

Supplementary material: Matlab code

This paper's supplementary materials include the Matlab codes that solve the system of equations (6), (8) and (14) for a perfectly wetting fluid ($\theta_e = 0^\circ$).

The input parameters to set are the radius of the large and the small fiber, a_1 and a_2 respectively.

- *wet_prog.m* is the program to run to compute the evolution of the wrapping angles α_1 and α_2 , the inter-fiber distance d and the radius of curvature R . The value of a_1 and a_2 can be modified in this code.
- *equation_wet.m* contains the system of equation to be solve. This program is used by the main program (*wet_prog.m*).
- *Cross_Section_Area.m* calculate and plot the evolution of the cross-section area $\tilde{A}$.
- *Plot_Cross_Section.m* plot the shape of the cross-section. It requires the presence of the code *circle.m* to run.

Wet_Prog.m

```

1 clear all; clc;
2
3 %
4 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
5 % AUTHORS: D. CEBRON, A. SAURET
6 % DATE: 31/03/2015
7 % DESCRIPTION: solve the 3 coupled algebraic eq. of Sauret et al. (2015)
8 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
9
10 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Geometrical Parameters %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
11 %—— Radius of fiber 1 (in meter)
12 a1=2.25e-04;
13 %—— Radius of fiber 2 (in meter)
14 a2=0.5e-04;
15
16 Nalpha1=500; %%%%%%%%% Number of point to solve
17 alpha1_tab=linspace(0,pi,Nalpha1);
18
19 %—— Solve equation_wet by varying the value of alpha1 between 0 and
20 %pi
21 NN=10; alpha2=zeros(NN,Nalpha1)*NaN; d=zeros(NN,Nalpha1)*NaN; R=zeros(
22 NN,Nalpha1)*NaN; Nsol=zeros(Nalpha1,1);
23 for k=1:length(alpha1_tab);
24     alpha1=alpha1_tab(k);
25     [alpha2s,ds,Rs]=equation_wet(a1,a2,alpha1);
26     Nsol(k)=length(alpha2s);
27     if (isempty(alpha2s)==0)
28         alpha2(1:length(alpha2s),k)=alpha2s; d(1:length(alpha2s),k)=ds
29         ; R(1:length(alpha2s),k)=Rs;
30     end
31 end
32
33 %—— Plot the evolution of alpha2 (wrapping angle around the small
34 %fiber)
35 %—— of the interfiber distance d
36 %—— of the radius of curvature R
37 %—— as a function of alpha1 (wrapping angle around the large fiber)
38
39 figure(1);
40 subplot(3,1,1);
41 plot(alpha1_tab.*180/pi,alpha2(1,:).*180/pi,'-r','Linewidth',2); hold
42 on; % sol 1
43 xlabel('\phi_1'); ylabel('\phi_2 (in degree)');
44 subplot(3,1,2);
45 plot(alpha1_tab.*180/pi,d(1,:), '-r', 'Linewidth',2); hold on;
46 xlabel('\phi_1 (in degree)'); ylabel('d', 'Linewidth',2);
47 subplot(3,1,3);
48 plot(alpha1_tab.*180/pi,R(1,:), '-r', 'Linewidth',2); hold on;
49 xlabel('\phi_1 (in degree)'); ylabel('R');

```

equation_wet.m

```

1 function [alpha2,d,R]=equation_wet(a1,a2,alpha1)
2 %
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
3 % AUTHORS: D. CEBRON, A. SAURET
4 % DATE: 31/03/2015
5 % DESCRIPTION: solve the 3 coupled algebraic eq. of Sauret et al. (2015)
6 %
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
7 % EQUATIONS: with  $K=\cos(\alpha_1)+\cos(\alpha_2)$ , the 3 eq. are:
8 % eq. 1:  $R-((a_1*(1-\cos(\alpha_1))+a_2*(1-\cos(\alpha_2))+2*d)/K)=0$ 
9 % eq. 2:  $(2*d+2*a_1)*\tan(\alpha_1/2)-(2*d+2*a_2)*\tan(\alpha_2/2)=0$ 
10 % eq. 3:  $R^2*a+R*b+c=0$  with  $(a,b,c)$  defined in Sauret et al.
   % (2015)
11 %
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
12 % METHOD: eq. 2 gives d, and eq. 3 gives the solution
13 %  $R = (1/2)*(-b+\delta*\sqrt{b^2-4*a*c})/a$  for  $\delta=-1$  &  $\delta=1$ 
14 % which are used to solve eq. 1 (for  $\delta=-1$  first and then for  $\delta=1$ )
15 %
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
16 tol=1e-14; % Tolerance allowed in the solution
17
18 if (alpha1==0) %%%% Obvious solution for alpha1=0
19     alpha2=0; d=0; R=0;
20 elseif (a1==a2) %%%% Obvious solution for a1=a2
21     f=2.*alpha1-sin(2.*alpha1);
22     R=(sqrt(pi./f)-1).^(-1); %%%% Radius of curvature
23     d=cos(phi).*(1+R)-1; %%%% interfiber distance
24 else
25     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 1ST PART
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
26     %—— Solving eq. 1 for delta=-1 (1st solution given by eq. 3 for R
   % ) ——
27     %
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
28     delta=-1;
29     % Eqtot is eq. 1 using the expressions of a,b,c, d (eq. 2) & R (eq
   % .3)
30     eqtot = @(alpha2) real(1/2*cos(alpha1)*a2-1/2*cos(alpha2)*a1+1/2*a2
        *cos(alpha1+alpha2)+1/8*a2*cos(3*alpha1+alpha2)-1/8*a2*cos(3*
        alpha1-alpha2)-1/2*a2*cos(alpha1-alpha2)-3/8*a2*cos(2*alpha1-2*
        alpha2)+3/8*a2*cos(3*alpha2+alpha1)-3/8*a2*cos(-3*alpha2+alpha1
        )+5/8*a2*cos(2*alpha1+2*alpha2)+alpha2*a2*sin(2*alpha1)+alpha2*
        a2*sin(alpha1-alpha2)+alpha2*a2*sin(alpha1+alpha2)-a2*alpha1*
        sin(alpha1-alpha2)+a2*alpha1*sin(2*alpha2)+a2*alpha1*sin(alpha1
        +alpha2)+1/8*a1-1/8*a2+2*a1*alpha2*sin(alpha1)+a2*pi*sin(alpha1
        )-a2*alpha1*sin(alpha1)-2*alpha2*a2*sin(alpha1)-2*a1*pi*sin(
        alpha1)+alpha1*a1*sin(alpha1)+2*a2*pi*sin(alpha2)-2*a2*alpha1*

