaf1]$	Km7=0.670203

Left-hand	<b>Right-hand Sides</b>	Initial
Sides		Concentrations (nM)
d[pMST2]/dt	$v_3 - v_2 - v_1$	0
d[pRaf1]/dt	$v_6 - v_7 - v_1$	0
d[pMST2-	v.	0
pRaf1]/dt		
d[MST2]/dt	$v_2 - v_3$	12
d[LATS1]/dt	<i>V</i> <sub>5</sub> - <i>V</i> <sub>4</sub>	250
d[LATS1*]/dt	<i>v</i> <sub>4</sub> - <i>v</i> <sub>5</sub>	0
d[Raf1*]/dt	$v_7 - v_6$	37

**Table S12.a. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S12.a..

b. Facilitates Biphasic Response



Table S12.b. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter values
number			
1	pMST2 + pRaf1	<i>v</i> <sub>1</sub>	$k_{1f} = 0.0119962,$
	$\leftrightarrow$ pMST2-	$= k_{1f}.[pMST2][pRaf1]$	$k_{1r} = 0.111344$
	pRaf1	$- k_{1r}.[pMST2.pRaf1]$	
2	$pMST2 \rightarrow MST2^*$	$v_2 = \frac{V_2.[MST2]}{V_2.[MST2]}$	$V_2 = 0.0363027,$
		$Km_2 + [MST2*]$	$Km_2 = 6.98519$
3	$MST2^* \rightarrow pMST2$	$v_{2} = \frac{V_{3}.[MST2*]}{V_{3}.[MST2*]}$	$V_3 = 0.147123,$
		$V_{3} = Km_{3} + [pMST2]$	Km <sub>3</sub> = 31.0557
4	$LATS1 \rightarrow LATS1^*$	$n_{\star} = \frac{kc_{4}. [MST2 *]. [LATS1]}{kc_{4}}$	$kc_4 = 0.307658,$
		$Km_4 + [LATS1]$	$Km_4 = 2.56876$
5	$LATS1^* \rightarrow LATS1$	$v_{r} = \frac{V_{5} \cdot [LATS1 *]}{V_{5} \cdot [LATS1 *]}$	$V_{5}=$ 3.20068,
		$Km_5 + [LATS1 *]$	$Km_5 = 0.0641184$
6	$Rafl^* \rightarrow pRafl$	$v_{c} = \frac{kc_{6}.[LATS1*].[Raf1*]}{kc_{6}.[LATS1*].[Raf1*]}$	$kc_6 = 0.00963654,$
		$Km_6 + [Raf1*]$	$Km_6 = 0.0688294$
7	$pRaf1 \rightarrow Raf1^*$	$v_7 = \frac{V_7 \cdot [pRaf1]}{V_7 \cdot [pRaf1]}$	$V_{7}=$ 0.207988,
		$Km_7 + [pRaf1]$	$Km_7 = 62.1023$

Left-hand	<b>Right-hand Sides</b>	Initial Concentrations (nM)
Sides		
d[pMST2]/dt	$v_3 - v_2 - v_1$	0
d[pRaf1]/dt	$v_6 - v_7 - v_1$	0
d[pMST2-	W <sub>1</sub>	0
pRaf1]/dt		0
d[MST2]/dt	$v_2 - v_3$	100
d[LATS1]/dt	<i>v</i> <sub>5</sub> - <i>v</i> <sub>4</sub>	85
d[LATS1*]/dt	<i>v</i> <sub>4</sub> - <i>v</i> <sub>5</sub>	0
d[Raf1*]/dt	<i>v</i> <sub>7</sub> - <i>v</i> <sub>6</sub>	38

