Supplementary material: Matlab code

This paper's supplementary materials include the Matlab codes that solve the system of equations (6), (8) and (14) for a perfectly wetting fluid ($\theta_e = 0^o$).

The input parameters to set are the radius of the large and the small fiber, a_1 and a_2 respectively.

- *wet_prog.m* is the program to run to compute the evolution of the wrapping angles α_1 and α_2, the inter-fiber distance d and the radius of curvature R. The value of a_1 and a_2 can be modified in this code.
- *equation_wet.m* contains the system of equation to be solve. This program is used by the main program (wet_prog.m).
- *Cross_Section_Area.m* calculate and plot the evolution of the cross-section area $\tilde{A}$.
- *Plot_Cross_Section.m* plot the shape of the cross-section. It requires the presence of the code circle.m to run.

Wet_Prog.m

```matlab
clear all; clc;

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% AUTHORS: D. CEBRON, A. SAURET
% DATE: 31/03/2015
% DESCRIPTION: solve the 3 coupled agebraic eq. of Sauret et al. (2015)
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%% Geometrical Parameters %%%%%%
%---- Radius of fiber 1 (in meter)
a1=2.25e-04;
%---- Radius of fiber 2 (in meter)
a2=0.5e-04;

Nalpha1=500; %%%% Number of point to solve
alpha1_tab=linspace(0,pi,Nalpha1);


%---- Solve equation_wet by varying the value of alpha1 between 0 and pi
NN=10; alpha2=zeros(NN,Nalpha1)*NaN; d=zeros(NN,Nalpha1)*NaN; R=zeros(NN,Nalpha1)*NaN; Nsol=zeros(Nalpha1,1);
for k=1:length(alpha1_tab);
    alpha1=alpha1_tab(k);
    [alpha2s,ds,Rs]=equation_wet(a1,a2,alpha1);
    Nsol(k)=length(alpha2s);
    if(isempty(alpha2s)==0)
        alpha2(1:length(alpha2s),k)=alpha2s;  d(1:length(alpha2s),k)=ds; R(1:length(alpha2s),k)=Rs;
    end
end


%---- Plot the evolution of alpha2 (wrapping angle around the small fiber)
%---- of the interfiber distance d
%---- of the radius of curvature R
%---- as a function of alpha1 (wrapping angle around the large fiber)

figure(1);
subplot(3,1,1);
plot(alpha1_tab.*180/pi,alpha2(1,:).*180/pi,'-r','Linewidth',2); hold on; % sol 1
xlabel('\phi_1'); ylabel('\phi_2 (in degree)');
subplot(3,1,2);
plot(alpha1_tab.*180/pi,d(1,:),'-r','Linewidth',2); hold on;
xlabel('\phi_1 (in degree)'); ylabel('d','Linewidth',2);
subplot(3,1,3);
plot(alpha1_tab.*180/pi,R(1,:),'-r','Linewidth',2); hold on;
xlabel('\phi_1 (in degree)'); ylabel('R');
```

equation_wet.m

```
function [alpha2,d,R]=equation_wet(a1,a2,alpha1)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% AUTHORS: D. CEBRON, A. SAURET
% DATE: 31/03/2015
% DESCRIPTION: solve the 3 coupled agebraic eq. of Sauret et al. (2015)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% EQUATIONS: with K=cos(alpha1)+cos(alpha2), the 3 eq. are:
% eq. 1: R-((a1*(1-cos(alpha1))+a2*(1-cos(alpha2))+2*d)/K)=0
% eq. 2: (2*d+2*a1)*tan(alpha1/2)-(2*d+2*a2)*tan(alpha2/2)=0
% eq. 3: R^2*a+R*b+c=0      with (a,b,c) defined in Sauret et al. (2015)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% METHOD: eq. 2 gives d, and eq. 3 gives the solution
% R = (1/2)*(-b+delta*sqrt(b^2-4*a*c))/a    for delta=-1 & delta=1
% which are used to solve eq. 1 (for delta=-1 first and then for delta=1)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
tol=1e-14; % Tolerance allowed in the solution

if(alpha1==0)                    %%% Obvious solution for alpha1=0
    alpha2=0; d=0; R=0;
elseif(a1==a2)                   %%% Obvious solution for a1=a2
    f=2.*alpha1-sin(2.*alpha1);
    R=(sqrt(pi./f)-1).^(-1);     %%% Radius of curvature
    d=cos(phi).*(1+R)-1;         %%% interfiber distance
else
    %%%%%%%%%%%%%%%%%%%%%%%%  1ST PART  %%%%%%%%%%%%%%%%%%%%%%%%%%%
    %---- Solving eq. 1 for delta=-1 (1st solution given by eq. 3 for R) --
    %-------------------------------------------------------------------------
    delta=-1;
    % Eqtot is eq. 1 using the expressions of a,b,c, d (eq. 2) & R (eq.3)
    eqtot = @(alpha2) real(1/2*cos(alpha1)*a2-1/2*cos(alpha2)*a1+1/2*a2*cos(alpha1+alpha2)+1/8*a2*cos(3*alpha1+alpha2)-1/8*a2*cos(3*alpha1-alpha2)-1/2*a2*cos(alpha1-alpha2)-3/8*a2*cos(2*alpha1-2*alpha2)+3/8*a2*cos(3*alpha2+alpha1)-3/8*a2*cos(-3*alpha2+alpha1)+5/8*a2*cos(2*alpha1+2*alpha2)+alpha2*a2*sin(2*alpha1)+alpha2*a2*sin(alpha1-alpha2)+alpha2*a2*sin(alpha1+alpha2)-a2*alpha1*sin(alpha1-alpha2)+a2*alpha1*sin(2*alpha2)+a2*alpha1*sin(alpha1+alpha2)+1/8*a1-1/8*a2+2*a1*alpha2*sin(alpha1)+a2*pi*sin(alpha1)-a2*alpha1*sin(alpha1)-2*alpha2*a2*sin(alpha1)-2*a1*pi*sin(alpha1)+alpha1*a1*sin(alpha1)+2*a2*pi*sin(alpha2)-2*a2*alpha1*
```

```
sin(alpha2)+a1*alpha2*sin(alpha2)-a1*pi*sin(alpha2)-alpha2*a2*
sin(alpha2)+2*alpha1*a1*sin(alpha2)-1/4*a1*cos(alpha1)+1/4*a2*
cos(alpha2)+1/2*a1*cos(alpha1-alpha2)-1/2*a1*cos(alpha1+alpha2)