```

$$\begin{aligned}
& \sin(\alpha_2) + a_1 \alpha_2 \sin(\alpha_2) - a_1 \pi \sin(\alpha_2) - \alpha_2 a_2 \sin(\alpha_2) + 2 \alpha_1 a_1 \sin(\alpha_2) - 1/4 a_1 \cos(\alpha_1) + 1/4 a_2 \cos(\alpha_2) \\
& + 1/2 a_1 \cos(\alpha_1 - \alpha_2) - 1/2 a_1 \cos(\alpha_1 + \alpha_2) + 3/8 a_1 \cos(2 \alpha_1 - 2 \alpha_2) - 5/8 a_1 \cos(2 \alpha_1 + 2 \alpha_2) \\
& - 1/8 \delta a (8 a_1 a_2 \cos(2 \alpha_1 - 2 \alpha_2) + 6 a_1^2 + 6 a_2^2 - 64 \alpha_1 a_2^2 \alpha_2 - 64 \alpha_2 a_1^2 \alpha_1 + 64 \pi a_1^2 \alpha_1 + 64 \pi a_2^2 \alpha_2 \\
& - 8 a_1 a_2 \cos(\alpha_1 + \alpha_2) + 4 a_1 a_2 \cos(3 \alpha_1 - \alpha_2) + 8 a_1 a_2 \cos(\alpha_1 - \alpha_2) - 4 a_1 a_2 \cos(3 \alpha_1 + \alpha_2) \\
& - 4 a_1 a_2 \cos(3 \alpha_2 + \alpha_1) + 32 \alpha_2 a_1^2 \sin(2 \alpha_1) - 32 a_1 \alpha_2 a_2 \sin(2 \alpha_1) - 32 a_1 \alpha_2 a_2 \sin(\alpha_1 + \alpha_2) \\
& - 32 a_1 \alpha_2 a_2 \sin(\alpha_1 - \alpha_2) - 32 a_2 \alpha_1 a_1 \sin(\alpha_1 + \alpha_2) + 32 a_2 \alpha_1 a_1 \sin(\alpha_1 - \alpha_2) - 32 a_2 \alpha_1 a_1 \sin(\alpha_1 + \alpha_2) \\
& (2 \alpha_2) - 12 a_1^2 \cos(\alpha_1 - \alpha_2) + 12 a_1^2 \cos(3 \alpha_1 + \alpha_2) - 12 a_2^2 \cos(\alpha_1 - \alpha_2) + 12 a_2^2 \cos(3 \alpha_2 + \alpha_1) \\
& - 2 a_1^2 \cos(4 \alpha_1) + 4 a_1^2 \cos(2 \alpha_2) - 4 a_1^2 \cos(2 \alpha_1) - 2 a_1^2 \cos(2 \alpha_1 - 2 \alpha_2) - 2 a_1^2 \cos(2 \alpha_1 + 2 \alpha_2) + 4 a_2^2 \cos(2 \alpha_1) \\
& - 4 a_2^2 \cos(2 \alpha_2) - 2 a_2^2 \cos(2 \alpha_1 - 2 \alpha_2) - 2 a_2^2 \cos(2 \alpha_1 + 2 \alpha_2) - 2 a_2^2 \cos(4 \alpha_2) - 32 \pi a_1^2 \sin(2 \alpha_1) \\
& - 8 a_1 a_2 \cos(2 \alpha_1 + 2 \alpha_2) + 32 a_1^2 \alpha_1 \sin(\alpha_1 + \alpha_2) - 32 a_1^2 \alpha_1 \sin(\alpha_1 - \alpha_2) + 32 a_2^2 \alpha_2 \sin(\alpha_1 + \alpha_2) + 32 a_2^2 \alpha_2 \sin(\alpha_1 - \alpha_2) \\
& - 32 \pi a_2^2 \sin(2 \alpha_2) + 32 \alpha_1 a_2^2 \sin(2 \alpha_2) + 4 a_1^2 \cos(\alpha_1 + \alpha_2) - 4 a_1^2 \cos(3 \alpha_1 - \alpha_2) + 4 a_2^2 \cos(\alpha_1 + \alpha_2) - 4 a_2^2 \cos(-3 \alpha_2 + \alpha_1) \\
& + 4 a_1 a_2 \cos(-3 \alpha_2 + \alpha_1) + 128 \alpha_1 a_1 \alpha_2 a_2)^{1/2} \sin(\alpha_1) - 1/8 \delta a (8 a_1 a_2 \cos(2 \alpha_1 - 2 \alpha_2) + 6 a_1^2 + 6 a_2^2 - 64 \alpha_1 a_2^2 \alpha_2 - 64 \alpha_2 a_1^2 \alpha_1 + 64 \pi a_1^2 \alpha_1 + 64 \pi a_2^2 \alpha_2 \\
& - 8 a_1 a_2 \cos(\alpha_1 + \alpha_2) + 4 a_1 a_2 \cos(3 \alpha_1 - \alpha_2) + 8 a_1 a_2 \cos(\alpha_1 - \alpha_2) - 4 a_1 a_2 \cos(3 \alpha_1 + \alpha_2) - 4 a_1 a_2 \cos(3 \alpha_2 + \alpha_1) \\
& + 32 \alpha_2 a_1^2 \sin(2 \alpha_1) - 32 a_1 \alpha_2 a_2 \sin(2 \alpha_1) - 32 a_1 \alpha_2 a_2 \sin(\alpha_1 + \alpha_2) - 32 a_1 \alpha_2 a_2 \sin(\alpha_1 - \alpha_2) - 32 a_2 \alpha_1 a_1 \sin(\alpha_1 + \alpha_2) \\
& + 32 a_2 \alpha_1 a_1 \sin(\alpha_1 - \alpha_2) - 32 a_2 \alpha_1 a_1 \sin(\alpha_1 + \alpha_2) (2 \alpha_2) - 12 a_1^2 \cos(\alpha_1 - \alpha_2) + 12 a_1^2 \cos(3 \alpha_1 + \alpha_2) - 12 a_2^2 \cos(\alpha_1 - \alpha_2) + 12 a_2^2 \cos(3 \alpha_2 + \alpha_1) \\
& - 2 a_1^2 \cos(4 \alpha_1) + 4 a_1^2 \cos(2 \alpha_2) - 4 a_1^2 \cos(2 \alpha_1) - 2 a_1^2 \cos(2 \alpha_1 - 2 \alpha_2) - 2 a_1^2 \cos(2 \alpha_1 + 2 \alpha_2) + 4 a_2^2 \cos(2 \alpha_1) - 4 a_2^2 \cos(2 \alpha_2) \\
& - 2 a_2^2 \cos(2 \alpha_1 - 2 \alpha_2) - 2 a_2^2 \cos(2 \alpha_1 + 2 \alpha_2) - 2 a_2^2 \cos(4 \alpha_2) - 32 \pi a_1^2 \sin(2 \alpha_1) - 8 a_1 a_2 \cos(2 \alpha_1 + 2 \alpha_2) + 32 a_1^2 \alpha_1 \sin(\alpha_1 + \alpha_2) - 32 a_1^2 \alpha_1 \sin(\alpha_1 - \alpha_2) \\
& + 32 a_2^2 \alpha_2 \sin(\alpha_1 + \alpha_2) + 32 a_2^2 \alpha_2 \sin(\alpha_1 - \alpha_2) - 32 \pi a_2^2 \sin(2 \alpha_2) + 32 \alpha_1 a_2^2 \sin(2 \alpha_2) + 4 a_1^2 \cos(\alpha_1 + \alpha_2) - 4 a_1^2 \cos(3 \alpha_1 - \alpha_2) + 4 a_2^2 \cos(\alpha_1 + \alpha_2) \\
& - 4 a_2^2 \cos(-3 \alpha_2 + \alpha_1) + 4 a_1 a_2 \cos(-3 \alpha_2 + \alpha_1) + 128 \alpha_1 a_1 \alpha_2 a_2)^{1/2} \sin(\alpha_1 - 2 \alpha_2) - \alpha_1 a_1 \sin(\alpha_1 - 2 \alpha_2) - a_2 \pi \sin(\alpha_1 - 2 \alpha_2) + a_2 \alpha_1 \sin(\alpha_1 - 2 \alpha_2) - \alpha_2 a_2 \sin(2 \alpha_1 - \alpha_2) \\
& + a_1 \cos(2 \alpha_1 + \alpha_2) - 1/8 a_1 \cos(4 \alpha_1 - \alpha_2) - 3/8 a_1 \cos(2 \alpha_1 - \alpha_2) + 1/8 \delta a (8 a_1 a_2 \cos(2 \alpha_1 - 2 \alpha_2) + 6 a_1^2 + 6 a_2^2 - 64 \alpha_1 a_2^2 \alpha_2 - 64 \alpha_2 a_1^2 \alpha_1 + 64 \pi a_1^2 \alpha_1 + 64 \pi a_2^2 \alpha_2 \\
& - 8 a_1 a_2 \cos(\alpha_1 + \alpha_2) + 4 a_1 a_2 \cos(3 \alpha_1 - \alpha_2) + 8 a_1 a_2 \cos(\alpha_1 - \alpha_2) - 4 a_1 a_2 \cos(3 \alpha_1 + \alpha_2) - 4 a_1 a_2 \cos(3 \alpha_2 + \alpha_1) + 32 \alpha_2 a_1^2 \sin(2 \alpha_1) - 32 a_1 \alpha_2 a_2 \sin(2 \alpha_1) - 32 a_1 \alpha_2 a_2 \sin(\alpha_1 + \alpha_2) \\
& - 32 a_1 \alpha_2 a_2 \sin(\alpha_1 - \alpha_2) - 32 a_2 \alpha_1 a_1 \sin(\alpha_1 + \alpha_2) - 32 a_2 \alpha_1 a_1 \sin(\alpha_1 - \alpha_2)
\end{aligned}$$

