Supplementary Material for Soft Matter

Entanglements in polymer nanocomposites containing spherical nanoparticles

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TABLE S1: Nanoparticle volume fraction ϕ (%), length of the simulation cell (*L*) measured in units of the monomer diameter σ_m , nanoparticles of radius *R* and *type*: repulsive (Re), attractive (A), polymer matrix: N = 200, N_e (S-coil, S-kink, modified S-coil, M-coil), number of "kinks" $\langle Z \rangle$ in the frozen limit, number of "kinks" $\langle Z \rangle$ (phantom) in the phantom limit, L_{pp} : contour length of the primitive path in the frozen limit, L_{pp} (phantom): contour length of the primitive path in the phantom limit. Tube diameter α_{pp} of polymers with N = 200 in the nanocomposites. The error bar is the standard deviation

ϕ (%)	$L(\sigma_m)$	type	(S-coil)	(S-kink)	(m.S-coil)	(M-coil)	< Z >	< Z >(phantom)	L_{pp}	L_{pp} (phantom)	α_{pp}
26.6	33.416	Re $(R = 4)$	$60.8{\pm}2.6$	32.9±1.6	80.5±4.6		5.1±0.3	$5{\pm}0.2$	37.7±0.7	-	-
26.95	33.343	A(R=4)	62.5±3.9	$32.6{\pm}1.5$	83.9±7.4	57.4	5.2±0.3	$4.9 {\pm} 0.1$	37.6±0.7	$36.9{\pm}0.5$	11.95
19.5	32.531	Re $(R = 4)$	$60.9{\pm}2.5$	32.6±1.4	80.2±4.6		5.5±0.3	$5.0{\pm}0.1$	36.7±1.0	-	-
19.6	32.458	A (R = 4)	$60{\pm}2.4$	32±1.3	78.3±4.1	66.4	5.3±0.3	$5.1 {\pm} 0.2$	37.6±0.6	37.4±0.6	10.987
10.3	31.534	${\rm Re}(R=4)$	63.2±1.7	33.7±0.9	83.6±3.2		4.9±0.2	$4.9 {\pm} 0.2$	36±0.6	-	-
10.7	31.386	A(R=4)	55.2±2.2	32.2±1.0	69.8±3.8		5.2±0.2	$5.1 {\pm} 0.2$	$37.2{\pm}0.5$	$36.9{\pm}0.6$	10.205
25.4	34.068	$\operatorname{Re}\left(R=2 ight)$	$60.9{\pm}2.7$	33.5±0.9	80±4.8		4.9±0.2	$4.9 {\pm} 0.1$	$36.3 \pm \!$	-	-
25.8	33.909	$A\left(R=2\right)$	66.1±2.4	31.8±1	90.7±4.7	64	5.3±0.2	$4.5 {\pm} 0.2$	39.5±0.6	$37.7{\pm}0.5$	12.825
18.7	32.963	$\operatorname{Re}\left(R=2 ight)$	$61.9{\pm}2.9$	33.8±1.1	81.6±5.1		4.9±0.2	$4.8{\pm}0.2$	36±0.6	-	-
19	32.780	$A\left(R=2\right)$	64.4±2.7	30.3±0.9	88.1±5.2	63.4	5.6±0.2	5.1±0.2	39.7±0.8	38.6±0.7	11.255
10.5	31.766	$\operatorname{Re}\left(R=2 ight)$	61±3	33.5±1.1	80.2±5.4		5±0.2	$4.9 {\pm} 0.2$	36.1±0.8	-	-
10.6	31.636	A(R=2)	57.6±2.2	32.3±1.0	74.5±3.8		5.2±0.2	5±0.2	37.5±0.6	37.1±0.5	10.393

TABLE S2: Nanoparticle volume fraction ϕ (%), nanoparticles of radius R = 1 and type: repulsive (Re), attractive (A), polymer matrix: N = 200, Entanglement length, N_e , extracted by the M-coil estimator, L_{pp} (phantom): contour length of the primitive path in the phantom limit, L_{pp} : contour length of the primitive path in the frozen limit, Tube diameter α_{pp} of polymers with N = 200 in the nanocomposites. The error bar is the standard deviation.

ϕ (%)	L	type	N_e (M-coil)	L_{pp} (phantom)	L_{pp}	α_{pp}
36	38.53	А	14	-	85.7	-
22.9	35.267	Re	-	33.1±0.5	63.6±2.6	11.058
24.2	34.653	А	21.2	37.5 ± 0.5	67.6±2.3	14.235
17.3	33.831	Re	-	36±0.5	46.1±4.7	10.436
18.2	33.282	А	33.6	38.1±0.7	56.5±4.4	13.1
13.8	33.117	Re	-	35.1±1	41.2±2.1	10.801
14.5	32.568	А	41.9	37.2±0.6	46.7±2.9	11.369
10	32.280	Re	-	36.5±0.7	40.5±1.0	10.375
10.3	31.863	А	49.2	37.5±0.6	42.1±0.9	10.608
5.4	31.398	Re	-	36.6±0.8	38.1±0.8	10.011
5.5	31.157	А	55.64	38.3±0.6	39.9±0.6	9.879

TABLE S3: Contour length of the primitive path in the frozen limit, L_{pp} , in nanocomposites containing attractive nanoparticles of radius R = 1 and polymer matrices N < 200. Nanoparticle volume fractions ϕ (%) and polymer matrices N.

$\phi~(\%)$	L_{pp}	L_{pp}	L_{pp}	L_{pp}
	(N = 10)	(N = 20)	(N = 50)	(N = 100)
36	4.6	8.7	21.2	44
24.2	4.1	7.3	16.8	33.6
18.2	3.8	5.9	13.7	29.4
14.5	3.9	6.4	13.2	25
10.3	3.9	6.2	12.5	22.8
5.5	3.8	6	11.7	21.6



FIG. S1: Entanglement length N_e predicted by Eq.2 for the nanocomposites as a function of the interparticle distance ID for different nanoparticle sizes. Predictions of the entanglement length by Eq. 1 follow the same trends.