+3/8*a1*cos(2*alpha1-2*alpha2)-5/8*a1*cos(2*alpha1+2*alpha2)
-1/8*delta*(8*a1*a2*cos(2*alpha1-2*alpha2)+6*a1^2+6*a2^2-64*
alpha1*a2^2*alpha2-64*alpha2*a1^2*alpha1+64*pi*a1^2*alpha1+64*
pi*a2^2*alpha2-8*a1*a2*cos(alpha1+alpha2)+4*a1*a2*cos(3*alpha1-
alpha2)+8*a1*a2*cos(alpha1-alpha2)-4*a1*a2*cos(3*alpha1+alpha2)
-4*a1*a2*cos(3*alpha2+alpha1)+32*alpha2*a1^2*sin(2*alpha1)-32*
a1*alpha2*a2*sin(2*alpha1)-32*a1*alpha2*a2*sin(alpha1+alpha2)
-32*a1*alpha2*a2*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin(alpha1+
alpha2)+32*a2*alpha1*a1*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin
(2*alpha2)-12*a1^2*cos(alpha1-alpha2)+12*a1^2*cos(3*alpha1+
alpha2)-12*a2^2*cos(alpha1-alpha2)+12*a2^2*cos(3*alpha2+alpha1)
-2*a1^2*cos(4*alpha1)+4*a1^2*cos(2*alpha2)-4*a1^2*cos(2*alpha1)
-2*a1^2*cos(2*alpha1-2*alpha2)-2*a1^2*cos(2*alpha1+2*alpha2)+4*
a2^2*cos(2*alpha1)-4*a2^2*cos(2*alpha2)-2*a2^2*cos(2*alpha1-2*
alpha2)-2*a2^2*cos(2*alpha1+2*alpha2)-2*a2^2*cos(4*alpha2)-32*
pi*a1^2*sin(2*alpha1)-8*a1*a2*cos(2*alpha1+2*alpha2)+32*a1^2*
alpha1*sin(alpha1+alpha2)-32*a1^2*alpha1*sin(alpha1-alpha2)+32*
a2^2*alpha2*sin(alpha1+alpha2)+32*a2^2*alpha2*sin(alpha1-alpha2
)-32*pi*a2^2*sin(2*alpha2)+32*alpha1*a2^2*sin(2*alpha2)+4*a1^2*
cos(alpha1+alpha2)-4*a1^2*cos(3*alpha1-alpha2)+4*a2^2*cos(
alpha1+alpha2)-4*a2^2*cos(-3*alpha2+alpha1)+4*a1*a2*cos(-3*
alpha2+alpha1)+128*alpha1*a1*alpha2*a2)^(1/2)*sin(alpha1)-1/8*
delta*(8*a1*a2*cos(2*alpha1-2*alpha2)+6*a1^2+6*a2^2-64*alpha1*
a2^2*alpha2-64*alpha2*a1^2*alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*
alpha2-8*a1*a2*cos(alpha1+alpha2)+4*a1*a2*cos(3*alpha1-alpha2)
+8*a1*a2*cos(alpha1-alpha2)-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*
a2*cos(3*alpha2+alpha1)+32*alpha2*a1^2*sin(2*alpha1)-32*a1*
alpha2*a2*sin(2*alpha1)-32*a1*alpha2*a2*sin(alpha1+alpha2)-32*
a1*alpha2*a2*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin(alpha1+
alpha2)+32*a2*alpha1*a1*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin
(2*alpha2)-12*a1^2*cos(alpha1-alpha2)+12*a1^2*cos(3*alpha1+
alpha2)-12*a2^2*cos(alpha1-alpha2)+12*a2^2*cos(3*alpha2+alpha1)
-2*a1^2*cos(4*alpha1)+4*a1^2*cos(2*alpha2)-4*a1^2*cos(2*alpha1)
-2*a1^2*cos(2*alpha1-2*alpha2)-2*a1^2*cos(2*alpha1+2*alpha2)+4*
a2^2*cos(2*alpha1)-4*a2^2*cos(2*alpha2)-2*a2^2*cos(2*alpha1-2*
alpha2)-2*a2^2*cos(2*alpha1+2*alpha2)-2*a2^2*cos(4*alpha2)-32*
pi*a1^2*sin(2*alpha1)-8*a1*a2*cos(2*alpha1+2*alpha2)+32*a1^2*
alpha1*sin(alpha1+alpha2)-32*a1^2*alpha1*sin(alpha1-alpha2)+32*
a2^2*alpha2*sin(alpha1+alpha2)+32*a2^2*alpha2*sin(alpha1-alpha2
)-32*pi*a2^2*sin(2*alpha2)+32*alpha1*a2^2*sin(2*alpha2)+4*a1^2*
cos(alpha1+alpha2)-4*a1^2*cos(3*alpha1-alpha2)+4*a2^2*cos(
alpha1+alpha2)-4*a2^2*cos(-3*alpha2+alpha1)+4*a1*a2*cos(-3*
alpha2+alpha1)+128*alpha1*a1*alpha2*a2)^(1/2)*sin(alpha1-2*
alpha2)-alpha1*a1*sin(alpha1-2*alpha2)-a2*pi*sin(alpha1-2*
alpha2)+a2*alpha1*sin(alpha1-2*alpha2)-alpha2*a2*sin(2*alpha1-
alpha2)+a1*cos(2*alpha1+alpha2)-1/8*a1*cos(4*alpha1-alpha2)
-3/8*a1*cos(2*alpha1-alpha2)+1/8*delta*(8*a1*a2*cos(2*alpha1-2*
alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*a1^2*
alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*alpha2-8*a1*a2*cos(alpha1+
alpha2)+4*a1*a2*cos(3*alpha1-alpha2)+8*a1*a2*cos(alpha1-alpha2)
-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*a2*cos(3*alpha2+alpha1)+32*
alpha2*a1^2*sin(2*alpha1)-32*a1*alpha2*a2*sin(2*alpha1)-32*a1*
alpha2*a2*sin(alpha1+alpha2)-32*a1*alpha2*a2*sin(alpha1-alpha2)
```

```
-32*a2*alpha1*a1*sin(alpha1+alpha2)+32*a2*alpha1*a1*sin(alpha1-
alpha2)-32*a2*alpha1*a1*sin(2*alpha2)-12*a1^2*cos(alpha1-alpha2
)+12*a1^2*cos(3*alpha1+alpha2)-12*a2^2*cos(alpha1-alpha2)+12*a2
^2*cos(3*alpha2+alpha1)-2*a1^2*cos(4*alpha1)+4*a1^2*cos(2*
alpha2)-4*a1^2*cos(2*alpha1)-2*a1^2*cos(2*alpha1-2*alpha2)-2*a1
^2*cos(2*alpha1+2*alpha2)+4*a2^2*cos(2*alpha1)-4*a2^2*cos(2*
alpha2)-2*a2^2*cos(2*alpha1-2*alpha2)-2*a2^2*cos(2*alpha1+2*
alpha2)-2*a2^2*cos(4*alpha2)-32*pi*a1^2*sin(2*alpha1)-8*a1*a2*
cos(2*alpha1+2*alpha2)+32*a1^2*alpha1*sin(alpha1+alpha2)-32*a1
^2*alpha1*sin(alpha1-alpha2)+32*a2^2*alpha2*sin(alpha1+alpha2)
+32*a2^2*alpha2*sin(alpha1-alpha2)-32*pi*a2^2*sin(2*alpha2)+32*
alpha1*a2^2*sin(2*alpha2)+4*a1^2*cos(alpha1+alpha2)-4*a1^2*cos
(3*alpha1-alpha2)+4*a2^2*cos(alpha1+alpha2)-4*a2^2*cos(-3*
alpha2+alpha1)+4*a1*a2*cos(-3*alpha2+alpha1)+128*alpha1*a1*
alpha2*a2)^(1/2)*sin(alpha2)-1/8*delta*(8*a1*a2*cos(2*alpha1-2*
alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*a1^2*
alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*alpha2-8*a1*a2*cos(alpha1+
alpha2)+4*a1*a2*cos(3*alpha1-alpha2)+8*a1*a2*cos(alpha1-alpha2)
-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*a2*cos(3*alpha2+alpha1)+32*
alpha2*a1^2*sin(2*alpha1)-32*a1*alpha2*a2*sin(2*alpha1)-32*a1*