$$\begin{aligned}
& -32*a2*alpha1*a1*\sin(alpha1+alpha2)+32*a2*alpha1*a1*\sin(alpha1- \\
& alpha2)-32*a2*alpha1*a1*\sin(2*alpha2)-12*a1^2*\cos(alpha1-alpha2) \\
& +12*a1^2*\cos(3*alpha1+alpha2)-12*a2^2*\cos(alpha1-alpha2)+12*a2 \\
& ^2*\cos(3*alpha2+alpha1)-2*a1^2*\cos(4*alpha1)+4*a1^2*\cos(2* \\
& alpha2)-4*a1^2*\cos(2*alpha1)-2*a1^2*\cos(2*alpha1-2*alpha2)-2*a1 \\
& ^2*\cos(2*alpha1+2*alpha2)+4*a2^2*\cos(2*alpha1)-4*a2^2*\cos(2* \\
& alpha2)-2*a2^2*\cos(2*alpha1-2*alpha2)-2*a2^2*\cos(2*alpha1+2* \\
& alpha2)-2*a2^2*\cos(4*alpha2)-32*\pi*a1^2*\sin(2*alpha1)-8*a1*a2* \\
& \cos(2*alpha1+2*alpha2)+32*a1^2*alpha1*\sin(alpha1+alpha2)-32*a1 \\
& ^2*alpha1*\sin(alpha1-alpha2)+32*a2^2*alpha2*\sin(alpha1+alpha2) \\
& +32*a2^2*alpha2*\sin(alpha1-alpha2)-32*\pi*a2^2*\sin(2*alpha2)+32* \\
& alpha1*a2^2*\sin(2*alpha2)+4*a1^2*\cos(alpha1+alpha2)-4*a1^2*\cos \\
& (3*alpha1-alpha2)+4*a2^2*\cos(alpha1+alpha2)-4*a2^2*\cos(-3* \\
& alpha2+alpha1)+4*a1*a2*\cos(-3*alpha2+alpha1)+128*alpha1*a1* \\
& alpha2*a2)^{(1/2)}*\sin(alpha2)-1/8*\delta*(8*a1*a2*\cos(2*alpha1-2* \\
& alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*a1^2* \\
& alpha1+64*\pi*a1^2*alpha1+64*\pi*a2^2*alpha2-8*a1*a2*\cos(alpha1+ \\
& alpha2)+4*a1*a2*\cos(3*alpha1-alpha2)+8*a1*a2*\cos(alpha1-alpha2) \\
& -4*a1*a2*\cos(3*alpha1+alpha2)-4*a1*a2*\cos(3*alpha2+alpha1)+32* \\
& alpha2*a1^2*\sin(2*alpha1)-32*a1*alpha2*a2*\sin(2*alpha1)-32*a1* \\
& alpha2*a2*\sin(alpha1+alpha2)-32*a1*alpha2*a2*\sin(alpha1-alpha2) \\
& -32*a2*alpha1*a1*\sin(alpha1+alpha2)+32*a2*alpha1*a1*\sin(alpha1- \\
& alpha2)-32*a2*alpha1*a1*\sin(2*alpha2)-12*a1^2*\cos(alpha1-alpha2) \\
& +12*a1^2*\cos(3*alpha1+alpha2)-12*a2^2*\cos(alpha1-alpha2)+12*a2 \\
& ^2*\cos(3*alpha2+alpha1)-2*a1^2*\cos(4*alpha1)+4*a1^2*\cos(2* \\
& alpha2)-4*a1^2*\cos(2*alpha1)-2*a1^2*\cos(2*alpha1-2*alpha2)-2*a1 \\
& ^2*\cos(2*alpha1+2*alpha2)+4*a2^2*\cos(2*alpha1)-4*a2^2*\cos(2* \\
& alpha2)-2*a2^2*\cos(2*alpha1-2*alpha2)-2*a2^2*\cos(2*alpha1+2* \\
& alpha2)-2*a2^2*\cos(4*alpha2)-32*\pi*a1^2*\sin(2*alpha1)-8*a1*a2* \\
& \cos(2*alpha1+2*alpha2)+32*a1^2*alpha1*\sin(alpha1+alpha2)-32*a1 \\
& ^2*alpha1*\sin(alpha1-alpha2)+32*a2^2*alpha2*\sin(alpha1+alpha2) \\
& +32*a2^2*alpha2*\sin(alpha1-alpha2)-32*\pi*a2^2*\sin(2*alpha2)+32* \\
& alpha1*a2^2*\sin(2*alpha2)+4*a1^2*\cos(alpha1+alpha2)-4*a1^2*\cos \\
& (3*alpha1-alpha2)+4*a2^2*\cos(alpha1+alpha2)-4*a2^2*\cos(-3* \\
& alpha2+alpha1)+4*a1*a2*\cos(-3*alpha2+alpha1)+128*alpha1*a1* \\
& alpha2*a2)^{(1/2)}*\sin(2*alpha1-alpha2)-a1*\pi*\sin(2*alpha1-alpha2) \\
& +a1*alpha2*\sin(2*alpha1-alpha2)-a1*alpha2*\sin(alpha1+alpha2)- \\
& a1*alpha2*\sin(alpha1-alpha2)+a1*\pi*\sin(alpha1+alpha2)+a1*\pi*\sin \\
& (alpha1-alpha2)-a2*\pi*\sin(alpha1+alpha2)+a2*\pi*\sin(alpha1- \\
& alpha2)-alpha1*a1*\sin(alpha1+alpha2)+alpha1*a1*\sin(alpha1- \\
& alpha2)+1/8*a2*\cos(-4*alpha2+alpha1)-a2*\cos(alpha1+2*alpha2) \\
& +3/8*a2*\cos(alpha1-2*alpha2)+1/4*\delta*(8*a1*a2*\cos(2*alpha1-2* \\
& alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*a1^2* \\
& alpha1+64*\pi*a1^2*alpha1+64*\pi*a2^2*alpha2-8*a1*a2*\cos(alpha1+ \\
& alpha2)+4*a1*a2*\cos(3*alpha1-alpha2)+8*a1*a2*\cos(alpha1-alpha2) \\
& -4*a1*a2*\cos(3*alpha1+alpha2)-4*a1*a2*\cos(3*alpha2+alpha1)+32* \\
& alpha2*a1^2*\sin(2*alpha1)-32*a1*alpha2*a2*\sin(2*alpha1)-32*a1* \\
& alpha2*a2*\sin(alpha1+alpha2)-32*a1*alpha2*a2*\sin(alpha1-alpha2) \\
& -32*a2*alpha1*a1*\sin(alpha1+alpha2)+32*a2*alpha1*a1*\sin(alpha1- \\
& alpha2)-32*a2*alpha1*a1*\sin(2*alpha2)-12*a1^2*\cos(alpha1-alpha2) \\
& +12*a1^2*\cos(3*alpha1+alpha2)-12*a2^2*\cos(alpha1-alpha2)+12*a2 \\
& ^2*\cos(3*alpha2+alpha1)-2*a1^2*\cos(4*alpha1)+4*a1^2*\cos(2* \\
& alpha2)-4*a1^2*\cos(2*alpha1)-2*a1^2*\cos(2*alpha1-2*alpha2)-2*a1 \\
& ^2*\cos(2*alpha1+2*alpha2)+4*a2^2*\cos(2*alpha1)-4*a2^2*\cos(2* \\
& alpha2)-2*a2^2*\cos(2*alpha1-2*alpha2)-2*a2^2*\cos(2*alpha1+2* \\
& alpha2)-2*a2^2*\cos(4*alpha2)-32*\pi*a1^2*\sin(2*alpha1)-8*a1*a2*
\end{aligned}$$