**Table S12.b. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S12.b..

S13.A. Raf- MEK-ERK signaling network facilitates oscillation



Table S13.A. Reactions ar	d rates of the kinetic model.
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Reaction	Reactions	Reaction rates	Parameter
number			values
1	$Raf \rightarrow Raf^*$	$V_1 \cdot [Raf]$	V <sub>3</sub> =1.227,
		$\nu_1 = \frac{1}{Km_1 + [Raf]}$	Km <sub>1</sub> =1218
2	$Raf^* \rightarrow Raf$	$V_2.[Raf*]$	V <sub>2</sub> =1.034,
		$v_2 = Km_2 + [Raf *]$	Km <sub>2</sub> =254
3	$MEK \rightarrow MEK^*$	$v_{2} = \frac{kc_{3}.[Raf *].[MEK]}{k}$	kc <sub>3</sub> =0.006,
		$Km_3 + [MEK]$	Km <sub>3</sub> =387
4	$MEK^* \rightarrow MEK$	$v_{4} = \frac{V_{4} \cdot [MEK *]}{V_{4} \cdot [MEK *]}$	V <sub>4</sub> =0.186,
		$v_4 = Km_4 + [MEK *]$	Km <sub>4</sub> =90
5	$ERK \rightarrow ERK^*$	$u_{7} = \frac{kc_{5}.[MEK *].[ERK]}{k}$	kc <sub>5</sub> =0.05,
		$V_5 = Km_5 + [ERK]$	Km <sub>5</sub> =2
6	$ERK^* \rightarrow ERK$	$v_{c} = \frac{V_{6} \cdot [ERK *]}{V_{6} \cdot [ERK *]}$	V <sub>6</sub> =3.305,
		$V_6 = Km_6 + [ERK *]$	Km <sub>6</sub> =1458
7	Raf → ipRaf	$v_{7} = \frac{kc_{7} \cdot [ERK *] \cdot [Raf]}{kc_{7} \cdot [ERK *] \cdot [Raf]}$	kc7=0.006,
		$Km_7 + [Raf]$	Km <sub>7</sub> =1
8	$ipRaf \rightarrow Raf$	$v_{0} = \frac{V_{8}.[ipRaf]}{V_{1}$	$V_8 = 0.404,$
		$Km_8 + [ipRaf]$	Km <sub>8</sub> =3

**Table S13.A. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S13.A.

Left-hand Sides	<b>Right-hand Sides</b>	Initial Concentrations (nM)
d[Raf]/dt	$v_1 + v_8 - v_2 - v_7$	100
d[Raf*]/dt	$v_2 - v_1$	0
d[ipRaf]/dt	<i>v</i> <sub>7</sub> - <i>v</i> <sub>8</sub>	0
d[MEK]/dt	$v_4 - v_3$	100
d[MEK*]/dt	$v_3 - v_4$	0
d[ERK]/dt	<i>v</i> <sub>6</sub> - <i>v</i> <sub>5</sub>	100
d[ERK*]/dt	<i>v</i> <sub>5</sub> - <i>v</i> <sub>6</sub>	0

S13.B. Raf- RKIP-MEK signaling network facilitates oscillation



Table S13.B. Reactions an	d rates of the ki	inetic model.
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Reaction	Reactions	Reaction rates	Parameter
number			values
1	$Raf \rightarrow Raf^*$	$V_1 = V_1 \cdot [Raf]$	V <sub>3</sub> =0.107,
		$\nu_1 = \frac{1}{Km_1 + [Raf]}$	Km1=130
2	$Raf^* \rightarrow Raf$	$V_2 = V_2 \cdot [Raf *]$	V <sub>2</sub> =0.198,
		$V_2 = Km_2 + [Raf *]$	Km <sub>2</sub> =111
3	$MEK \rightarrow MEK^*$	$n_{2} = \frac{kc_{3}.[Raf *].[MEK]}{k}$	kc <sub>3</sub> =0.616,
		$V_3 = Km_3 + [MEK]$	Km3=0.1
4	$MEK^* \rightarrow MEK$	$V_4.[MEK *]$	V <sub>4</sub> =4.572,
		$v_4 = \frac{1}{Km_4 + [MEK *]}$	Km4=0.1
5	RKIP + MEK	$v_5 = k_{5f}.[RKIP][MEK]$	k <sub>5f</sub> =0.005,
	$\leftrightarrow$	$-k_{5r}$ . [RKIP. MEK]	k <sub>5r</sub> =0.001
	RKIP.MEK		
6	RKIP + Raf	$v_6 = k_{6f}.[RKIP][Raf]$	k <sub>6f</sub> =0.005,
	$\leftarrow \rightarrow$ RKIP.Raf	$-k_{6r}$ . [RKIP. Raf]	k <sub>6r</sub> =0.00009

**Table S13.B. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S13.B.

Left-hand Sides	<b>Right-hand Sides</b>	Initial Concentrations (nM)
d[Raf]/dt	$v_1 - v_2 - v_6$	100
d[Raf*]/dt	$v_2 - v_1$	0
d[MEK]/dt	$v_4 - v_3 - v_5$	100
d[MEK*]/dt	$v_3 - v_4$	0
d[RKIP.Raf]/dt	<i>v</i> <sub>6</sub>	0
d[RKIP.MEK]/dt	<i>v</i> <sub>5</sub>	0
d[RKIP]/dt	$-v_6 - v_5$	100