alpha2*a2*sin(alpha1+alpha2)-32*a1*alpha2*a2*sin(alpha1-alpha2)
-32*a2*alpha1*a1*sin(alpha1+alpha2)+32*a2*alpha1*a1*sin(alpha1-
alpha2)-32*a2*alpha1*a1*sin(2*alpha2)-12*a1^2*cos(alpha1-alpha2
)+12*a1^2*cos(3*alpha1+alpha2)-12*a2^2*cos(alpha1-alpha2)+12*a2
^2*cos(3*alpha2+alpha1)-2*a1^2*cos(4*alpha1)+4*a1^2*cos(2*
alpha2)-4*a1^2*cos(2*alpha1)-2*a1^2*cos(2*alpha1-2*alpha2)-2*a1
^2*cos(2*alpha1+2*alpha2)+4*a2^2*cos(2*alpha1)-4*a2^2*cos(2*
alpha2)-2*a2^2*cos(2*alpha1-2*alpha2)-2*a2^2*cos(2*alpha1+2*
alpha2)-2*a2^2*cos(4*alpha2)-32*pi*a1^2*sin(2*alpha1)-8*a1*a2*
cos(2*alpha1+2*alpha2)+32*a1^2*alpha1*sin(alpha1+alpha2)-32*a1
^2*alpha1*sin(alpha1-alpha2)+32*a2^2*alpha2*sin(alpha1+alpha2)
+32*a2^2*alpha2*sin(alpha1-alpha2)-32*pi*a2^2*sin(2*alpha2)+32*
alpha1*a2^2*sin(2*alpha2)+4*a1^2*cos(alpha1+alpha2)-4*a1^2*cos
(3*alpha1-alpha2)+4*a2^2*cos(alpha1+alpha2)-4*a2^2*cos(-3*
alpha2+alpha1)+4*a1*a2*cos(-3*alpha2+alpha1)+128*alpha1*a1*
alpha2*a2)^(1/2)*sin(2*alpha1-alpha2)-a1*pi*sin(2*alpha1-alpha2
)+a1*alpha2*sin(2*alpha1-alpha2)-a1*alpha2*sin(alpha1+alpha2)-
a1*alpha2*sin(alpha1-alpha2)+a1*pi*sin(alpha1+alpha2)+a1*pi*sin
(alpha1-alpha2)-a2*pi*sin(alpha1+alpha2)+a2*pi*sin(alpha1-
alpha2)-alpha1*a1*sin(alpha1+alpha2)+alpha1*a1*sin(alpha1-
alpha2)+1/8*a2*cos(-4*alpha2+alpha1)-a2*cos(alpha1+2*alpha2)
+3/8*a2*cos(alpha1-2*alpha2)+1/4*delta*(8*a1*a2*cos(2*alpha1-2*
alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*a1^2*
alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*alpha2-8*a1*a2*cos(alpha1+
alpha2)+4*a1*a2*cos(3*alpha1-alpha2)+8*a1*a2*cos(alpha1-alpha2)
-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*a2*cos(3*alpha2+alpha1)+32*
alpha2*a1^2*sin(2*alpha1)-32*a1*alpha2*a2*sin(2*alpha1)-32*a1*
alpha2*a2*sin(alpha1+alpha2)-32*a1*alpha2*a2*sin(alpha1-alpha2)
-32*a2*alpha1*a1*sin(alpha1+alpha2)+32*a2*alpha1*a1*sin(alpha1-
alpha2)-32*a2*alpha1*a1*sin(2*alpha2)-12*a1^2*cos(alpha1-alpha2
)+12*a1^2*cos(3*alpha1+alpha2)-12*a2^2*cos(alpha1-alpha2)+12*a2
^2*cos(3*alpha2+alpha1)-2*a1^2*cos(4*alpha1)+4*a1^2*cos(2*
alpha2)-4*a1^2*cos(2*alpha1)-2*a1^2*cos(2*alpha1-2*alpha2)-2*a1
^2*cos(2*alpha1+2*alpha2)+4*a2^2*cos(2*alpha1)-4*a2^2*cos(2*
alpha2)-2*a2^2*cos(2*alpha1-2*alpha2)-2*a2^2*cos(2*alpha1+2*
alpha2)-2*a2^2*cos(4*alpha2)-32*pi*a1^2*sin(2*alpha1)-8*a1*a2*
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cos(2*alpha1+2*alpha2)+32*a1^2*alpha1*sin(alpha1+alpha2)-32*a1
^2*alpha1*sin(alpha1-alpha2)+32*a2^2*alpha2*sin(alpha1+alpha2)
+32*a2^2*alpha2*sin(alpha1-alpha2)-32*pi*a2^2*sin(2*alpha2)+32*
alpha1*a2^2*sin(2*alpha2)+4*a1^2*cos(alpha1+alpha2)-4*a1^2*cos
(3*alpha1-alpha2)+4*a2^2*cos(alpha1+alpha2)-4*a2^2*cos(-3*
alpha2+alpha1)+4*a1*a2*cos(-3*alpha2+alpha1)+128*alpha1*a1*
alpha2*a2)^(1/2)*sin(alpha1-alpha2)+1/8*delta*(8*a1*a2*cos(2*
alpha1-2*alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*
a1^2*alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*alpha2-8*a1*a2*cos(
alpha1+alpha2)+4*a1*a2*cos(3*alpha1-alpha2)+8*a1*a2*cos(alpha1-
alpha2)-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*a2*cos(3*alpha2+
alpha1)+32*alpha2*a1^2*sin(2*alpha1)-32*a1*alpha2*a2*sin(2*
alpha1)-32*a1*alpha2*a2*sin(alpha1+alpha2)-32*a1*alpha2*a2*sin(
alpha1-alpha2)-32*a2*alpha1*a1*sin(alpha1+alpha2)+32*a2*alpha1*
a1*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin(2*alpha2)-12*a1^2*cos
(alpha1-alpha2)+12*a1^2*cos(3*alpha1+alpha2)-12*a2^2*cos(alpha1
-alpha2)+12*a2^2*cos(3*alpha2+alpha1)-2*a1^2*cos(4*alpha1)+4*a1
^2*cos(2*alpha2)-4*a1^2*cos(2*alpha1)-2*a1^2*cos(2*alpha1-2*
alpha2)-2*a1^2*cos(2*alpha1+2*alpha2)+4*a2^2*cos(2*alpha1)-4*a2
^2*cos(2*alpha2)-2*a2^2*cos(2*alpha1-2*alpha2)-2*a2^2*cos(2*
alpha1+2*alpha2)-2*a2^2*cos(4*alpha2)-32*pi*a1^2*sin(2*alpha1)
-8*a1*a2*cos(2*alpha1+2*alpha2)+32*a1^2*alpha1*sin(alpha1+
alpha2)-32*a1^2*alpha1*sin(alpha1-alpha2)+32*a2^2*alpha2*sin(
alpha1+alpha2)+32*a2^2*alpha2*sin(alpha1-alpha2)-32*pi*a2^2*sin
(2*alpha2)+32*alpha1*a2^2*sin(2*alpha2)+4*a1^2*cos(alpha1+
alpha2)-4*a1^2*cos(3*alpha1-alpha2)+4*a2^2*cos(alpha1+alpha2)
-4*a2^2*cos(-3*alpha2+alpha1)+4*a1*a2*cos(-3*alpha2+alpha1)
+128*alpha1*a1*alpha2*a2)^(1/2)*sin(2*alpha1)-a1*alpha2*sin(2*
alpha1)-alpha1*a1*sin(2*alpha2)-3/4*a2*cos(2*alpha1+alpha2)+a1*
pi*sin(2*alpha1)+3/4*a1*cos(alpha1+2*alpha2)+3/8*a1*cos(3*
alpha1-alpha2)-3/8*a1*cos(3*alpha1+alpha2)+1/8*a1*cos(-3*alpha2
+alpha1)-1/8*a1*cos(3*alpha2+alpha1)-1/8*delta*(8*a1*a2*cos(2*
alpha1-2*alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*
a1^2*alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*alpha2-8*a1*a2*cos(
alpha1+alpha2)+4*a1*a2*cos(3*alpha1-alpha2)+8*a1*a2*cos(alpha1-
alpha2)-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*a2*cos(3*alpha2+
alpha1)+32*alpha2*a1^2*sin(2*alpha1)-32*a1*alpha2*a2*sin(2*
alpha1)-32*a1*alpha2*a2*sin(alpha1+alpha2)-32*a1*alpha2*a2*sin(