$$\begin{aligned}
& \cos(2*\alpha_1+2*\alpha_2)+32*a_1^2*\alpha_1*\sin(\alpha_1+\alpha_2)-32*a_1^2*\alpha_1*\sin(\alpha_1-\alpha_2)+32*a_2^2*\alpha_2*\sin(\alpha_1+\alpha_2) \\
& +32*a_2^2*\alpha_2*\sin(\alpha_1-\alpha_2)-32*\pi*a_2^2*\sin(2*\alpha_2)+32*\alpha_1*a_2^2*\sin(2*\alpha_2)+4*a_1^2*\cos(\alpha_1+\alpha_2)-4*a_1^2*\cos \\
& (3*\alpha_1-\alpha_2)+4*a_2^2*\cos(\alpha_1+\alpha_2)-4*a_2^2*\cos(-3*\alpha_2+\alpha_1)+4*a_1*a_2*\cos(-3*\alpha_2+\alpha_1)+128*\alpha_1*a_1* \\
& \alpha_2*a_2)^{(1/2)}*\sin(\alpha_1-\alpha_2)+1/8*\delta*(8*a_1*a_2*\cos(2*\alpha_1-2*\alpha_2)+6*a_1^2+6*a_2^2-64*\alpha_1*a_2^2*\alpha_2-64*\alpha_2* \\
& a_1^2*\alpha_1+64*\pi*a_1^2*\alpha_1+64*\pi*a_2^2*\alpha_2-8*a_1*a_2*\cos(\alpha_1+\alpha_2)+4*a_1*a_2*\cos(3*\alpha_1-\alpha_2)+8*a_1*a_2*\cos(\alpha_1- \\
& \alpha_2)-4*a_1*a_2*\cos(3*\alpha_1+\alpha_2)-4*a_1*a_2*\cos(3*\alpha_2+ \\
& \alpha_1)+32*\alpha_2*a_1^2*\sin(2*\alpha_1)-32*a_1*\alpha_2*a_2*\sin(2*\alpha_1)-32*a_1*\alpha_2*a_2*\sin(\alpha_1+\alpha_2)-32*a_1*\alpha_2*a_2*\sin(\alpha_1-\alpha_2)-32*a_2*\alpha_1*a_1*\sin(\alpha_1+\alpha_2)+32*a_2*\alpha_1*a_1*\sin(\alpha_1-\alpha_2)-32*a_2*\alpha_1*a_1*\sin(2*\alpha_2)-12*a_1^2*\cos(\alpha_1-\alpha_2)+12*a_1^2*\cos(3*\alpha_1+\alpha_2)-12*a_2^2*\cos(\alpha_1-\alpha_2)+12*a_2^2*\cos(3*\alpha_2+\alpha_1)-2*a_1^2*\cos(4*\alpha_1)+4*a_1^2*\cos(2*\alpha_2)-4*a_1^2*\cos(2*\alpha_1)-2*a_1^2*\cos(2*\alpha_1-2*\alpha_2)-2*a_1^2*\cos(2*\alpha_1+2*\alpha_2)+4*a_2^2*\cos(2*\alpha_1)-4*a_2^2*\cos(2*\alpha_2)-2*a_2^2*\cos(2*\alpha_1-2*\alpha_2)-2*a_2^2*\cos(2*\alpha_1+2*\alpha_2)-2*a_2^2*\cos(4*\alpha_2)-32*\pi*a_1^2*\sin(2*\alpha_1)-8*a_1*a_2*\cos(2*\alpha_1+2*\alpha_2)+32*a_1^2*\alpha_1*\sin(\alpha_1+\alpha_2)-32*a_1^2*\alpha_1*\sin(\alpha_1-\alpha_2)+32*a_2^2*\alpha_2*\sin(\alpha_1+\alpha_2)+32*a_2^2*\alpha_2*\sin(\alpha_1-\alpha_2)-32*\pi*a_2^2*\sin(2*\alpha_2)+32*\alpha_1*a_2^2*\sin(2*\alpha_2)+4*a_1^2*\cos(\alpha_1+\alpha_2)-4*a_1^2*\cos(3*\alpha_1-\alpha_2)+4*a_2^2*\cos(\alpha_1+\alpha_2)-4*a_2^2*\cos(-3*\alpha_2+\alpha_1)+4*a_1*a_2*\cos(-3*\alpha_2+\alpha_1)+128*\alpha_1*a_1*\alpha_2*a_2)^{(1/2)}*\sin(2*\alpha_1)-a_1*\alpha_2*\sin(2*\alpha_1)-\alpha_1*a_1*\sin(2*\alpha_2)-3/4*a_2*\cos(2*\alpha_1+\alpha_2)+a_1*\pi*\sin(2*\alpha_1)+3/4*a_1*\cos(\alpha_1+2*\alpha_2)+3/8*a_1*\cos(3*\alpha_1-\alpha_2)-3/8*a_1*\cos(3*\alpha_1+\alpha_2)+1/8*a_1*\cos(-3*\alpha_2+\alpha_1)-1/8*a_1*\cos(3*\alpha_2+\alpha_1)-1/8*\delta*(8*a_1*a_2*\cos(2*\alpha_1-2*\alpha_2)+6*a_1^2+6*a_2^2-64*\alpha_1*a_2^2*\alpha_2-64*\alpha_2*a_1^2*\alpha_1+64*\pi*a_1^2*\alpha_1+64*\pi*a_2^2*\alpha_2-8*a_1*a_2*\cos(\alpha_1+\alpha_2)+4*a_1*a_2*\cos(3*\alpha_1-\alpha_2)+8*a_1*a_2*\cos(\alpha_1-\alpha_2)-4*a_1*a_2*\cos(3*\alpha_1+\alpha_2)-4*a_1*a_2*\cos(3*\alpha_2+\alpha_1)+32*\alpha_2*a_1^2*\sin(2*\alpha_1)-32*a_1*\alpha_2*a_2*\sin(2*\alpha_1)-32*a_1*\alpha_2*a_2*\sin(\alpha_1+\alpha_2)-32*a_1*\alpha_2*a_2*\sin(\alpha_1-\alpha_2)-32*a_2*\alpha_1*a_1*\sin(\alpha_1+\alpha_2)+32*a_2*\alpha_1*a_1*\sin(\alpha_1-\alpha_2)-32*a_2*\alpha_1*a_1*\sin(2*\alpha_2)-12*a_1^2*\cos(\alpha_1-\alpha_2)+12*a_1^2*\cos(3*\alpha_1+\alpha_2)-12*a_2^2*\cos(\alpha_1-\alpha_2)+12*a_2^2*\cos(3*\alpha_2+\alpha_1)-2*a_1^2*\cos(4*\alpha_1)+4*a_1^2*\cos(2*\alpha_2)-4*a_1^2*\cos(2*\alpha_1)-2*a_1^2*\cos(2*\alpha_1-2*\alpha_2)-2*a_1^2*\cos(2*\alpha_1+2*\alpha_2)+4*a_2^2*\cos(2*\alpha_1)-4*a_2^2*\cos(2*\alpha_2)-2*a_2^2*\cos(2*\alpha_1-2*\alpha_2)-2*a_2^2*\cos(2*\alpha_1+2*\alpha_2)-2*a_2^2*\cos(4*\alpha_2)-32*\pi*a_1^2*\sin(2*\alpha_1)-8*a_1*a_2*\cos(2*\alpha_1+2*\alpha_2)+32*a_1^2*\alpha_1*\sin(\alpha_1+\alpha_2)-32*a_1^2*\alpha_1*\sin(\alpha_1-\alpha_2)+32*a_2^2*\alpha_2*\sin(\alpha_1+\alpha_2)+32*a_2^2*\alpha_2*\sin(\alpha_1-\alpha_2)-32*\pi*a_2^2*\sin(2*\alpha_2)+32*\alpha_1*a_2^2*\sin(2*\alpha_2)+4*a_1^2*\cos(\alpha_1+\alpha_2)-4*a_1^2*\cos(3*\alpha_1-\alpha_2)+4*a_2^2*\cos(\alpha_1+\alpha_2)-4*a_2^2*\cos(-3*\alpha_2+\alpha_1)+4*a_1*a_2*\cos(-3*\alpha_2+\alpha_1)+128*\alpha_1*a_1*\alpha_2*a_2)^{(1/2)}*\sin(2*\alpha_2)-a_2*\pi*\sin(2*\alpha_2)+1/4*a_1*\cos(2*\alpha_2)-1/4*a_1*\cos(2*\alpha_1)-1/4*a_2*\cos(2*\alpha_1)+1/4*a_2*\cos(2*\alpha_2)-1/8*a_1*\cos(2*\alpha_1-3*\alpha_2)+1/8*a_2*\cos(3*\alpha_1-2*\alpha_2)-1/2*a_1*\cos(\alpha_1-2*\alpha_2)-1/4*a_1*
\end{aligned}$$

```

cos(3*alpha1-2*alpha2)+1/2*a2*cos(2*alpha1-alpha2)+1/4*a2*cos
(2*alpha1-3*alpha2)+1/4*a1*cos(3*alpha1)-1/4*a2*cos(3*alpha2)
+1/8*a1*cos(3*alpha2)-1/8*a2*cos(3*alpha1)-1/8*a2*cos(4*alpha2)
+1/8*a1*cos(4*alpha1));
31 guess=linspace(0,pi,20); % Various starting guess for the solver
32 racine=[]; % will allow to discard solutions too closed from 0
33 options=optimset('Display','off','FunValCheck','on','TolFun',1e-14,
'TolX',1e-14,'MaxFunEvals',1e4,'MaxIter',1e4);
34 for kh=1:length(guess) % Solving with fzero for each starting guess
35 [x,fval,exitflag] = fzero(eqtot,guess(kh),options);
36 if(exitflag==1)
37     flagij=0;
38     for kkh=1:length(racine) % test if the solution is new
39         if(abs(x-racine(kkh))<=1e-10), flagij=1; end;
40     end
41     if(flagij==0), racine=[racine x]; end; %only store new
        solution
42     end
43 end
44 alpha2=racine;
45 dd = -(-tan((1./2).*alpha1).*a1+tan((1./2).*alpha2).*a2)./(-tan
((1./2).*alpha1)+tan((1./2).*alpha2)); % d associated with the
roots
46 RR=1./2.*(-a1.*cos(alpha1).*sin(alpha1)-a1.*sin(alpha1).*cos(alpha2)
)-a2.*sin(alpha2).*cos(alpha1)-a2.*cos(alpha2).*sin(alpha2)+2.*
alpha1.*a1+2.*alpha2.*a2+delta.*(4.*a1.*cos(alpha1).*sin(alpha1)
).*a2.*cos(alpha2).*sin(alpha2)+2.*a1.*cos(alpha1).^2.*sin(
alpha1).*a2.*sin(alpha2)-4.*a1.*cos(alpha1).*sin(alpha1).*
alpha2.*a2+2.*a1.*sin(alpha1).*cos(alpha2).^2.*a2.*sin(alpha2)
-4.*a1.*sin(alpha1).*cos(alpha2).*alpha2.*a2-4.*a2.*sin(alpha2)
.*cos(alpha1).*alpha1.*a1-4.*a2.*cos(alpha2).*sin(alpha2).*
alpha1.*a1-8.*sin(1./2.*alpha1+1./2.*alpha2).*cos(1./2.*alpha1
+1./2.*alpha2).*a1.^2.*sin(alpha1).*cos(alpha1)-8.*sin(1./2.*
alpha1+1./2.*alpha2).*cos(1./2.*alpha1+1./2.*alpha2).*a2.^2.*
sin(alpha2).*cos(alpha2)-4.*alpha1.*a2.^2.*alpha2-4.*alpha2.*a1
.^2.*alpha1+4.*pi.*a1.^2.*alpha1+4.*pi.*a2.^2.*alpha2+2.*a1
.^2.*cos(alpha1).*cos(alpha2)+2.*a2.^2.*cos(alpha1).*cos(alpha2)
)+8.*alpha1.*a1.*alpha2.*a2-4.*a1.^2.*sin(alpha1).*cos(alpha2)
.*alpha1-4.*a2.^2.*sin(alpha2).*cos(alpha1).*alpha2-4.*pi.*a1
.^2.*sin(alpha1).*cos(alpha1)-4.*pi.*a2.^2.*sin(alpha2).*cos(
alpha2)+4.*alpha1.*a2.^2.*sin(alpha2).*cos(alpha2)+4.*alpha2.*
a1.^2.*sin(alpha1).*cos(alpha1)+8.*sin(1./2.*alpha1+1./2.*
alpha2).*cos(1./2.*alpha1+1./2.*alpha2).*a1.^2.*alpha1+8.*sin
(1./2.*alpha1+1./2.*alpha2).*cos(1./2.*alpha1+1./2.*alpha2).*a2
.^2.*alpha2+a2.^2.*cos(alpha2).^2-a2.^2.*cos(alpha2).^4-a1.^2.*
cos(alpha1).^4+a1.^2.*cos(alpha2).^2+a2.^2.*cos(alpha1).^2+a1
.^2.*cos(alpha1).^2-2.*a1.^2.*cos(alpha1).^3.*cos(alpha2)-2.*a2
.^2.*cos(alpha1).*cos(alpha2).^3-a1.^2.*cos(alpha2).^2.*cos(
alpha1).^2-a2.^2.*cos(alpha1).^2.*cos(alpha2).^2).^((1./2))./(pi
-alpha1-alpha2+2.*sin(1./2.*alpha1+1./2.*alpha2).*cos(1./2.*
alpha1+1./2.*alpha2));
47
48 %— Loop testing that the found roots are indeed solutions of the 3
eq.
49 nn=0; alpha2_sol=[]; d_sol=[]; R_sol=[]; % nn count the valid
solutions