S13.C. Raf- RKIP-MEK-ERK signaling network facilitates oscillation



Table S13.C. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter
number			values
1	$Raf \rightarrow Raf^*$	$V_1.[Raf]$	V <sub>3</sub> =0.368,
		$\nu_1 = Km_1 + [Raf]$	Km1=0.1
2	$Raf^* \rightarrow Raf$	$v_{2} = \frac{V_{2} \cdot [Raf *]}{V_{2} \cdot [Raf *]}$	V <sub>2</sub> =61.37,
		$Km_2 + [Raf *]$	Km <sub>2</sub> =3
3	$MEK \rightarrow MEK^*$	$v_2 = \frac{V_3.[MEK]}{V_3.[MEK]}$	V <sub>3</sub> =0.152,
		$Km_3 + [MEK]$	Km <sub>3</sub> =5494
4	$MEK^* \rightarrow MEK$	$v_4 = \frac{V_4 \cdot [MEK *]}{V_4 \cdot [MEK *]}$	V <sub>4</sub> =8.48,
		$Km_4 + [MEK *]$	Km <sub>4</sub> =25485
5	$ERK \rightarrow ERK^*$	$v_{r} = \frac{kc_{5}.[MEK *].[ERK]}{k}$	kc <sub>5</sub> =0.977,
		$Km_5 + [ERK]$	Km5=2
6	$ERK^* \rightarrow ERK$	$v_c = \frac{V_6.[ERK*]}{V_6.[ERK*]}$	V <sub>6</sub> =0.432,
		$Km_6 + [ERK *]$	Km <sub>6</sub> =0.02
7	Raf → ipRaf	$v_7 = \frac{kc_7 \cdot [ERK *] \cdot [Raf]}{kc_7 \cdot [ERK *] \cdot [Raf]}$	kc <sub>7</sub> =1.6,
		$Km_7 + [Raf]$	Km <sub>7</sub> =530
8	ipRaf → Raf	$v_{0} = \frac{V_{8}.[ipRaf]}{V_{1}$	V <sub>8</sub> =0.678,
		$Km_8 + [ipRaf]$	Km <sub>8</sub> =1
9	RKIP + MEK	$v_9 = k_{9f}.[RKIP][MEK]$	k <sub>9f</sub> =0.0007,
	$\leftrightarrow$	$-k_{9r}$ . [RKIP. MEK]	k <sub>9r</sub> =0.00004
	RKIP.MEK		
10	RKIP + Raf	$v_{10} = k_{10f} \cdot [RKIP][Raf]$	$k_{10f} = 0.001,$
	$\leftarrow \rightarrow$ RKIP.Raf	$- k_{10r}.[RKIP.Raf]$	k <sub>10r</sub> =0.002

Left-hand Sides	<b>Right-hand Sides</b>	Initial Concentrations (nM)
d[Raf]/dt	$v_1 + v_8 - v_2 - v_7 - v_{10}$	100
d[Raf*]/dt	$v_2 - v_1$	0
d[ipRaf]/dt	<i>v</i> <sub>7</sub> - <i>v</i> <sub>8</sub>	0
d[MEK]/dt	$v_4 - v_3 - v_9$	100
d[MEK*]/dt	$v_3 - v_4$	0
d[ERK]/dt	<i>v</i> <sub>6</sub> - <i>v</i> <sub>5</sub>	100
d[ERK*]/dt	$v_5 - v_6$	0
d[RKIP.Raf]/dt	<i>v</i> <sub>10</sub>	0
d[RKIP.MEK]/dt	<i>V</i> 9	0
d[RKIP]/dt	$-v_9 - v_{10}$	100

**Table S13.C. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S13.C.

S14. Akt-YAP-14.3.3 signaling network facilitates Bistability



Table S14. Reactions and rates of the kinetic model.