alpha1-alpha2)-32*a2*alpha1*a1*sin(alpha1+alpha2)+32*a2*alpha1*
a1*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin(2*alpha2)-12*a1^2*cos
(alpha1-alpha2)+12*a1^2*cos(3*alpha1+alpha2)-12*a2^2*cos(alpha1
-alpha2)+12*a2^2*cos(3*alpha2+alpha1)-2*a1^2*cos(4*alpha1)+4*a1
^2*cos(2*alpha2)-4*a1^2*cos(2*alpha1)-2*a1^2*cos(2*alpha1-2*
alpha2)-2*a1^2*cos(2*alpha1+2*alpha2)+4*a2^2*cos(2*alpha1)-4*a2
^2*cos(2*alpha2)-2*a2^2*cos(2*alpha1-2*alpha2)-2*a2^2*cos(2*
alpha1+2*alpha2)-2*a2^2*cos(4*alpha2)-32*pi*a1^2*sin(2*alpha1)
-8*a1*a2*cos(2*alpha1+2*alpha2)+32*a1^2*alpha1*sin(alpha1+
alpha2)-32*a1^2*alpha1*sin(alpha1-alpha2)+32*a2^2*alpha2*sin(
alpha1+alpha2)+32*a2^2*alpha2*sin(alpha1-alpha2)-32*pi*a2^2*sin
(2*alpha2)+32*alpha1*a2^2*sin(2*alpha2)+4*a1^2*cos(alpha1+
alpha2)-4*a1^2*cos(3*alpha1-alpha2)+4*a2^2*cos(alpha1+alpha2)
-4*a2^2*cos(-3*alpha2+alpha1)+4*a1*a2*cos(-3*alpha2+alpha1)
+128*alpha1*a1*alpha2*a2)^(1/2)*sin(2*alpha2)-a2*pi*sin(2*
alpha2)+1/4*a1*cos(2*alpha2)-1/4*a1*cos(2*alpha1)-1/4*a2*cos(2*
alpha1)+1/4*a2*cos(2*alpha2)-1/8*a1*cos(2*alpha1-3*alpha2)+1/8*
a2*cos(3*alpha1-2*alpha2)-1/2*a1*cos(alpha1-2*alpha2)-1/4*a1*
```

```
    cos(3*alpha1-2*alpha2)+1/2*a2*cos(2*alpha1-alpha2)+1/4*a2*cos
    (2*alpha1-3*alpha2)+1/4*a1*cos(3*alpha1)-1/4*a2*cos(3*alpha2)
    +1/8*a1*cos(3*alpha2)-1/8*a2*cos(3*alpha1)-1/8*a2*cos(4*alpha2)
    +1/8*a1*cos(4*alpha1));
guess=linspace(0,pi,20); % Various starting guess for the solver
racine=[]; % will allow to discard solutions too closed from 0
options=optimset('Display','off','FunValCheck','on','TolFun',1e-14,
    'TolX',1e-14,'MaxFunEvals',1e4,'MaxIter',1e4);
for kh=1:length(guess) % Solving with fzero for each starting guess
    [x,fval,exitflag] = fzero(eqtot,guess(kh),options);
    if(exitflag==1)
        flagij=0;
        for kkh=1:length(racine) % test if the solution is new
            if(abs(x-racine(kkh))<=1e-10), flagij=1; end;
        end
        if(flagij==0), racine=[racine x]; end; %only store new
            solution
    end
end
alpha2=racine;
dd = -(-tan((1./2).*alpha1).*a1+tan((1./2).*alpha2).*a2)./(-tan
    ((1./2).*alpha1)+tan((1./2).*alpha2)); % d associated with the
    roots
RR=1./2.*(-a1.*cos(alpha1).*sin(alpha1)-a1.*sin(alpha1).*cos(alpha2
    )-a2.*sin(alpha2).*cos(alpha1)-a2.*cos(alpha2).*sin(alpha2)+2.*
    alpha1.*a1+2.*alpha2.*a2+delta.*(4.*a1.*cos(alpha1).*sin(alpha1
    ).*a2.*cos(alpha2).*sin(alpha2)+2.*a1.*cos(alpha1).^2.*sin(
    alpha1).*a2.*sin(alpha2)-4.*a1.*cos(alpha1).*sin(alpha1).*
    alpha2.*a2+2.*a1.*sin(alpha1).*cos(alpha2).^2.*a2.*sin(alpha2)
    -4.*a1.*sin(alpha1).*cos(alpha2).*alpha2.*a2-4.*a2.*sin(alpha2)
    .*cos(alpha1).*alpha1.*a1-4.*a2.*cos(alpha2).*sin(alpha2).*
    alpha1.*a1-8.*sin(1./2.*alpha1+1./2.*alpha2).*cos(1./2.*alpha1
    +1./2.*alpha2).*a1.^2.*sin(alpha1).*cos(alpha1)-8.*sin(1./2.*
    alpha1+1./2.*alpha2).*cos(1./2.*alpha1+1./2.*alpha2).*a2.^2.*
    sin(alpha2).*cos(alpha2)-4.*alpha1.*a2.^2.*alpha2-4.*alpha2.*a1
    .^2.*alpha1+4.*pi.*a1.^2.*alpha1+4.*pi.*a2.^2.*alpha2+2.*a1
    .^2.*cos(alpha1).*cos(alpha2)+2.*a2.^2.*cos(alpha1).*cos(alpha2
    )+8.*alpha1.*a1.*alpha2.*a2-4.*a1.^2.*sin(alpha1).*cos(alpha2)
    .*alpha1-4.*a2.^2.*sin(alpha2).*cos(alpha1).*alpha2-4.*pi.*a1
    .^2.*sin(alpha1).*cos(alpha1)-4.*pi.*a2.^2.*sin(alpha2).*cos(
    alpha2)+4.*alpha1.*a2.^2.*sin(alpha2).*cos(alpha2)+4.*alpha2.*
    a1.^2.*sin(alpha1).*cos(alpha1)+8.*sin(1./2.*alpha1+1./2.*
    alpha2).*cos(1./2.*alpha1+1./2.*alpha2).*a1.^2.*alpha1+8.*sin
    (1./2.*alpha1+1./2.*alpha2).*cos(1./2.*alpha1+1./2.*alpha2).*a2
    .^2.*alpha2+a2.^2.*cos(alpha2).^2-a2.^2.*cos(alpha2).^4-a1.^2.*
    cos(alpha1).^4+a1.^2.*cos(alpha2).^2+a2.^2.*cos(alpha1).^2+a1
    .^2.*cos(alpha1).^2-2.*a1.^2.*cos(alpha1).^3.*cos(alpha2)-2.*a2
    .^2.*cos(alpha1).*cos(alpha2).^3-a1.^2.*cos(alpha2).^2.*cos(
    alpha1).^2-a2.^2.*cos(alpha1).^2.*cos(alpha2).^2).^(1./2))./(pi
    -alpha1-alpha2+2.*sin(1./2.*alpha1+1./2.*alpha2).*cos(1./2.*
    alpha1+1./2.*alpha2));

%-- Loop testing that the found roots are indeed solutions of the 3
    eq.
nn=0; alpha2_sol=[]; d_sol=[]; R_sol=[]; % nn count the valid
    solutions
```

```
for kk=1:length(racine) % test after calculating a,b,c,d for each root
    alpha2=racine(kk); d=dd(kk); R=RR(kk);
    a=2*(pi/2-(alpha1+alpha2)/2+sin((alpha1+alpha2)/2)*cos((alpha1+alpha2)/2)); b=((a1*sin(alpha1)+a2*sin(alpha2))*(cos(alpha1)+cos(alpha2))-2*(alpha1*a1+alpha2*a2)); c=-a1^2*(alpha1-sin(alpha1)*cos(alpha1))-a2^2*(alpha2-sin(alpha2)*cos(alpha2));
    dD=-(-tan((1./2).*alpha1).*a1+tan((1./2).*alpha2).*a2)./(-tan((1./2).*alpha1)+tan((1./2).*alpha2));
    if((abs(R-((a1*(1-cos(alpha1))+a2*(1-cos(alpha2))+2*d)/(cos(alpha1)+cos(alpha2))))<=tol)&&(abs((2*d+2*a1)*tan(alpha1/2)-(2*d+2*a2)*tan(alpha2/2))<=tol) ...
