Supporting Information

1. Derivation of configuration energy balance equation (11)

We start by taking variation of the contact line (see figure below)

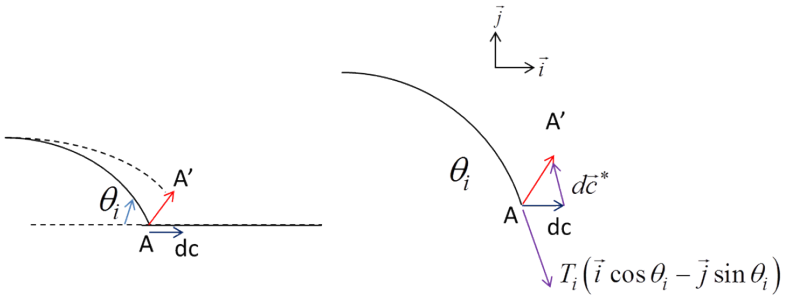

Figure S1 Marginal change in total free energy is computed by making a small variation in the location of the contact line.

The pressure does not do work in this process since it is an internal force and we are considering a constant volume process where the volume of the drop is held constant during the variation. Let us first consider the part of the system consisting of the film (the deflected plus the undeflected portions). Due to the virtual motion of the contact line by *dc*, the point *A* will move to *A'*. The change in energy on the inner portion (the one that bulges out), ΔU_i , as $c \to c + dc$, equals the work done by the force T_i acting on this point, that is,

$$\Delta U_i = -T_i\left(-\cos\theta_i \mathbf{i} + \sin\theta_i \mathbf{j}\right)\bullet \overline{A}\mathbf{A}' = -T_i\left(-\cos\theta_i \mathbf{i} + \sin\theta_i \mathbf{j}\right)\bullet\left(dc\,\mathbf{i} + dc^*\left(-\cos\theta_i \mathbf{i} + \sin\theta_i \mathbf{j}\right)\right) \quad \text{(S1)}$$
$$= T_i\cos\theta_i dc - T_i dc^*$$

Since

$$\frac{dc^*}{dc} = \frac{dc^*}{d\rho}\frac{d\rho}{dc} \quad \text{(S2)}$$

where $d\rho$ is the length of the material element associate with dc. By definition,

$$\frac{dc^*}{d\rho} = \lambda_i, \quad \frac{dc}{d\rho} = \lambda_0 \Rightarrow \frac{dc^*}{dc} = \lambda_i / \lambda_0 \quad \text{(S3)}$$

Substituting (S3) in (S1), we have

$$\Delta U_i = T_i\left[\cos\theta_i - \lambda_i / \lambda_0\right]dc \quad \text{(S4)}$$

On the outer flat portion, the change in energy ΔU_0 as $c \to c + dc$ is

$$\Delta U_0 = \left[W(\lambda_i) - W(\lambda_0) \right] d\rho \qquad \text{(S5)}$$

Using $d\rho = dc / \lambda_0$ (S3), the total change in energy in the film as the contact line advances by dc is

$$\Delta U_1 + \Delta U_2 = T_i \left[\cos\theta_i - \lambda_i / \lambda_0 \right] dc + \frac{W(\lambda_i) - W(\lambda_0)}{\lambda_0} dc \qquad \text{(S6)}$$

During this time, the change in energy of the liquid is

$$\Delta U_L = \gamma_{LV} \cos\theta_c dc \qquad \text{(S7)}$$

The total change in energy is zero, that is

$$T_i \left[\cos\theta_i - \lambda_i / \lambda_0 \right] + \frac{W(\lambda_i) - W(\lambda_0)}{\lambda_0} + \gamma_{LV} \cos\theta_c = 0 \qquad \text{(S8)}$$

which is the same as equation (11) in the main text of the manuscript.

2. Derivation of Equation 12a

The normalized elastic energy densities are:

$$\begin{aligned} \bar{W}(\lambda_i) &\equiv \frac{W(\lambda_i)}{T_{R0}} = (\bar{\gamma}_{SV} + \bar{\gamma}_{SL}) + \bar{T}_{Ri}(\lambda_i - 1) + \frac{1}{2\beta}\left(\lambda_i^2 + \frac{1}{\lambda_i^2} - 2 \right), \\ \bar{W}(\lambda_0) &\equiv \frac{W(\lambda_0)}{T_{R0}} \equiv 2\bar{\gamma}_{SV} + (\lambda_0 - 1) + \frac{1}{2\beta}\left(\lambda_0^2 + \frac{1}{\lambda_0^2} - 2 \right) \end{aligned} \qquad \text{(S9)}$$

Subtracting, we have

$$\begin{aligned} \bar{W}(\lambda_i) - \bar{W}(\lambda_0) &= (-\bar{\gamma}_{SV} + \bar{\gamma}_{SL}) + \bar{T}_{Ri}(\lambda_i - 1) - (\lambda_0 - 1) + \frac{1}{2\beta}\left(\lambda_i^2 + \frac{1}{\lambda_i^2} - \lambda_0^2 - \frac{1}{\lambda_0^2} \right) \\ &= \underbrace{\left(-\bar{\gamma}_{SV} + \bar{\gamma}_{SL} - \bar{T}_{Ri} + 1 \right)}_{\varepsilon} + \bar{T}_{Ri}\lambda_i - \lambda_0 + \frac{1}{2\beta}\left(\lambda_i^2 + \frac{1}{\lambda_i^2} - \lambda_0^2 - \frac{1}{\lambda_0^2} \right) \end{aligned} \qquad \text{S10}$$

where

$$\varepsilon = -\bar{\gamma}_{SV} + \bar{\gamma}_{SL} - \bar{T}_{Ri} + 1 = \frac{-\gamma_{SV} + \gamma_{SL} - \sigma_{SL}}{2\sigma_{SV}} + \frac{1}{2} \qquad \text{S11}$$