```

```

50   for kk=1:length(racine) % test after calculating a,b,c,d for each
      root
51       alpha2=racine(kk); d=dd(kk); R=RR(kk);
52       a=2*(pi/2-(alpha1+alpha2)/2)+sin((alpha1+alpha2)/2)*cos((alpha1+
          alpha2)/2)); b=((a1*sin(alpha1)+a2*sin(alpha2))*(cos(alpha1
          )+cos(alpha2))-2*(alpha1*a1+alpha2*a2)); c=a1^2*(alpha1-
          sin(alpha1)*cos(alpha1))-a2^2*(alpha2-sin(alpha2)*cos(
          alpha2));
53       dD=(-tan((1./2).*alpha1).*a1+tan((1./2).*alpha2).*a2)./(-tan
          ((1./2).*alpha1)+tan((1./2).*alpha2));
54       if ((abs(R-((a1*(1-cos(alpha1))+a2*(1-cos(alpha2))+2*d)/(cos(
          alpha1)+cos(alpha2))))<=tol)&&(abs((2*d+2*a1)*tan(alpha1/2)
          -(2*d+2*a2)*tan(alpha2/2))<=tol) ...
55           &&(abs(R^2*a+R*b+c)<=tol)&&(alpha2>=0)&&(alpha2<=pi)&&(
          dD>0)) % Check eq. 3 bounds on alpha2 (0<=alpha2<=
          pi)
56       nn=nn+1;      alpha2_sol(nn)=alpha2; d_sol(nn) = dD;
57       R_sol(nn)=(1/2)*(-b+delta*a*sqrt(b.^2-4*a*c))./a;
58   end
59 end
60 %

```

```

61  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 2ND PART
62  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
63  %----- Solving eq. 1 for delta=1 (1st solution given by eq. 3 for R)

```

```

64   delta=1;
65   % Eqtot is eq. 1 using the expressions of a,b,c, d (eq. 2) & R (eq
      .3)
66   eqtot = @(alpha2) real(1/2*cos(alpha1)*a2-1/2*cos(alpha2)*a1+1/2*a2
      *cos(alpha1+alpha2)+1/8*a2*cos(3*alpha1+alpha2)-1/8*a2*cos(3*
      alpha1-alpha2)-1/2*a2*cos(alpha1-alpha2)-3/8*a2*cos(2*alpha1-2*
      alpha2)+3/8*a2*cos(3*alpha2+alpha1)-3/8*a2*cos(-3*alpha2+alpha1
      )+5/8*a2*cos(2*alpha1+2*alpha2)+alpha2*a2*sin(2*alpha1)+alpha2*
      a2*sin(alpha1-alpha2)+alpha2*a2*sin(alpha1+alpha2)-a2*alpha1*
      sin(alpha1-alpha2)+a2*alpha1*sin(2*alpha2)+a2*alpha1*sin(alpha1
      +alpha2)+1/8*a1-1/8*a2+2*a1*alpha2*sin(alpha1)+a2*pi*sin(alpha1
      )-a2*alpha1*sin(alpha1)-2*alpha2*a2*sin(alpha1)-2*a1*pi*sin(
      alpha1)+alpha1*a1*sin(alpha1)+2*a2*pi*sin(alpha2)-2*a2*alpha1*
      sin(alpha2)+a1*alpha2*sin(alpha2)-a1*pi*sin(alpha2)-alpha2*a2*
      sin(alpha2)+2*alpha1*a1*sin(alpha2)-1/4*a1*cos(alpha1)+1/4*a2*
      cos(alpha2)+1/2*a1*cos(alpha1-alpha2)-1/2*a1*cos(alpha1+alpha2)
      +3/8*a1*cos(2*alpha1-2*alpha2)-5/8*a1*cos(2*alpha1+2*alpha2)
      -1/8*delta*(8*a1*a2*cos(2*alpha1-2*alpha2)+6*a1^2+6*a2^2-64*
      alpha1*a2^2*alpha2-64*alpha2*a1^2*alpha1+64*pi*a1^2*alpha1+64*
      pi*a2^2*alpha2-8*a1*a2*cos(alpha1+alpha2)+4*a1*a2*cos(3*alpha1-
      alpha2)+8*a1*a2*cos(alpha1-alpha2)-4*a1*a2*cos(3*alpha1+alpha2)
      -4*a1*a2*cos(3*alpha2+alpha1)+32*alpha2*a1^2*sin(2*alpha1)-32*
      a1*alpha2*a2*sin(2*alpha1)-32*a1*alpha2*a2*sin(alpha1+alpha2)
      -32*a1*alpha2*a2*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin(alpha1+
      alpha2)+32*a2*alpha1*a1*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin
      (2*alpha2)-12*a1^2*cos(alpha1-alpha2)+12*a1^2*cos(3*alpha1+