            &&(abs(R^2*a+R*b+c)<=tol)&&(alpha2>=0)&&(alpha2<=pi)&&(dD>0)) % Check eq. & bounds on alpha2 (0<=alpha2<=pi)
        nn=nn+1;        alpha2_sol(nn)=alpha2; d_sol(nn) = dD;
        R_sol(nn)=(1/2)*(-b+delta*sqrt(b.^2-4*a*c))./a;
    end
end
%-------------------------------------------------------------------------
%%%%%%%%%%%%%%%%%%%%%%%%    2ND PART    %%%%%%%%%%%%%%%%%%%%%%%%%%%%
%---- Solving eq. 1 for delta=1 (1st solution given by eq. 3 for R) ----
%-------------------------------------------------------------------------
delta=1;
% Eqtot is eq. 1 using the expressions of a,b,c, d (eq. 2) & R (eq.3)
eqtot = @(alpha2)  real(1/2*cos(alpha1)*a2-1/2*cos(alpha2)*a1+1/2*a2*cos(alpha1+alpha2)+1/8*a2*cos(3*alpha1+alpha2)-1/8*a2*cos(3*alpha1-alpha2)-1/2*a2*cos(alpha1-alpha2)-3/8*a2*cos(2*alpha1-2*alpha2)+3/8*a2*cos(3*alpha2+alpha1)-3/8*a2*cos(-3*alpha2+alpha1)+5/8*a2*cos(2*alpha1+2*alpha2)+alpha2*a2*sin(2*alpha1)+alpha2*a2*sin(alpha1-alpha2)+alpha2*a2*sin(alpha1+alpha2)-a2*alpha1*sin(alpha1-alpha2)+a2*alpha1*sin(2*alpha2)+a2*alpha1*sin(alpha1+alpha2)+1/8*a1-1/8*a2+2*a1*alpha2*sin(alpha1)+a2*pi*sin(alpha1)-a2*alpha1*sin(alpha1)-2*alpha2*a2*sin(alpha1)-2*a1*pi*sin(alpha1)+alpha1*a1*sin(alpha1)+2*a2*pi*sin(alpha2)-2*a2*alpha1*sin(alpha2)+a1*alpha2*sin(alpha2)-a1*pi*sin(alpha2)-alpha2*a2*sin(alpha2)+2*alpha1*a1*sin(alpha2)-1/4*a1*cos(alpha1)+1/4*a2*cos(alpha2)+1/2*a1*cos(alpha1-alpha2)-1/2*a1*cos(alpha1+alpha2)+3/8*a1*cos(2*alpha1-2*alpha2)-5/8*a1*cos(2*alpha1+2*alpha2)-1/8*delta*(8*a1*a2*cos(2*alpha1-2*alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*a1^2*alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*alpha2-8*a1*a2*cos(alpha1+alpha2)+4*a1*a2*cos(3*alpha1-alpha2)+8*a1*a2*cos(alpha1-alpha2)-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*a2*cos(3*alpha2+alpha1)+32*alpha2*a1^2*sin(2*alpha1)-32*a1*alpha2*a2*sin(2*alpha1)-32*a1*alpha2*a2*sin(alpha1+alpha2)-32*a1*alpha2*a2*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin(alpha1+alpha2)+32*a2*alpha1*a1*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin(2*alpha2)-12*a1^2*cos(alpha1-alpha2)+12*a1^2*cos(3*alpha1+
```

```
alpha2)-12*a2^2*cos(alpha1-alpha2)+12*a2^2*cos(3*alpha2+alpha1)
-2*a1^2*cos(4*alpha1)+4*a1^2*cos(2*alpha2)-4*a1^2*cos(2*alpha1)
-2*a1^2*cos(2*alpha1-2*alpha2)-2*a1^2*cos(2*alpha1+2*alpha2)+4*
a2^2*cos(2*alpha1)-4*a2^2*cos(2*alpha2)-2*a2^2*cos(2*alpha1-2*
alpha2)-2*a2^2*cos(2*alpha1+2*alpha2)-2*a2^2*cos(4*alpha2)-32*
pi*a1^2*sin(2*alpha1)-8*a1*a2*cos(2*alpha1+2*alpha2)+32*a1^2*
alpha1*sin(alpha1+alpha2)-32*a1^2*alpha1*sin(alpha1-alpha2)+32*
a2^2*alpha2*sin(alpha1+alpha2)+32*a2^2*alpha2*sin(alpha1-alpha2
)-32*pi*a2^2*sin(2*alpha2)+32*alpha1*a2^2*sin(2*alpha2)+4*a1^2*
cos(alpha1+alpha2)-4*a1^2*cos(3*alpha1-alpha2)+4*a2^2*cos(
alpha1+alpha2)-4*a2^2*cos(-3*alpha2+alpha1)+4*a1*a2*cos(-3*
alpha2+alpha1)+128*alpha1*a1*alpha2*a2)^(1/2)*sin(alpha1)-1/8*
delta*(8*a1*a2*cos(2*alpha1-2*alpha2)+6*a1^2+6*a2^2-64*alpha1*
a2^2*alpha2-64*alpha2*a1^2*alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*
alpha2-8*a1*a2*cos(alpha1+alpha2)+4*a1*a2*cos(3*alpha1-alpha2)
+8*a1*a2*cos(alpha1-alpha2)-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*
a2*cos(3*alpha2+alpha1)+32*alpha2*a1^2*sin(2*alpha1)-32*a1*
alpha2*a2*sin(2*alpha1)-32*a1*alpha2*a2*sin(alpha1+alpha2)-32*
a1*alpha2*a2*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin(alpha1+
alpha2)+32*a2*alpha1*a1*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin
(2*alpha2)-12*a1^2*cos(alpha1-alpha2)+12*a1^2*cos(3*alpha1+
alpha2)-12*a2^2*cos(alpha1-alpha2)+12*a2^2*cos(3*alpha2+alpha1)
-2*a1^2*cos(4*alpha1)+4*a1^2*cos(2*alpha2)-4*a1^2*cos(2*alpha1)
-2*a1^2*cos(2*alpha1-2*alpha2)-2*a1^2*cos(2*alpha1+2*alpha2)+4*
a2^2*cos(2*alpha1)-4*a2^2*cos(2*alpha2)-2*a2^2*cos(2*alpha1-2*
alpha2)-2*a2^2*cos(2*alpha1+2*alpha2)-2*a2^2*cos(4*alpha2)-32*
pi*a1^2*sin(2*alpha1)-8*a1*a2*cos(2*alpha1+2*alpha2)+32*a1^2*
alpha1*sin(alpha1+alpha2)-32*a1^2*alpha1*sin(alpha1-alpha2)+32*
a2^2*alpha2*sin(alpha1+alpha2)+32*a2^2*alpha2*sin(alpha1-alpha2
)-32*pi*a2^2*sin(2*alpha2)+32*alpha1*a2^2*sin(2*alpha2)+4*a1^2*
cos(alpha1+alpha2)-4*a1^2*cos(3*alpha1-alpha2)+4*a2^2*cos(
alpha1+alpha2)-4*a2^2*cos(-3*alpha2+alpha1)+4*a1*a2*cos(-3*
alpha2+alpha1)+128*alpha1*a1*alpha2*a2)^(1/2)*sin(alpha1-2*
alpha2)-alpha1*a1*sin(alpha1-2*alpha2)-a2*pi*sin(alpha1-2*
alpha2)+a2*alpha1*sin(alpha1-2*alpha2)-alpha2*a2*sin(2*alpha1-
alpha2)+a1*cos(2*alpha1+alpha2)-1/8*a1*cos(4*alpha1-alpha2)
-3/8*a1*cos(2*alpha1-alpha2)+1/8*delta*(8*a1*a2*cos(2*alpha1-2*
alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*a1^2*
alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*alpha2-8*a1*a2*cos(alpha1+
alpha2)+4*a1*a2*cos(3*alpha1-alpha2)+8*a1*a2*cos(alpha1-alpha2)
-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*a2*cos(3*alpha2+alpha1)+32*
