Supplementary Information for ***Topologically protected Dirac plasmons and their evolution across the Quantum Phase Transition in $(Bi_{1-x}In_x)_2Se_3$ Topological Insulator***

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Attribution of single particle response to Dirac carriers

In order to attribute the single particle Drude response and the collective plasmon behavior to Dirac carriers in Bi_2Se_3, we calculate the surface density of carriers participating to the electronic response at 10 K, using the sum rule for the real part of the conductance $G_1(\omega) = \frac{e^2}{\hbar}\frac{\omega_e \Gamma}{\omega^2 + \Gamma^2}$:

$$\frac{2}{\pi}\int G_1(\omega)d\omega = \frac{e^2}{\hbar}\omega_e = \frac{n_{2D}e^2}{m^*}$$

where $\omega=2\pi\nu$, n_{2D} is the surface density of carriers (*per* surface). Assuming that only Dirac carriers participate to the conduction one has $m^* = \hbar k_F / v_F$ and $v_F = \partial E_F / \hbar \partial k_F$. The energy/momentum dispersion has been measured and shows a quadratic correction to the linear term:

$E_F = Ak_F + Bk_F^2 = 2.02k_F + 10.44k_F^2$.[1] Moreover, for a single surface state one has $n_{2D} = k_F^2/4\pi$. Assuming that each surface has the same charge density and contributes equally to the conductance one obtains from the sum-rule:

$$\omega_e \frac{e^2}{\hbar} = \frac{k_F(A + 2Bk_F)e^2}{2\pi\hbar^2}$$

Then, setting $\nu_e = \frac{\omega_e}{2\pi} = 34.1$ THz as obtained from Drude-Lorentz fit one achieves $n_{2D}=1.8\ 10^{13}$ cm^{-2}, in good agreement with Dirac surface density estimated from transport measurements.[2] This result suggests that, at low-T, the major part of the optical spectral weight and therefore the corresponding conductivity determining the plasmon response is due to Dirac carriers.

[1] L. Wu, M. Brahlek, R. V. Aguilar, A. V. Stier, C. M. Morris, Y. Lubashevsky, L. S. Bilbro, N. Bansal, S. Oh, and N. P. Armitage, *Nature Phys.* 9, 410 (2013).

[2] N. Bansal, Y. S. Kim, M. Brahlek, E. Edrey, and S. Oh, *Phys. Rev. Lett. 109*, 116804 (2012).