## Supplementary Information

# Lateral capillary interactions between colloids beneath an oil-water interface that are driven by out-of-plane electrostatic double-layer interactions 

Bum Jun Park ${ }^{\text {** }}$, Mina Lee ${ }^{\text {a }}$, Bomsock Lee ${ }^{\text {a }}$, and Eric M. Furst ${ }^{\text {b }}$


#### Abstract

${ }^{\text {a}}$ Department of Chemical Engineering, Kyung Hee University, Yongin, 17104, South Korea ${ }^{\mathrm{b}}$ Department of Chemical and Biomolecular Engineering and Center for Molecular \& Engineering Thermodynamics, University of Delaware, Newark, Delaware 19716, United States


Corresponding authors: Bum Jun Park: (Tel) +82-31-201-2429, (email) bjpark@khu.ac.kr

## Initial Value Problem

The solution to the initial value problem (IVP) in Fig. 3a is numerically calculated using the algorithm proposed by Chan et al. ${ }^{1}$ Figure S 1 shows an example of the nonlinear PB solution. At $h=0$, we set the potentials of $\psi_{m}=-10,-25,-50 \mathrm{mV}$ and no potential gradient $d \psi / d h=0$ at the midplane. From this minimum potential plane, the potential is integrated until it reaches the given surface potential of the left plane $\psi_{a}=-80 \mathrm{mV}$, at which point, the distance of the plane becomes $\left|h_{a}\right|$. This same procedure is repeated for the right plane with $\psi_{b}=-80 \mathrm{mV}$, until the distance of the plane becomes $\left|h_{b}\right|$. The potential $\psi_{m}$ at the zero gradient plane is eventually related to the separation between the two planes $h=\left|h_{a}\right|+\left|h_{b}\right|$.

Knowing the relation between $\psi_{m}$ and $h$, the disjoining pressure is readily calculated using Eq. 3. This result is consistent with that calculated from BVP, as shown in Fig. 3a. Note that this method is useful when the two planes have different surface charges.


Fig. S1 Example solutions of the nonlinear PB equation (IVP) with the same charge between two parallel planes (i.e., $\psi_{P W}=\psi_{D W}=-80 \mathrm{mV}$ ) separated by $h=\left|h_{a}\right|+\left|h_{b}\right|$. At $h=0$, the potential and the potential gradient are set as $\psi_{m}=-10,-25,-50 \mathrm{mV}$ and $d \psi / d h=0$, respectively (see the text).

## Reference

1. D. Y. C. Chan, R. M. Pashley and L. R. White, J. Colloid Interface Sci., 1980, 77, 283-285.