alpha2*a1^2*sin(2*alpha1)-32*a1*alpha2*a2*sin(2*alpha1)-32*a1*
alpha2*a2*sin(alpha1+alpha2)-32*a1*alpha2*a2*sin(alpha1-alpha2)
-32*a2*alpha1*a1*sin(alpha1+alpha2)+32*a2*alpha1*a1*sin(alpha1-
alpha2)-32*a2*alpha1*a1*sin(2*alpha2)-12*a1^2*cos(alpha1-alpha2
)+12*a1^2*cos(3*alpha1+alpha2)-12*a2^2*cos(alpha1-alpha2)+12*a2
^2*cos(3*alpha2+alpha1)-2*a1^2*cos(4*alpha1)+4*a1^2*cos(2*
alpha2)-4*a1^2*cos(2*alpha1)-2*a1^2*cos(2*alpha1-2*alpha2)-2*a1
^2*cos(2*alpha1+2*alpha2)+4*a2^2*cos(2*alpha1)-4*a2^2*cos(2*
alpha2)-2*a2^2*cos(2*alpha1-2*alpha2)-2*a2^2*cos(2*alpha1+2*
alpha2)-2*a2^2*cos(4*alpha2)-32*pi*a1^2*sin(2*alpha1)-8*a1*a2*
cos(2*alpha1+2*alpha2)+32*a1^2*alpha1*sin(alpha1+alpha2)-32*a1
^2*alpha1*sin(alpha1-alpha2)+32*a2^2*alpha2*sin(alpha1+alpha2)
+32*a2^2*alpha2*sin(alpha1-alpha2)-32*pi*a2^2*sin(2*alpha2)+32*
alpha1*a2^2*sin(2*alpha2)+4*a1^2*cos(alpha1+alpha2)-4*a1^2*cos
(3*alpha1-alpha2)+4*a2^2*cos(alpha1+alpha2)-4*a2^2*cos(-3*
```

```
alpha2+alpha1)+4*a1*a2*cos(-3*alpha2+alpha1)+128*alpha1*a1*
alpha2*a2)^(1/2)*sin(alpha2)-1/8*delta*(8*a1*a2*cos(2*alpha1-2*
alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*a1^2*
alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*alpha2-8*a1*a2*cos(alpha1+
alpha2)+4*a1*a2*cos(3*alpha1-alpha2)+8*a1*a2*cos(alpha1-alpha2)
-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*a2*cos(3*alpha2+alpha1)+32*
alpha2*a1^2*sin(2*alpha1)-32*a1*alpha2*a2*sin(2*alpha1)-32*a1*
alpha2*a2*sin(alpha1+alpha2)-32*a1*alpha2*a2*sin(alpha1-alpha2)
-32*a2*alpha1*a1*sin(alpha1+alpha2)+32*a2*alpha1*a1*sin(alpha1-
alpha2)-32*a2*alpha1*a1*sin(2*alpha2)-12*a1^2*cos(alpha1-alpha2
)+12*a1^2*cos(3*alpha1+alpha2)-12*a2^2*cos(alpha1-alpha2)+12*a2
^2*cos(3*alpha2+alpha1)-2*a1^2*cos(4*alpha1)+4*a1^2*cos(2*
alpha2)-4*a1^2*cos(2*alpha1)-2*a1^2*cos(2*alpha1-2*alpha2)-2*a1
^2*cos(2*alpha1+2*alpha2)+4*a2^2*cos(2*alpha1)-4*a2^2*cos(2*
alpha2)-2*a2^2*cos(2*alpha1-2*alpha2)-2*a2^2*cos(2*alpha1+2*
alpha2)-2*a2^2*cos(4*alpha2)-32*pi*a1^2*sin(2*alpha1)-8*a1*a2*
cos(2*alpha1+2*alpha2)+32*a1^2*alpha1*sin(alpha1+alpha2)-32*a1
^2*alpha1*sin(alpha1-alpha2)+32*a2^2*alpha2*sin(alpha1+alpha2)
+32*a2^2*alpha2*sin(alpha1-alpha2)-32*pi*a2^2*sin(2*alpha2)+32*
alpha1*a2^2*sin(2*alpha2)+4*a1^2*cos(alpha1+alpha2)-4*a1^2*cos
(3*alpha1-alpha2)+4*a2^2*cos(alpha1+alpha2)-4*a2^2*cos(-3*
alpha2+alpha1)+4*a1*a2*cos(-3*alpha2+alpha1)+128*alpha1*a1*
alpha2*a2)^(1/2)*sin(2*alpha1-alpha2)-a1*pi*sin(2*alpha1-alpha2
)+a1*alpha2*sin(2*alpha1-alpha2)-a1*alpha2*sin(alpha1+alpha2)-
a1*alpha2*sin(alpha1-alpha2)+a1*pi*sin(alpha1+alpha2)+a1*pi*sin
(alpha1-alpha2)-a2*pi*sin(alpha1+alpha2)+a2*pi*sin(alpha1-
alpha2)-alpha1*a1*sin(alpha1+alpha2)+alpha1*a1*sin(alpha1-
alpha2)+1/8*a2*cos(-4*alpha2+alpha1)-a2*cos(alpha1+2*alpha2)
+3/8*a2*cos(alpha1-2*alpha2)+1/4*delta*(8*a1*a2*cos(2*alpha1-2*
alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*a1^2*
alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*alpha2-8*a1*a2*cos(alpha1+
alpha2)+4*a1*a2*cos(3*alpha1-alpha2)+8*a1*a2*cos(alpha1-alpha2)
-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*a2*cos(3*alpha2+alpha1)+32*
alpha2*a1^2*sin(2*alpha1)-32*a1*alpha2*a2*sin(2*alpha1)-32*a1*
alpha2*a2*sin(alpha1+alpha2)-32*a1*alpha2*a2*sin(alpha1-alpha2)
-32*a2*alpha1*a1*sin(alpha1+alpha2)+32*a2*alpha1*a1*sin(alpha1-
alpha2)-32*a2*alpha1*a1*sin(2*alpha2)-12*a1^2*cos(alpha1-alpha2
)+12*a1^2*cos(3*alpha1+alpha2)-12*a2^2*cos(alpha1-alpha2)+12*a2
^2*cos(3*alpha2+alpha1)-2*a1^2*cos(4*alpha1)+4*a1^2*cos(2*
alpha2)-4*a1^2*cos(2*alpha1)-2*a1^2*cos(2*alpha1-2*alpha2)-2*a1
^2*cos(2*alpha1+2*alpha2)+4*a2^2*cos(2*alpha1)-4*a2^2*cos(2*
alpha2)-2*a2^2*cos(2*alpha1-2*alpha2)-2*a2^2*cos(2*alpha1+2*
alpha2)-2*a2^2*cos(4*alpha2)-32*pi*a1^2*sin(2*alpha1)-8*a1*a2*
cos(2*alpha1+2*alpha2)+32*a1^2*alpha1*sin(alpha1+alpha2)-32*a1
^2*alpha1*sin(alpha1-alpha2)+32*a2^2*alpha2*sin(alpha1+alpha2)
+32*a2^2*alpha2*sin(alpha1-alpha2)-32*pi*a2^2*sin(2*alpha2)+32*
alpha1*a2^2*sin(2*alpha2)+4*a1^2*cos(alpha1+alpha2)-4*a1^2*cos
(3*alpha1-alpha2)+4*a2^2*cos(alpha1+alpha2)-4*a2^2*cos(-3*
alpha2+alpha1)+4*a1*a2*cos(-3*alpha2+alpha1)+128*alpha1*a1*
alpha2*a2)^(1/2)*sin(alpha1-alpha2)+1/8*delta*(8*a1*a2*cos(2*
alpha1-2*alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*
a1^2*alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*alpha2-8*a1*a2*cos(
alpha1+alpha2)+4*a1*a2*cos(3*alpha1-alpha2)+8*a1*a2*cos(alpha1-
alpha2)-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*a2*cos(3*alpha2+
alpha1)+32*alpha2*a1^2*sin(2*alpha1)-32*a1*alpha2*a2*sin(2*
alpha1)-32*a1*alpha2*a2*sin(alpha1+alpha2)-32*a1*alpha2*a2*sin(
```

```
        alpha1-alpha2)-32*a2*alpha1*a1*sin(alpha1+alpha2)+32*a2*alpha1*
        a1*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin(2*alpha2)-12*a1^2*cos
        (alpha1-alpha2)+12*a1^2*cos(3*alpha1+alpha2)-12*a2^2*cos(alpha1