```


$$\begin{aligned}
& \alpha_2) - 12a^2 \cos(\alpha_1 - \alpha_2) + 12a^2 \cos(3\alpha_2 + \alpha_1) \\
& - 2a^2 \cos(4\alpha_1) + 4a^2 \cos(2\alpha_2) - 4a^2 \cos(2\alpha_1) \\
& - 2a^2 \cos(2\alpha_1 - 2\alpha_2) - 2a^2 \cos(2\alpha_1 + 2\alpha_2) + 4a^2 \cos(2\alpha_1) \\
& - 4a^2 \cos(2\alpha_2) - 2a^2 \cos(2\alpha_1 - 2\alpha_2) - 2a^2 \cos(2\alpha_1 + 2\alpha_2) - 2a^2 \cos(4\alpha_2) - 32\pi a^2 \sin(2\alpha_1) \\
& - 8a^2 \cos(2\alpha_1 + 2\alpha_2) + 32a^2 \alpha_1 \sin(\alpha_1 + \alpha_2) - 32a^2 \alpha_1 \sin(\alpha_1 - \alpha_2) + 32a^2 \alpha_2 \sin(\alpha_1 + \alpha_2) \\
& + 32a^2 \alpha_2 \sin(\alpha_1 - \alpha_2) - 32\pi a^2 \sin(2\alpha_2) + 32\alpha_1 a^2 \sin(2\alpha_2) + 4a^2 \cos(\alpha_1 + \alpha_2) - 4a^2 \cos(3\alpha_1 - \alpha_2) \\
& + 4a^2 \cos(\alpha_1 + \alpha_2) - 4a^2 \cos(-3\alpha_2 + \alpha_1) + 4a^2 \cos(-3\alpha_2 + \alpha_1) + 128\alpha_1 a^2 \alpha_2 \sin(\alpha_1) - 1/8 \delta * (8a^2 \cos(2\alpha_1 - 2\alpha_2) + 6a^2 + 6a^2 - 64\alpha_1 a^2 \alpha_2 - 64\alpha_2 a^2 \alpha_1 + 64\pi a^2 \alpha_1 + 64\pi a^2 \alpha_2 - 8a^2 \cos(\alpha_1 + \alpha_2) + 4a^2 \cos(3\alpha_1 - \alpha_2) + 8a^2 \cos(\alpha_1 - \alpha_2) - 4a^2 \cos(3\alpha_1 + \alpha_2) - 4a^2 \cos(3\alpha_2 + \alpha_1) + 32\alpha_2 a^2 \sin(2\alpha_1) - 32a^2 \alpha_1 \sin(2\alpha_1) - 32a^2 \alpha_1 \sin(\alpha_1 + \alpha_2) - 32a^2 \alpha_1 \sin(\alpha_1 - \alpha_2) - 32a^2 \alpha_2 \sin(\alpha_1 + \alpha_2) + 32a^2 \alpha_2 \sin(\alpha_1 - \alpha_2) - 32a^2 \alpha_1 a^2 \sin(2\alpha_2) - 12a^2 \cos(\alpha_1 - \alpha_2) + 12a^2 \cos(3\alpha_1 + \alpha_2) - 12a^2 \cos(\alpha_1 - \alpha_2) + 12a^2 \cos(3\alpha_2 + \alpha_1) - 2a^2 \cos(4\alpha_1) + 4a^2 \cos(2\alpha_2) - 4a^2 \cos(2\alpha_1) - 2a^2 \cos(2\alpha_1 - 2\alpha_2) - 2a^2 \cos(2\alpha_1 + 2\alpha_2) + 4a^2 \cos(2\alpha_1) - 4a^2 \cos(2\alpha_2) - 2a^2 \cos(2\alpha_1 - 2\alpha_2) - 2a^2 \cos(2\alpha_1 + 2\alpha_2) - 2a^2 \cos(4\alpha_2) - 32\pi a^2 \sin(2\alpha_1) - 8a^2 \cos(2\alpha_1 + 2\alpha_2) + 32a^2 \alpha_1 \sin(\alpha_1 + \alpha_2) - 32a^2 \alpha_1 \sin(\alpha_1 - \alpha_2) + 32a^2 \alpha_2 \sin(\alpha_1 + \alpha_2) + 32a^2 \alpha_2 \sin(\alpha_1 - \alpha_2) - 32\pi a^2 \sin(2\alpha_2) + 32\alpha_1 a^2 \sin(2\alpha_2) + 4a^2 \cos(\alpha_1 + \alpha_2) - 4a^2 \cos(3\alpha_1 - \alpha_2) + 4a^2 \cos(\alpha_1 + \alpha_2) - 4a^2 \cos(-3\alpha_2 + \alpha_1) + 4a^2 \cos(-3\alpha_2 + \alpha_1) + 128\alpha_1 a^2 \alpha_2 \sin(\alpha_1 - 2\alpha_2) - \alpha_1 a^2 \sin(\alpha_1 - 2\alpha_2) - a^2 \pi \sin(\alpha_1 - 2\alpha_2) + a^2 \alpha_1 \sin(\alpha_1 - 2\alpha_2) - \alpha_2 a^2 \sin(2\alpha_1 - \alpha_2) + a^2 \cos(2\alpha_1 + \alpha_2) - 1/8 a^2 \cos(4\alpha_1 - \alpha_2) - 3/8 a^2 \cos(2\alpha_1 - \alpha_2) + 1/8 \delta * (8a^2 \cos(2\alpha_1 - 2\alpha_2) + 6a^2 + 6a^2 - 64\alpha_1 a^2 \alpha_2 - 64\alpha_2 a^2 \alpha_1 + 64\pi a^2 \alpha_1 + 64\pi a^2 \alpha_2 - 8a^2 \cos(\alpha_1 + \alpha_2) + 4a^2 \cos(3\alpha_1 - \alpha_2) + 8a^2 \cos(\alpha_1 - \alpha_2) - 4a^2 \cos(3\alpha_1 + \alpha_2) - 4a^2 \cos(3\alpha_2 + \alpha_1) + 32\alpha_2 a^2 \sin(2\alpha_1) - 32a^2 \alpha_1 \sin(2\alpha_1) - 32a^2 \alpha_1 \sin(\alpha_1 + \alpha_2) - 32a^2 \alpha_1 \sin(\alpha_1 - \alpha_2) - 32a^2 \alpha_2 \sin(\alpha_1 + \alpha_2) + 32a^2 \alpha_2 \sin(\alpha_1 - \alpha_2) - 32a^2 \alpha_1 a^2 \sin(2\alpha_2) - 12a^2 \cos(\alpha_1 - \alpha_2) + 12a^2 \cos(3\alpha_1 + \alpha_2) - 12a^2 \cos(\alpha_1 - \alpha_2) + 12a^2 \cos(3\alpha_2 + \alpha_1) - 2a^2 \cos(4\alpha_1) + 4a^2 \cos(2\alpha_2) - 4a^2 \cos(2\alpha_1) - 2a^2 \cos(2\alpha_1 - 2\alpha_2) - 2a^2 \cos(2\alpha_1 + 2\alpha_2) + 4a^2 \cos(2\alpha_1) - 4a^2 \cos(2\alpha_2) - 2a^2 \cos(2\alpha_1 - 2\alpha_2) - 2a^2 \cos(2\alpha_1 + 2\alpha_2) - 2a^2 \cos(4\alpha_2) - 32\pi a^2 \sin(2\alpha_1) - 8a^2 \cos(2\alpha_1 + 2\alpha_2) + 32a^2 \alpha_1 \sin(\alpha_1 + \alpha_2) - 32a^2 \alpha_1 \sin(\alpha_1 - \alpha_2) + 32a^2 \alpha_2 \sin(\alpha_1 + \alpha_2) + 32a^2 \alpha_2 \sin(\alpha_1 - \alpha_2) - 32\pi a^2 \sin(2\alpha_2) + 32\alpha_1 a^2 \sin(2\alpha_2) + 4a^2 \cos(\alpha_1 + \alpha_2) - 4a^2 \cos(3\alpha_1 - \alpha_2) + 4a^2 \cos(\alpha_1 + \alpha_2) - 4a^2 \cos(-3\alpha_2 + \alpha_1)
\end{aligned}$$

