Electronic Supplementary Material (ESI) for Nanoscale.

Calculation of the thermal energy of each part:

$$\frac{dQ_{fluid,outside}}{dt} = \int_0^\infty \rho A C_{v,fluid} T\omega \cdot 2\pi r^2 dr$$

$$\frac{dQ_{fluid,outside}}{dt} = \int_0^\infty \rho A C_{v,fluid} T_0 \omega \exp(-k_1 r) \times \sin(\omega t - k_1 r) \cdot 2\pi r^2 dr$$

$$\frac{dQ_{fluid,outside}}{dt} = 2\pi\rho A C_{v,fluid} T_0 \omega \cdot \left. \frac{e^{-k_1 r}(1+k_1 r)((1+k_1 r)\cos(\omega t - k_1 r) + (1-k_1 r)\sin(\omega t - k_1 r))}{2k_1^3} \right|_0^\infty$$

$$\frac{dQ_{fluid,outside}}{dt} = \rho A C_{v,fluid} T_0 \omega \cdot \frac{2\pi}{\sqrt{2} k_1^3} \sin\left(\omega t - \frac{3\pi}{4}\right)$$

$$\frac{dQ_{source}}{dt} = \rho_s A C_s T_0 \omega d_s \sin(\omega t)$$

$$\frac{dQ_{fluid,inside}}{dt} = \rho A C_{v,fluid} T_0 \omega d_{gap} \sin\left(\omega t - \frac{3\pi}{4}\right)$$

Calculation of Eqn (6):

The conservation equation of thermal energy

$$dQ_{tot} = dQ_{fluid,outside} + dQ_{fluid,inside} + dQ_{source}$$

$$dQ_{fluid,PCS} = \frac{dQ_{fluid,outside} + dQ_{fluid,inside}}{dQ_{fluid,outside} + dQ_{fluid,inside} + dQ_{source}} \cdot dQ_{tot}$$

From the thermal energy of each part, we can get

$$dQ_{fluid,PCS} = \frac{I^2 R \sin^2(\omega t) dt}{1 + \dfrac{\rho_s C_s d_s \sin(\omega t)}{\rho C_{v,fluid} \cdot \left(\dfrac{\sqrt{2}\pi}{k_1^3} + d_{gap,PCS}\right) \sin\left(\omega t - \dfrac{3\pi}{4}\right)}}$$

The $\dot{Q}_{fluid}$ is defined as $\dot{Q}_{fluid} \equiv \frac{1}{T}\int dQ_{fluid}$, and if we set $\xi_{PCS} = \dfrac{\rho_s C_s d_s}{\rho C_{v,fluid} \cdot \left(\dfrac{\sqrt{2}\pi}{k_1^3} + d_{gap,PCS}\right)}$, we will get

$$\dot{Q}_{fluid,\ PCS} = \frac{(2 - 2\sqrt{2}\xi_{PCS} + 2\xi_{PCS}^2 - \sqrt{2}\xi_{PCS}^3)}{4(1 - \sqrt{2}\xi_{PCS} + \xi_{PCS}^2)^2} \cdot P_e$$

For a small ξ, the equation can be simplified as

$$\dot{Q}_{fluid,PCS} = \frac{1}{2} \cdot \frac{1}{1 - \sqrt{2}\xi_{PCS} + \xi_{PCS}^2} \cdot P_e$$

Calculation of Eqn (10):

$$\frac{dQ_{fluid,outside}}{dt} = \int_0^{\infty} \rho A C_{v,fluid} T \omega dr$$

$$\frac{dQ_{fluid,outside}}{dt} = \rho A C_{v,fluid} T_0 \omega \cdot \frac{2\pi}{\sqrt{2} k_1^{\ 3}} \sin\left(\omega t - \frac{\pi}{4} \right)$$

$$\dot{Q}_{fluid} = \frac{1}{T} \int_0^T \frac{\sin(\omega t)}{\dfrac{1}{\sin(\omega t)} + \dfrac{\xi_{AW}}{\sin\left(\omega t - \dfrac{\pi}{4} \right)}} \cdot I^2 R dt$$

$$\dot{Q}_{fluid} = \frac{(2 + 2\sqrt{2}\xi_{AW} + 2\xi_{AW}^2 + \sqrt{2}\xi_{AW}^3)}{4(1 + \sqrt{2}\xi_{AW} + \xi_{AW}^2)^2} \cdot P_e$$

Where $\xi_{AW} = \dfrac{\rho_s C_s d_s}{\rho C_{v,fluid} \cdot \left(\dfrac{1}{\sqrt{2}} \left(\dfrac{\alpha_v}{\pi f} \right)^{1/2} + d_{gap,AW} \right)}$. Here the ξ_{AW} is not small and we cannot ignore the high order of the ξ_{AW}.