## Supplementary Information for detail procedure to derive Surface free energy from Wulff Construction:

The consequence of Wulff's Theorem is "For an equilibrium crystal there is a point in the interior such that its perpendicular distance $r_{i}$ from the $i^{\text {th }}$ face is proportional to the surface free energy $\gamma_{i}$ "

The procedure to derive surface free energy (SE) for a given shape (3D or hole or pore) is as follows. In Figure 1 (a hole of the same shape derived experimentally in this work), point $O$ can be considered as Wulff point, which is center of mass and can be identified as the intersection point of two bisectors of two different facets. Assuming that $\gamma_{A}$ and $\gamma_{B}$ are the SE of facet A and B , respectively, $\gamma_{A}$ and $\gamma_{B}$ are proportional to the perpendicular distances from $O$ to the facet i.e. $r_{A}$ and $r_{B}$. Thereby, if the SE of one facet is known, the other one can be derived from the equations given below.

$$
\frac{\gamma_{A}}{r_{A}}=\text { const. }=\frac{\gamma_{B}}{r_{B}} \quad \gamma_{i}=\gamma_{\text {known }} \frac{r_{i}}{r_{\text {known }}}
$$

To get a complete information about all the facets, one needs to plot $r_{i}(\theta)$ as a function of $\theta$. Here $r_{i}(\theta)$ is the distance from any point on the surface of the hole to point O. For convenience, the plot is started from one arbitrary point on the hole, which can be considered as $\theta=0$. Subsequently, $r_{i}(\theta)$ is recorded at an angle interval ( $5^{\circ}$ in the present case) for the entire hole (i.e. $360^{\circ}$ ) and finally one can get a plot like figure 2 .

For the sake of convenience, $r_{i}(\theta)$ can be normalized with respect to the $r$ value corresponding to the facet with known SE. In the present experiments, $\gamma(10-10)$ is known, taken from the theoretical value. Hence, $r_{i}(\theta)$ has been normalized with respect to the $r_{(10-10)}$ value. Therefore, the minimum $r(i)$ (perpendicular distance) for each facet has to be multiplied by $\gamma(10-10)$ in order to get $\gamma(\theta)$, i.e. $\gamma(\theta)=\gamma(10-10) \times r(j) / r(i)$ where $r(j)$ is the $r$ value of unknown facets and in this example $j$ corresponds to $(10-1-3)$.


Figure 1


Figure 2