Reaction	Reactions	Reaction rates	Parameter
number			values
1	$Akt \rightarrow Akt^*$	$n_{1} = \frac{kc_{1} \cdot [PDK1] \cdot [Akt]}{kc_{1} \cdot [PDK1] \cdot [Akt]}$	$kc_1 = 0.02,$
		$V_1 = Km_1 + [Akt]$	$Km_1 = 3259$
2	$Akt^* \rightarrow Akt$	$v_2 = \frac{V_2 \cdot [B]}{V_2 \cdot [B]}$	$V_2 = 14.79,$
		$Km_2 + [B]$	$Km_2 = 806$
3	$YAP^* \rightarrow pYAP$	$v_{2} = \frac{kc_{3}.\left[Akt *\right].\left[YAP *\right]}{kc_{3}}$	kc <sub>3</sub> =7.39,
		$Km_3 + [YAP *]$	$Km_3 = 203$
4	$pYAP \rightarrow YAP^*$	$v_{4} = \frac{V_{4} \cdot [pYAP]}{V_{4} \cdot [pYAP]}$	V <sub>4</sub> =0.257,
		$V_4 = Km_4 + [pYAP]$	$Km_4 = 0.6$
5	$14-3-3 + pYAP \leftrightarrow \rightarrow$	$v_5 = k_{5f}.[14.3.3][pYAP]$	k <sub>5f</sub> =0.094,
	14-3-3.pYAP	$-k_{5r}$ . [14.3.3	k <sub>5r</sub> =0.014
		- pYAP]	
6	$14-3-3 + PDK1 \leftarrow \rightarrow$	$v_6 = k_{6f}.[14.3.3][PDK1]$	k <sub>6f</sub> =0.019,
	14-3-3.PDK1	$-k_{6r}$ . [14.3.3	k <sub>6r</sub> =0.059
		-PDK1]	

Left-hand Sides	<b>Right-hand Sides</b>	Initial Concentrations (nM)
d[PDK1]/dt	$-v_{6}$	300
d[Akt]/dt	$v_2 - v_1$	10
d[Akt*]/dt	$v_1 - v_2$	0
d[pYAP]/dt	$v_3 - v_4 - v_5$	0
d[YAP*]/dt	$v_4 - v_3$	2981
d[14-3-3]/dt	$-v_5 - v_6$	1512
d[14-3-3.pYAP]/dt	$-v_{5}$	0
d[14-3-3.PDK1]/dt	$-v_{6}$	0

**Table S14. Ordinary differential equations of the kinetic model.**The reaction rates aregiven in Table S14.

#### S15. Alternate PPI coupling strategies



S16. Analytical Solution for PPI Dynamic Regulation Pattern

#### a. PPI event with no intermediates



#### b. PPI event with one intermediate



$$\frac{d[C]}{dt} = -ka2 \cdot \frac{1}{[A]} \cdot X \cdot [C] + kd2 \cdot [BC] \qquad \longrightarrow 4$$
$$\Rightarrow \frac{d[C]}{dt} \alpha - \frac{1}{[A]}$$

#### c. PPI event with two intermediates



#### d. PPI event with three intermediates

$$\begin{array}{l} & \overset{AB}{}_{B} & \overset{AB}{}_{L} & \overset{AB}{}_{L} \\ & \overset{B}{}_{C} & \overset{D}{}_{D} \\ & \overset{CD}{}_{D} & \overset{D}{}_{E} \\ \end{array} \\ & \overset{d[E]}{dt} = -ka4. \ [D]. \ [E] + kd4. \ [DE] & \longrightarrow \ (8) \\ \end{array}$$
From (7),  

$$[D] = \frac{1}{[A]} \cdot \frac{1}{Y.ka3} \left( kd3. \ [CD] - \frac{d[D]}{dt} \right)$$

Let 
$$\frac{1}{Y.ka3} \left( kd3. [CD] - \frac{d[D]}{dt} \right) = Z$$
  
 $[D] = \frac{1}{[A]} Z \longrightarrow 9$   
Substituting  $9$  in  $(8)$ ,

$$\frac{d[E]}{dt} = -k\alpha 4. \frac{1}{[A]}. Z. [E] + kd4. [DE]$$
$$\Rightarrow \frac{d[E]}{dt}\alpha - \frac{1}{[A]}$$

### 17. Protein Interaction Domains

Туре	Motif	Example Proteins
		ΡLCγ
		STAT
	SH2 Domain	PI3K
	PTB Domain	IRS
	14-3-3 Domain	
	SH3 Domain	Grb2
		PSD-95
		PSD-93
		SAP102
		SAP97
	PDZ Domain	GRIP1
	WW Domain	ITCH
		Eph receptor
	SAM Domain	STIM
	CH Domain	parvin
		talin
	FERM Domain	FAK
		FcRγ
		TCR
		FCeRI
	ITAM	Ig
		LIN11
		ISL1
		MEC3
		Lasp-1
Protein-Protein		PINCH
Interactions	LIM Domain	paxillin
	BAR Domain	SNX family
		PTEN
		РКС
	C2 Domain	synaptotagmins
	ENTH Domain	Actin
		PLC
		РКВ
		PLD
		Btk
	PH Domain	IRS
	PX Domain	SNX family
		EEA1
Protein-Lipid Interactions	FYVE Domain	PIKfyve