        -alpha2)+12*a2^2*cos(3*alpha2+alpha1)-2*a1^2*cos(4*alpha1)+4*a1
        ^2*cos(2*alpha2)-4*a1^2*cos(2*alpha1)-2*a1^2*cos(2*alpha1-2*
        alpha2)-2*a1^2*cos(2*alpha1+2*alpha2)+4*a2^2*cos(2*alpha1)-4*a2
        ^2*cos(2*alpha2)-2*a2^2*cos(2*alpha1-2*alpha2)-2*a2^2*cos(2*
        alpha1+2*alpha2)-2*a2^2*cos(4*alpha2)-32*pi*a1^2*sin(2*alpha1)
        -8*a1*a2*cos(2*alpha1+2*alpha2)+32*a1^2*alpha1*sin(alpha1+
        alpha2)-32*a1^2*alpha1*sin(alpha1-alpha2)+32*a2^2*alpha2*sin(
        alpha1+alpha2)+32*a2^2*alpha2*sin(alpha1-alpha2)-32*pi*a2^2*sin
        (2*alpha2)+32*alpha1*a2^2*sin(2*alpha2)+4*a1^2*cos(alpha1+
        alpha2)-4*a1^2*cos(3*alpha1-alpha2)+4*a2^2*cos(alpha1+alpha2)
        -4*a2^2*cos(-3*alpha2+alpha1)+4*a1*a2*cos(-3*alpha2+alpha1)
        +128*alpha1*a1*alpha2*a2)^(1/2)*sin(2*alpha1)-a1*alpha2*sin(2*
        alpha1)-alpha1*a1*sin(2*alpha2)-3/4*a2*cos(2*alpha1+alpha2)+a1*
        pi*sin(2*alpha1)+3/4*a1*cos(alpha1+2*alpha2)+3/8*a1*cos(3*
        alpha1-alpha2)-3/8*a1*cos(3*alpha1+alpha2)+1/8*a1*cos(-3*alpha2
        +alpha1)-1/8*a1*cos(3*alpha2+alpha1)-1/8*delta*(8*a1*a2*cos(2*
        alpha1-2*alpha2)+6*a1^2+6*a2^2-64*alpha1*a2^2*alpha2-64*alpha2*
        a1^2*alpha1+64*pi*a1^2*alpha1+64*pi*a2^2*alpha2-8*a1*a2*cos(
        alpha1+alpha2)+4*a1*a2*cos(3*alpha1-alpha2)+8*a1*a2*cos(alpha1-
        alpha2)-4*a1*a2*cos(3*alpha1+alpha2)-4*a1*a2*cos(3*alpha2+
        alpha1)+32*alpha2*a1^2*sin(2*alpha1)-32*a1*alpha2*a2*sin(2*
        alpha1)-32*a1*alpha2*a2*sin(alpha1+alpha2)-32*a1*alpha2*a2*sin(
        alpha1-alpha2)-32*a2*alpha1*a1*sin(alpha1+alpha2)+32*a2*alpha1*
        a1*sin(alpha1-alpha2)-32*a2*alpha1*a1*sin(2*alpha2)-12*a1^2*cos
        (alpha1-alpha2)+12*a1^2*cos(3*alpha1+alpha2)-12*a2^2*cos(alpha1
        -alpha2)+12*a2^2*cos(3*alpha2+alpha1)-2*a1^2*cos(4*alpha1)+4*a1
        ^2*cos(2*alpha2)-4*a1^2*cos(2*alpha1)-2*a1^2*cos(2*alpha1-2*
        alpha2)-2*a1^2*cos(2*alpha1+2*alpha2)+4*a2^2*cos(2*alpha1)-4*a2
        ^2*cos(2*alpha2)-2*a2^2*cos(2*alpha1-2*alpha2)-2*a2^2*cos(2*
        alpha1+2*alpha2)-2*a2^2*cos(4*alpha2)-32*pi*a1^2*sin(2*alpha1)
        -8*a1*a2*cos(2*alpha1+2*alpha2)+32*a1^2*alpha1*sin(alpha1+
        alpha2)-32*a1^2*alpha1*sin(alpha1-alpha2)+32*a2^2*alpha2*sin(
        alpha1+alpha2)+32*a2^2*alpha2*sin(alpha1-alpha2)-32*pi*a2^2*sin
        (2*alpha2)+32*alpha1*a2^2*sin(2*alpha2)+4*a1^2*cos(alpha1+
        alpha2)-4*a1^2*cos(3*alpha1-alpha2)+4*a2^2*cos(alpha1+alpha2)
        -4*a2^2*cos(-3*alpha2+alpha1)+4*a1*a2*cos(-3*alpha2+alpha1)
        +128*alpha1*a1*alpha2*a2)^(1/2)*sin(2*alpha2)-a2*pi*sin(2*
        alpha2)+1/4*a1*cos(2*alpha2)-1/4*a1*cos(2*alpha1)-1/4*a2*cos(2*
        alpha1)+1/4*a2*cos(2*alpha2)-1/8*a1*cos(2*alpha1-3*alpha2)+1/8*
        a2*cos(3*alpha1-2*alpha2)-1/2*a1*cos(alpha1-2*alpha2)-1/4*a1*
        cos(3*alpha1-2*alpha2)+1/2*a2*cos(2*alpha1-alpha2)+1/4*a2*cos
        (2*alpha1-3*alpha2)+1/4*a1*cos(3*alpha1)-1/4*a2*cos(3*alpha2)
        +1/8*a1*cos(3*alpha2)-1/8*a2*cos(3*alpha1)-1/8*a2*cos(4*alpha2)
        +1/8*a1*cos(4*alpha1));
67
68      guess=linspace(0,pi,20);
69      racine2=[]; % will allow to discard solutions too closed from 0
70      options=optimset('Display','off','FunValCheck','on','TolFun',1e-14,
            'TolX',1e-14,'MaxFunEvals',1e4,'MaxIter',1e4);
71      for kh=1:length(guess) % Solving with fzero for each starting guess
72          [x,fval,exitflag] = fzero(eqtot,guess(kh),options);
73          if(exitflag==1)
74              flagij=0;
```

```
75              for kkh=1:length(racine2)
76                  if(abs(x-racine2(kkh))<=1e-10), flagij=1; end;
77              end
78              if(flagij==0), racine2=[racine2 x]; end;
79          end
80      end
81      alpha2=racine2;
82      dd2 = -(-tan((1./2).*alpha1).*a1+tan((1./2).*alpha2).*a2)./(-tan
            ((1./2).*alpha1)+tan((1./2).*alpha2));
83      RR2=1./2.*(-a1.*cos(alpha1).*sin(alpha1)-a1.*sin(alpha1).*cos(
            alpha2)-a2.*sin(alpha2).*cos(alpha1)-a2.*cos(alpha2).*sin(
            alpha2)+2.*alpha1.*a1+2.*alpha2.*a2+delta.*(4.*a1.*cos(alpha1)
            .*sin(alpha1).*a2.*cos(alpha2).*sin(alpha2)+2.*a1.*cos(alpha1)
            .^2.*sin(alpha1).*a2.*sin(alpha2)-4.*a1.*cos(alpha1).*sin(
            alpha1).*alpha2.*a2+2.*a1.*sin(alpha1).*cos(alpha2).^2.*a2.*sin
            (alpha2)-4.*a1.*sin(alpha1).*cos(alpha2).*alpha2.*a2-4.*a2.*sin
            (alpha2).*cos(alpha1).*alpha1.*a1-4.*a2.*cos(alpha2).*sin(
            alpha2).*alpha1.*a1-8.*sin(1./2.*alpha1+1./2.*alpha2).*cos
            (1./2.*alpha1+1./2.*alpha2).*a1.^2.*sin(alpha1).*cos(alpha1)
            -8.*sin(1./2.*alpha1+1./2.*alpha2).*cos(1./2.*alpha1+1./2.*
            alpha2).*a2.^2.*sin(alpha2).*cos(alpha2)-4.*alpha1.*a2.^2.*
            alpha2-4.*alpha2.*a1.^2.*alpha1+4.*pi.*a1.^2.*alpha1+4.*pi.*a2
