

## Supplementary Information

The deduction of formula (3) is as follows:

According to ideal gas equation

$$P_g M = \rho RT \quad (1)$$

and considering the gas pressure and the composition proportion in the bubble changed with the CO<sub>2</sub> diffusion, resulting in the change of the mixed gas density.

$$\frac{d\rho}{dt} = \frac{M}{RT} \frac{dP_g}{dt} + \frac{P_g}{RT} \frac{dM}{dt} \quad (2)$$

where  $P_g$  is the pressure in the bubble, according to Young-Laplace equation, written as

$$P_g = P_l + \frac{2\sigma}{R_b} \quad (3)$$

$M$  is the molecular weight of mixed gas, given by

$$M = x_{CO_2} M_{CO_2} + x_{Air} M_{Air} \quad (4)$$

Combining Equations (2), (3) and (4) yields the resulting expression,

$$\begin{aligned} \frac{d\rho}{dt} &= -\frac{2\sigma M}{RTR_b^2} \frac{dR_b}{dt} + \frac{P_g}{RT} \left( M_{CO_2} \frac{dx_{CO_2}}{dt} + M_{Air} \frac{dx_{Air}}{dt} \right) \\ &= -\frac{2\sigma M}{RTR_b^2} \frac{dR_b}{dt} + \frac{P_g}{RT} (M_{CO_2} - M_{Air}) \frac{dx_{CO_2}}{dt} \\ &= -\frac{2\sigma M}{RTR_b^2} \frac{dR_b}{dt} + x_{Air} (M_{CO_2} - M_{Air}) \left( \frac{4\pi R_b^2 P_l}{RT} + \frac{16\pi\sigma R_b}{3RT} \right) \frac{dR_b}{dt} \\ &= \left[ -\frac{2\sigma M}{RTR_b^2} + (M_{CO_2} - M) \left( \frac{3P_l}{RTR_b} + \frac{4\sigma}{RTR_b^2} \right) \right] \frac{dR_b}{dt} \end{aligned} \quad (5)$$

The change rate of gas mass in the bubble can be calculated as:

$$\begin{aligned} \frac{dm}{dt} &= \frac{d\left(\frac{4}{3}\pi R_b^3 \rho\right)}{dt} = 4\pi R_b^2 \rho \frac{dR_b}{dt} + \frac{4}{3}\pi R_b^3 \frac{d\rho}{dt} \\ &= 4\pi R_b^2 \rho \frac{dR_b}{dt} + \frac{4}{3}\pi R_b^3 \left[ -\frac{2\sigma M}{RTR_b^2} + (M_{CO_2} - M) \left( \frac{3P_l}{RTR_b} + \frac{4\sigma}{RTR_b^2} \right) \right] \frac{dR_b}{dt} \\ &= \left[ 4\pi R_b^2 \rho - \frac{8\pi\sigma MR_b}{3RT} + (M_{CO_2} - M) \left( \frac{4\pi R_b^2 P_l}{RT} + \frac{16\pi\sigma R_b}{3RT} \right) \right] \frac{dR_b}{dt} \end{aligned}$$

$$= \left\{ \rho - \left[ \frac{2\sigma}{R_b} - \left( \frac{M_{CO_2}}{M} - 1 \right) P_l + \frac{M_{CO_2}}{M} \cdot \frac{4\sigma}{3R_b} \right] \frac{M}{RT} \right\} 4\pi R_b^2 \frac{dR_b}{dt} \quad (6)$$

Considering the change rate of gas mass in the bubble is equal to the CO<sub>2</sub> diffusion rate through the gas-liquid interface

$$\frac{dm}{dt} = -4\pi R_b^2 j_{R_b} \quad (7)$$

where  $j_{R_b}$  is the CO<sub>2</sub> diffusion flux through the gas-liquid interface, which can be obtained by applying Fick's first law in the microalgae suspension as

$$j_{R_b} = -M_{CO_2} D_{CO_2} \frac{\partial C}{\partial r} \Big|_{r=R_b} \quad (8)$$

Combining formula (6), (7) and (8), the decreasing rate of the bubble radius can be obtained by

$$\frac{dR_b}{dt} = \left\{ \rho - \left[ \frac{2\sigma}{R_b} - \left( \frac{M_{CO_2}}{M} - 1 \right) P_l + \frac{M_{CO_2}}{M} \cdot \frac{4\sigma}{3R_b} \right] \frac{M}{RT} \right\}^{-1} M_{CO_2} D_{CO_2} \frac{\partial C}{\partial r} \Big|_{r=R_b} \quad (9)$$