$$\begin{aligned}
& \alpha_1 - \alpha_2) - 32a_2\alpha_1a_1\sin(\alpha_1 + \alpha_2) + 32a_2\alpha_1a_1\sin(\alpha_1 - \alpha_2) - 32a_2\alpha_1a_1\sin(2\alpha_2) - 12a_1^2\cos(\alpha_1 - \alpha_2) \\
& + 12a_1^2\cos(3\alpha_1 + \alpha_2) - 12a_2^2\cos(\alpha_1 - \alpha_2) + 12a_2^2\cos(3\alpha_2 + \alpha_1) - 2a_1^2\cos(4\alpha_1) + 4a_1^2\cos(2\alpha_2) \\
& - 4a_1^2\cos(2\alpha_1) - 2a_1^2\cos(2\alpha_1 - 2\alpha_2) - 2a_1^2\cos(2\alpha_1 + 2\alpha_2) + 4a_2^2\cos(2\alpha_1) - 4a_2^2\cos(2\alpha_2) \\
& - 2a_2^2\cos(2\alpha_1 - 2\alpha_2) - 2a_2^2\cos(2\alpha_1 + 2\alpha_2) - 2a_2^2\cos(4\alpha_2) - 32\pi a_1^2\sin(2\alpha_1) - 8a_1a_2\cos(2\alpha_1 + 2\alpha_2) \\
& + 32a_1^2\alpha_1\sin(\alpha_1 + \alpha_2) - 32a_1^2\alpha_1\sin(\alpha_1 - \alpha_2) + 32a_2^2\alpha_2\sin(\alpha_1 + \alpha_2) + 32a_2^2\alpha_2\sin(\alpha_1 - \alpha_2) \\
& - 32\pi a_2^2\sin(2\alpha_2) + 32\alpha_1a_2^2\sin(2\alpha_2) + 4a_1^2\cos(\alpha_1 + \alpha_2) - 4a_1^2\cos(3\alpha_1 - \alpha_2) \\
& + 4a_2^2\cos(\alpha_1 + \alpha_2) - 4a_2^2\cos(-3\alpha_2 + \alpha_1) + 4a_1a_2\cos(-3\alpha_2 + \alpha_1) + 128\alpha_1a_1\alpha_2a_2^2)^{(1/2)}\sin(2\alpha_1) - a_1\alpha_2\sin(2\alpha_1) \\
& - \alpha_1a_1\sin(2\alpha_2) - 3/4a_2\cos(2\alpha_1 + \alpha_2) + a_1\pi\sin(2\alpha_1) + 3/4a_1\cos(\alpha_1 + 2\alpha_2) + 3/8a_1\cos(3\alpha_1 - \alpha_2) \\
& - 3/8a_1\cos(3\alpha_1 + \alpha_2) + 1/8a_1\cos(-3\alpha_2 + \alpha_1) - 1/8a_1\cos(3\alpha_2 + \alpha_1) - 1/8\delta a(8a_1a_2\cos(2\alpha_1 - 2\alpha_2) \\
& + 6a_1^2 + 6a_2^2 - 64\alpha_1a_2^2\alpha_2 - 64\alpha_2a_2^2a_1^2\alpha_1 + 64\pi a_1^2\alpha_1 + 64\pi a_2^2\alpha_2 - 8a_1a_2\cos(\alpha_1 + \alpha_2) \\
& + 4a_1a_2\cos(3\alpha_1 - \alpha_2) + 8a_1a_2\cos(\alpha_1 - \alpha_2) - 4a_1a_2\cos(3\alpha_1 + \alpha_2) - 4a_1a_2\cos(3\alpha_2 + \alpha_1) \\
& + 32\alpha_2a_1^2\sin(2\alpha_1) - 32a_1\alpha_2a_2^2\sin(2\alpha_1) - 32a_1\alpha_2a_2^2\sin(\alpha_1 + \alpha_2) - 32a_1\alpha_2a_2^2\sin(\alpha_1 - \alpha_2) \\
& - 32a_2\alpha_1a_1\sin(\alpha_1 + \alpha_2) + 32a_2\alpha_1a_1\sin(\alpha_1 - \alpha_2) - 32a_2\alpha_1a_1\sin(2\alpha_2) - 12a_1^2\cos(\alpha_1 - \alpha_2) \\
& + 12a_1^2\cos(3\alpha_1 + \alpha_2) - 12a_2^2\cos(\alpha_1 - \alpha_2) + 12a_2^2\cos(3\alpha_2 + \alpha_1) - 2a_1^2\cos(4\alpha_1) + 4a_1^2\cos(2\alpha_2) \\
& - 4a_1^2\cos(2\alpha_1) - 2a_1^2\cos(2\alpha_1 - 2\alpha_2) - 2a_1^2\cos(2\alpha_1 + 2\alpha_2) + 4a_2^2\cos(2\alpha_1) - 4a_2^2\cos(2\alpha_2) \\
& - 2a_2^2\cos(2\alpha_1 - 2\alpha_2) - 2a_2^2\cos(2\alpha_1 + 2\alpha_2) - 2a_2^2\cos(4\alpha_2) - 32\pi a_1^2\sin(2\alpha_1) - 8a_1a_2\cos(2\alpha_1 + 2\alpha_2) \\
& + 32a_1^2\alpha_1\sin(\alpha_1 + \alpha_2) - 32a_1^2\alpha_1\sin(\alpha_1 - \alpha_2) + 32a_2^2\alpha_2\sin(\alpha_1 + \alpha_2) + 32a_2^2\alpha_2\sin(\alpha_1 - \alpha_2) \\
& - 32\pi a_2^2\sin(2\alpha_2) + 32\alpha_1a_2^2\sin(2\alpha_2) + 4a_1^2\cos(\alpha_1 + \alpha_2) - 4a_1^2\cos(3\alpha_1 - \alpha_2) \\
& + 4a_2^2\cos(\alpha_1 + \alpha_2) - 4a_2^2\cos(-3\alpha_2 + \alpha_1) + 4a_1a_2\cos(-3\alpha_2 + \alpha_1) + 128\alpha_1a_1\alpha_2a_2^2)^{(1/2)}\sin(2\alpha_2) - a_2\pi\sin(2\alpha_2) \\
& + 1/4a_1\cos(2\alpha_2) - 1/4a_1\cos(2\alpha_1) - 1/4a_2\cos(2\alpha_1) + 1/4a_2\cos(2\alpha_2) - 1/8a_1\cos(2\alpha_1 - 3\alpha_2) \\
& + 1/8a_2\cos(3\alpha_1 - 2\alpha_2) - 1/2a_1\cos(\alpha_1 - 2\alpha_2) - 1/4a_1\cos(3\alpha_1 - 2\alpha_2) + 1/2a_2\cos(2\alpha_1 - \alpha_2) \\
& + 1/4a_2\cos(2\alpha_1 - 3\alpha_2) + 1/4a_1\cos(3\alpha_1) - 1/4a_2\cos(3\alpha_2) + 1/8a_1\cos(3\alpha_2) - 1/8a_2\cos(3\alpha_1) \\
& - 1/8a_2\cos(4\alpha_2) + 1/8a_1\cos(4\alpha_1));
\end{aligned}$$

67

68 guess=linspace(0,pi,20);

69 racine2=[]; % will allow to discard solutions too closed from 0

70 options=optimset('Display','off','FunValCheck','on','TolFun',1e-14,'TolX',1e-14,'MaxFunEvals',1e4,'MaxIter',1e4);

71 for kh=1:length(guess) % Solving with fzero for each starting guess

72 [x,fval,exitflag] = fzero(eqtot,guess(kh),options);

73 if(exitflag==1)

