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## Hydrodynamic instabilities in shear flows of dry cohesive granular particles

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In this Electronic Supplementary Information (ESI), we compare our model predictions with our previous molecular dynamics (MD) simulations of cohesive granular shear flows in Ref. [62] (S. Takada et al. Phys. Rev. E **90**, 062207 (2014)).

Figure 1 displays the stability diagram obtained from our MD simulations [62], where the red (blue) shaded area represents unstable (stable) region. The solid line is the neutral curve derived from our linear stability analysis, where we confirm its qualitative agreement with the MD simulations, except for the data with  $s = 10^{-3}$  and  $\zeta^* = 10^{-4}$ , and  $s = \zeta^* = 10^0$ . In our MD simulations, the inelasticity is quantified by the microscopic viscosity coefficient,  $\zeta^*$ , i.e. the proportionality constant for the damping force between cohesive granular particles in contact, where the restitution coefficient can be roughly estimated by a linear function of  $\zeta^*$  as  $e = 1 - C\zeta^*$  with a constant  $C \approx 5 \times 10^{-3}$  (see Fig. 1 in Ref. [62]).



**Figure 1.** A double logarithmic plot of the phase diagram obtained from the MD simulations, where the symbols are as in Fig. 4 (in the paper). The solid line is the neutral curve derived from our linear stability analysis, i.e. Eq. (37) in the paper. In both the MD simulations and the neutral curve, the mean volume fraction is fixed to be  $\phi_0 = 0.31$ , while the microscopic viscosity coefficient for the MD simulations,  $\zeta^*$ , is related to the restitution coefficient by  $e = 1 - C\zeta^*$  with a constant,  $C = 5 \times 10^{-3}$ .



**Figure 2.** Profiles of (a) the volume fraction,  $\bar{\phi}$ , and (b) the dimensionless velocity field,  $\bar{u}_x$ , defined as Eqs. (21) and (22) in the paper, respectively, where we scale  $y = \tilde{y}/d$  and  $\bar{u}_x$  by the dimensionless system size,  $L^* = L/d$ , and the dimensionless relative speed,  $U^* = sL^*$ , respectively. The (blue) dotted and (red) solid lines are the results of the dynamic van der Waals model in the initial (t = 0) and steady (t = 8000) states, respectively (as listed in the legend in (b)), where we have used  $\phi_0 = 0.31$ ,  $s = 5 \times 10^{-4}$ ,  $1 - e^2 = 7 \times 10^{-7}$ , and  $L^* = 50$ . The green broken line in (a) is the scaled volume fraction, Eq. (1), where the fitting constant is given by  $\alpha = 2.25$ . The open circles are the results of the MD simulations [62] (as listed in the legend in (b)), where the mean volume fraction, the dimensionless shear rate, and the dimensionless system size are given by  $\phi_{MD} = 0.31$ ,  $s_{MD} = 10^{-0.2}$ , and  $L^*_{MD} = 32$ , respectively. In the MD simulations, the inelasticity is quantified by the microscopic viscosity coefficient,  $\zeta^* = 10^{0.2}$ .

Figure 2 shows the profiles of (a) the volume fraction,  $\bar{\phi}$ , and (b) the dimensionless velocity field,  $\bar{u}_x$ , where we scale  $y = \tilde{y}/d$  and  $\bar{u}_x$  by the dimensionless system size,  $L^* = L/d$ , and the dimensionless relative speed,  $U^* = sL^*$ , respectively. The volume fraction obtained from the dynamic van der Waals model (the red solid line in Fig. 2(a)) deviates from the MD simulations (the open circles) since both the shear rate and the inelasticity used in the MD simulations are much higher than those used in our model. Note that our model becomes numerically unstable for such a high inelasticity as mentioned in Sec. 3.2. However, a scaled volume fraction,

$$\bar{\phi}^* = \alpha \bar{\phi} + (1 - \alpha) \phi_0 , \qquad (1)$$

well agrees with the MD simulations (the green broken line) if the fitting constant is fixed to be  $\alpha = 2.25$ . Therefore, both the shape and positions of the interfaces are well reproduced by the dynamic van der Waals model. On the other hand, our model well describes the velocity field obtained from the MD simulations (the red solid line and the open circles in Fig. 2(b)).