# Supporting Information 

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Table S1 $E_{\mathrm{f}} \mathrm{s}(\mathrm{eV})$ of certain configurations observed in the NEMD simulation of $\mathrm{V}_{n} \mathrm{~s}$, for $n=4,6$, and 8 . The optimizations were performed with both REBO2 potential and DFT (in parentheses). The 5-, rotated 6-, 7-, and 8-membered rings and hole-structures are emphasized with yellow, purple, green, red, and dark purple colors, respectively. The haeckelites are distinguished by merged (M-) or separated (S-) with the total number of $5 \mid 7$ pairs (see the main text).

| $\mathrm{V}_{n} \mathrm{~s}$ | Representative configurations and $E_{\mathrm{f}} \mathrm{S}$ |
| :---: | :---: |
| $\mathrm{V}_{4}$ |  |
| $\mathrm{V}_{6}$ |  |
| $\mathrm{V}_{8}$ |  |

Table S2 The area of pentagon and heptagon of the 57 pair of certain haeckelites, and the resulted area compensation by 57 pair relative to two hexagons in the perfect graphene.

| MVs | $\mathrm{V}_{4}$ |  |  | $\mathrm{V}_{6}$ | $\mathrm{V}_{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Configurations |  | $\begin{aligned} & 88888 \\ & 888 \% \\ & 888 \\ & 8080 \end{aligned}$ | $\begin{aligned} & 888888 \\ & 8888888 \\ & 8888 \end{aligned}$ |  |  |
| Average area of pentagons ( $\AA^{2}$ ) | 3.59 | 3.61 | 3.62 | 3.56 | 3.66 |
| Average area of heptagons ( $\AA^{2}$ ) | 7.76 | 7.87 | 7.75 | 7.67 | 7.72 |
| $\mathrm{S}_{5}+\mathrm{S}_{7}=n \mathrm{~S}_{6}$ | 2.15 | 2.18 | 2.16 | 2.13 | 2.16 |
| Compensation(\%) | 45 | 54 | 56 | 39 | 48 |

Note: In the perfect graphene, C-C length is $1.424 \AA$, and the area of a hexagon is $5.27 \AA^{2}$.


Fig. S1 Probability distributions of bond lengths in representative configurations of $V_{n} s: V_{2}(a), V_{4}$ (b), $\mathrm{V}_{6}(\mathrm{c})$, and $\mathrm{V}_{8}(\mathrm{~d})$. The configurations and partial enlarged regions with bond length exceeds $1.49 \AA$ are also shown as insets.