74 flagij=0;

```

75         for kkh=1:length(racine2)
76             if (abs(x-racine2(kkh))<=1e-10), flagij=1; end;
77         end
78         if (flagij==0), racine2=[racine2 x]; end;
79     end
80 end
81 alpha2=racine2;
82 dd2 = -(-tan((1./2).*alpha1).*a1+tan((1./2).*alpha2).*a2)./(-tan
    ((1./2).*alpha1)+tan((1./2).*alpha2));
83 RR2=1./2.*(-a1.*cos(alpha1).*sin(alpha1)-a1.*sin(alpha1).*cos(
    alpha2)-a2.*sin(alpha2).*cos(alpha1)-a2.*cos(alpha2).*sin(
    alpha2)+2.*alpha1.*a1+2.*alpha2.*a2+delta.*(4.*a1.*cos(alpha1)
    .*sin(alpha1).*a2.*cos(alpha2).*sin(alpha2)+2.*a1.*cos(alpha1)
    .*^2.*sin(alpha1).*a2.*sin(alpha2)-4.*a1.*cos(alpha1).*sin(
    alpha1).*alpha2.*a2+2.*a1.*sin(alpha1).*cos(alpha2).^2.*a2.*sin
    (alpha2)-4.*a1.*sin(alpha1).*cos(alpha2).*alpha2.*a2-4.*a2.*sin
    (alpha2).*cos(alpha1).*alpha1.*a1-4.*a2.*cos(alpha2).*sin(
    alpha2).*alpha1.*a1-8.*sin(1./2.*alpha1+1./2.*alpha2).*cos
    (1./2.*alpha1+1./2.*alpha2).*a1.^2.*sin(alpha1).*cos(alpha1)
    -8.*sin(1./2.*alpha1+1./2.*alpha2).*cos(1./2.*alpha1+1./2.*
    alpha2).*a2.^2.*sin(alpha2).*cos(alpha2)-4.*alpha1.*a2.^2.*
    alpha2-4.*alpha2.*a1.^2.*alpha1+4.*pi.*a1.^2.*alpha1+4.*pi.*a2
    .*^2.*alpha2+2.*a1.^2.*cos(alpha1).*cos(alpha2)+2.*a2.^2.*cos(
    alpha1).*cos(alpha2)+8.*alpha1.*a1.*alpha2.*a2-4.*a1.^2.*sin(
    alpha1).*cos(alpha2).*alpha1-4.*a2.^2.*sin(alpha2).*cos(alpha1)
    .*alpha2-4.*pi.*a1.^2.*sin(alpha1).*cos(alpha1)-4.*pi.*a2.^2.*
    sin(alpha2).*cos(alpha2)+4.*alpha1.*a2.^2.*sin(alpha2).*cos(
    alpha2)+4.*alpha2.*a1.^2.*sin(alpha1).*cos(alpha1)+8.*sin
    (1./2.*alpha1+1./2.*alpha2).*cos(1./2.*alpha1+1./2.*alpha2).*a1
    .*^2.*alpha1+8.*sin(1./2.*alpha1+1./2.*alpha2).*cos(1./2.*alpha1
    +1./2.*alpha2).*a2.^2.*alpha2+a2.^2.*cos(alpha2).^2-a2.^2.*cos(
    alpha2).^4-a1.^2.*cos(alpha1).^4+a1.^2.*cos(alpha2).^2+a2.^2.*
    cos(alpha1).^2+a1.^2.*cos(alpha1).^2-2.*a1.^2.*cos(alpha1).^3.*
    cos(alpha2)-2.*a2.^2.*cos(alpha1).*cos(alpha2).^3-a1.^2.*cos(
    alpha2).^2.*cos(alpha1).^2-a2.^2.*cos(alpha1).^2.*cos(alpha2)
    .*^2).^2)./(pi-alpha1-alpha2+2.*sin(1./2.*alpha1+1./2.*
    alpha2).*cos(1./2.*alpha1+1./2.*alpha2));
84
85 %— Loop testing that the found roots are indeed solutions of the 3
    eq.
86 nn2=0; alpha2_sol2=[]; d_sol2=[]; R_sol2=[];
87 for kk=1:length(racine2)% test after calculating a,b,c,d for each
    root
88     alpha2=racine2(kk); d=dd2(kk); R=RR2(kk);
89     a=2*(pi/2-(alpha1+alpha2)/2+sin((alpha1+alpha2)/2)*cos((alpha1+
        alpha2)/2)); b=((a1*sin(alpha1)+a2*sin(alpha2))*(cos(alpha1)
        )+cos(alpha2))-2*(alpha1*a1+alpha2*a2)); c=a1^2*(alpha1-
        sin(alpha1)*cos(alpha1))-a2^2*(alpha2-sin(alpha2)*cos(
        alpha2));
90     dD=-(-tan((1./2).*alpha1).*a1+tan((1./2).*alpha2).*a2)./(-tan
        ((1./2).*alpha1)+tan((1./2).*alpha2));
91     if ((abs(R-((a1*(1-cos(alpha1))+a2*(1-cos(alpha2))+2*d)/(cos(
        alpha1)+cos(alpha2))))<=tol)&&(abs((2*d+2*a1)*tan(alpha1/2)
        -(2*d+2*a2)*tan(alpha2/2))<=tol) ...
92         &&(abs(R^2*a+R*b+c)<=tol)&&(alpha2>=0)&&(alpha2<=pi)&&(
            dD>=0)) % Check eq. 83 bounds on alpha2 (0<=alpha2<=

```

```

93          $\pi$ )
           nn2=nn2+1;           alpha2_sol2(nn2)=alpha2; d_sol2(nn2) = dD
           ;
94         R_sol2(nn2)=(1/2)*(-b+delta*sqrt(b.^2-4*a*c))./a;
95     end
96 end
97 %

```

```

98 % FINAL SOLUTIONS FOR alpha2, d and R
99 clear alpha2 d R
100 alpha2=[alpha2_sol alpha2_sol2]; d=[d_sol d_sol2]; R=[R_sol R_sol2
    ];
101 end

```

Cross_Section_Area.m

```

1 %
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

2 %%%%%%%%%% Calculation of the cross-section area
   %%%%%%%%%%
3 %%%%%%%%%% as a function of alpha1 (wrapping angle)
   %%%%%%%%%%
4 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
5
6 %
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

7 % AUTHORS: D. CEBRON, A. SAURET
8 % DATE: 31/03/2015
9 % DESCRIPTION: Calculation of the cross-section area as a function of
10 % alpha1 (wrapping angle)
11 %
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

12
13 close all
14
15 dd=d(1,:); %%% Inter-fiber distance
16 RR=R(1,:); %%% Radius of curvature
17
18 %%% Delete NaN points
19 alpha1=alpha1_tab;
20 RR(isnan(dd))=[];
21 alpha1(isnan(dd))=[];
22 dd(isnan(dd))=[];
23
24 %%% Calculate wrapping angle alpha2 on the small fiber
25 alpha2=2.*atan((2.*dd+2*a1)./(2.*dd+2*a2).*tan(alpha1./2));
26
27 %%% Calculate the cross-section area
28 Area=(a1.*sin(alpha1)+a2.*sin(alpha2)).*(a1.*(1-cos(alpha1))+a2.*(1-cos
   (alpha2))+2.*dd)-a1.^2.*(alpha1-sin(alpha1).*cos(alpha1))-a2.^2.*(
   alpha2-sin(alpha2).*cos(alpha2))-2.*RR.^2.*(pi./2-sin((alpha1+
   alpha2)./2).*cos((alpha1+alpha2)./2)-(alpha1+alpha2)./2);
29
30 %%% Plot the dimensionless area A/a1^2 as a function of the
   dimensionless
31 %%% inter-fiber distance d/a1
32 plot(dd./a1,Area./a1.^2)

```

Plot_Cross_Section.m

```

1 %
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

2 % AUTHORS: D. CEBRON, A. SAURET
3 % DATE: 31/03/2015
4 % DESCRIPTION: Plot the cross-section of a column
5 %
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

6
7 %%%% !! Require circle.m !! %%%%
8
9 close all
10
11 %%%% Collect the values of the wrapping angles, the interfiber distance
    and
12 %%%% the radius of curvature
13 clear alpha1;
14 alpha1=alpha1_tab;
15 alpha2=alpha2(1,:);
16 d=d(1,:);
17 R=R(1,:);
18
19 NN=size(d,2)-1; %%% Position to plot the cross-section
20 d(NN)./a1 %%% Value of the dimensionless inter-fiber distance
21
22 %%%%%%%%% Plot the cross-section using the radius of curvature
23 circle((a1+R(NN)).*cos(alpha1(NN)),-(a1+R(NN)).*sin(alpha1(NN)),R(NN))
24
25 hold on
26 circle((a1+R(NN)).*cos(alpha1(NN)),(a1+R(NN)).*sin(alpha1(NN)),R(NN))
27 hold on
28
29 %%%%%%%%% Plot the fibers %%%%%%%%%
30 circle(0,0,a1); %%% Largest fiber
31 hold on
32 circle(a1+2.*d(NN)+a2,0,a2); %%% Smallest fibers
33
34
35 axis equal

```