            .^2.*alpha2+2.*a1.^2.*cos(alpha1).*cos(alpha2)+2.*a2.^2.*cos(
            alpha1).*cos(alpha2)+8.*alpha1.*a1.*alpha2.*a2-4.*a1.^2.*sin(
            alpha1).*cos(alpha2).*alpha1-4.*a2.^2.*sin(alpha2).*cos(alpha1)
            .*alpha2-4.*pi.*a1.^2.*sin(alpha1).*cos(alpha1)-4.*pi.*a2.^2.*
            sin(alpha2).*cos(alpha2)+4.*alpha1.*a2.^2.*sin(alpha2).*cos(
            alpha2)+4.*alpha2.*a1.^2.*sin(alpha1).*cos(alpha1)+8.*sin
            (1./2.*alpha1+1./2.*alpha2).*cos(1./2.*alpha1+1./2.*alpha2).*a1
            .^2.*alpha1+8.*sin(1./2.*alpha1+1./2.*alpha2).*cos(1./2.*alpha1
            +1./2.*alpha2).*a2.^2.*alpha2+a2.^2.*cos(alpha2).^2-a2.^2.*cos(
            alpha2).^4-a1.^2.*cos(alpha1).^4+a1.^2.*cos(alpha2).^2+a2.^2.*
            cos(alpha1).^2+a1.^2.*cos(alpha1).^2-2.*a1.^2.*cos(alpha1).^3.*
            cos(alpha2)-2.*a2.^2.*cos(alpha1).*cos(alpha2).^3-a1.^2.*cos(
            alpha2).^2.*cos(alpha1).^2-a2.^2.*cos(alpha1).^2.*cos(alpha2)
            .^2).^(1./2))./(pi-alpha1-alpha2+2.*sin(1./2.*alpha1+1./2.*
            alpha2).*cos(1./2.*alpha1+1./2.*alpha2));
84
85      %-- Loop testing that the found roots are indeed solutions of the 3
            eq.
86      nn2=0; alpha2_sol2=[]; d_sol2=[]; R_sol2=[];
87      for kk=1:length(racine2)% test after calculating a,b,c,d for each
            root
88          alpha2=racine2(kk); d=dd2(kk); R=RR2(kk);
89          a=2*(pi/2-(alpha1+alpha2)/2+sin((alpha1+alpha2)/2)*cos((alpha1+
                alpha2)/2)); b=((a1*sin(alpha1)+a2*sin(alpha2))*(cos(alpha1
                )+cos(alpha2))-2*(alpha1*a1+alpha2*a2)); c=-a1^2*(alpha1-
                sin(alpha1)*cos(alpha1))-a2^2*(alpha2-sin(alpha2)*cos(
                alpha2));
90          dD=-(-tan((1./2).*alpha1).*a1+tan((1./2).*alpha2).*a2)./(-tan
                ((1./2).*alpha1)+tan((1./2).*alpha2));
91          if((abs(R-((a1*(1-cos(alpha1))+a2*(1-cos(alpha2))+2*d)/(cos(
                alpha1)+cos(alpha2))))<=tol)&&(abs((2*d+2*a1)*tan(alpha1/2)
                -(2*d+2*a2)*tan(alpha2/2))<=tol)...
92                  &&(abs(R^2*a+R*b+c)<=tol)&&(alpha2>=0)&&(alpha2<=pi)&&(
                        dD>=0)) % Check eq. & bounds on alpha2 (0<=alpha2<=
```

```
                    pi)
93                  nn2=nn2+1;          alpha2_sol2(nn2)=alpha2; d_sol2(nn2) = dD
                        ;
94                  R_sol2(nn2)=(1/2)*(-b+delta*sqrt(b.^2-4*a*c))./a;
95              end
96          end
97          %
```

```
98          % FINAL SOLUTIONS FOR alpha2, d and R
99          clear alpha2 d R
100         alpha2=[alpha2_sol alpha2_sol2]; d=[d_sol d_sol2]; R=[R_sol R_sol2
                ];
101 end
```

Cross_Section_Area.m

```matlab
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%% Calculation of the cross-section area %%%%%%%%%%%%%%%%%
%%%%%%%%%%% as a function of alpha1 (wrapping angle) %%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% AUTHORS: D. CEBRON, A. SAURET
% DATE: 31/03/2015
% DESCRIPTION: Calculation of the cross-section area as a function of
% alpha1 (wrapping angle)
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

close all

dd=d(1,:); %%% Inter-fiber distance
RR=R(1,:); %%% Radius of curvature

%%% Delete NaN points
alpha1=alpha1_tab;
RR(isnan(dd))=[];
alpha1(isnan(dd))=[];
dd(isnan(dd))=[];

%%% Calculate wrapping angle alpha2 on the small fiber
alpha2=2.*atan((2.*dd+2*a1)./(2.*dd+2.*a2).*tan(alpha1./2));

%%% Calculate the cross-section area
Area=(a1.*sin(alpha1)+a2.*sin(alpha2)).*(a1.*(1-cos(alpha1))+a2.*(1-cos(alpha2))+2.*dd)-a1.^2.*(alpha1-sin(alpha1).*cos(alpha1))-a2.^2.*(alpha2-sin(alpha2).*cos(alpha2))-2.*RR.^2.*(pi./2-sin((alpha1+alpha2)./2).*cos((alpha1+alpha2)./2)-(alpha1+alpha2)./2);

%%% Plot the dimensionless area A/a1^2 as a function of the dimensionless
%%% inter-fiber distance d/a1
plot(dd./a1,Area./a1.^2)
```

Plot_Cross_Section.m

```
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% AUTHORS: D. CEBRON, A. SAURET
% DATE: 31/03/2015
% DESCRIPTION: Plot the cross-section of a column
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%% !! Require circle.m !! %%%%%

close all

%%% Collect the values of the wrapping angles, the interfiber distance and
%%% the radius of curvature
clear alpha1;
alpha1=alpha1_tab;
alpha2=alpha2(1,:);
d=d(1,:);
R=R(1,:);

NN=size(d,2)-1; %%% Position to plot the cross-section
d(NN)./a1 %%% Value of the dimensionless inter-fiber distance

%%%% Plot the cross-section using the radius of curvature
circle((a1+R(NN)).*cos(alpha1(NN)),-(a1+R(NN)).*sin(alpha1(NN)),R(NN))

hold on
circle((a1+R(NN)).*cos(alpha1(NN)),(a1+R(NN)).*sin(alpha1(NN)),R(NN))
hold on

%%%%%%%%%%%% Plot the fibers %%%%%%%%%%%%%%
circle(0,0,a1); %%% Largest fiber
hold on
circle(a1+2.*d(NN)+a2,0,a2); %%% Smallest fibers


axis equal
```