Substituting S10 into the normalized form of (11), we have

$$\overline{T}_i\left[\cos\theta_i-\lambda_i/\lambda_0\right]+\frac{\varepsilon}{\lambda_0}+\overline{T}_{Ri}\frac{\lambda_i}{\lambda_0}-1+\frac{1}{2\beta\lambda_0}\left(\lambda_i^2+\frac{1}{\lambda_i^2}-\lambda_0^2-\frac{1}{\lambda_0^2}\right)+\overline{\gamma}_{LV}\cos\theta_c=0 \quad \text{S12}$$

Substituting (14) into S12, S12 becomes:

$$\left(\overline{T}_0-1\right)-\left(\overline{T}_i-\overline{T}_{Ri}\right)\frac{\lambda_i}{\lambda_0}+\frac{\varepsilon}{\lambda_0}+\frac{1}{2\beta\lambda_0}\left(\lambda_i^2+\frac{1}{\lambda_i^2}-\lambda_0^2-\frac{1}{\lambda_0^2}\right)=0. \quad \text{S13}$$

Equations (4a,b) in normalized form are:

$$\left(\overline{T}_0-1\right)=\frac{1}{\beta}\left(\lambda_0-\left(\frac{1}{\lambda_0}\right)^3\right),\left(\overline{T}_i-\overline{T}_{Ri}\right)=\frac{1}{\beta}\left(\lambda_i-\left(\frac{1}{\lambda_i}\right)^3\right) \quad \text{S14}$$

Substituting S14 into S13, we have

$$\begin{aligned}&\frac{1}{\beta}\left(\lambda_0-\left(\frac{1}{\lambda_0}\right)^3\right)-\frac{1}{\beta}\left(\lambda_i-\left(\frac{1}{\lambda_i}\right)^3\right)\frac{\lambda_i}{\lambda_0}+\frac{\varepsilon}{\lambda_0}+\frac{1}{2\beta\lambda_0}\left(\lambda_i^2+\frac{1}{\lambda_i^2}-\lambda_0^2-\frac{1}{\lambda_0^2}\right)=0\Rightarrow\\&\frac{1}{2\beta\lambda_0}\left(\lambda_0^2-\lambda_i^2+\frac{3}{\lambda_i^2}-\frac{3}{\lambda_0^2}\right)+\frac{\varepsilon}{\lambda_0}=0\Rightarrow\frac{1}{2\beta}\left(\lambda_0^2-\lambda_i^2+\frac{3}{\lambda_i^2}-\frac{3}{\lambda_0^2}\right)+\varepsilon=0\end{aligned} \quad \text{S15}$$

which is (12a).

3. Behavior of Energy balance equation

The solution of (12a) is

$$\lambda_0^2=\frac{-\left(\frac{3}{\lambda_i^2}-\lambda_i^2+2\varepsilon\beta\right)+\sqrt{\left(\frac{3}{\lambda_i^2}-\lambda_i^2+2\varepsilon\beta\right)^2+12}}{2} \quad \text{S16}$$

The choice of the positive square root sign in S8 is based on the fact that

$$\sqrt{\left(\frac{3}{\lambda_i^2}-\lambda_i^2+2\varepsilon\beta\right)^2+12}>\left|\frac{3}{\lambda_i^2}-\lambda_i^2+2\varepsilon\beta\right|, \quad \text{S17}$$

so λ_0 is complex if the negative sign is chosen. Simple algebra shows that S16 can be written as

$$\lambda_0^2 = \frac{-\left(\frac{3}{\lambda_i^2} - \lambda_i^2 + 2\varepsilon\beta\right) + \sqrt{\left(\frac{3}{\lambda_i^2} + \lambda_i^2 + 2\varepsilon\beta\right)^2 - 4\beta\varepsilon\lambda_i^2}}{2} \qquad \text{S18}$$

Equation S16 implies that for $\varepsilon > 0$, $\lambda_i \geq \lambda_0$ and the reverse is true for $\varepsilon < 0$. Indeed, if $\varepsilon < 0$ the relation between λ_i, λ_0 can be obtained by reflecting S16 about the line $\lambda_i = \lambda_0$. Therefore, it is sufficient to study the case of $\varepsilon > 0$. Figure S2 plots an example α = +/-200. A simple analysis shows that

$$\lambda_0^2 \approx \lambda_i^2 - \frac{\alpha\lambda_i^4}{\left(3 + \alpha\lambda_i^2\right)} \qquad \text{for} \quad \lambda_i^2 << 1 \qquad \text{S19a}$$

$$\lambda_0^2 \approx \lambda_i^2 - \frac{\alpha\lambda_i^2}{\left(\lambda_i^2 + \alpha\right)} \qquad \text{for} \quad \lambda_i^2 >> 1 \qquad \text{S19b}$$

Note for large α, S19a implies that $\lambda_0^2 \approx \lambda_i^2$ only for very small values of λ_i^2. Also, for $\lambda_i^2 >> \alpha$, S10b implies that $\lambda_0^2 \approx \lambda_i^2 - \alpha$. The range of validity of S19a can be very small for large α as shown in Fig.S2. Fig. S2 shows that, for large α, there is rapid transition of behavior at $\lambda_i^2 \approx \alpha$. In this neighborhood, $\lambda_i \approx \sqrt{\alpha} >> 1$ while λ_0 is of order 1. The situation is reversed for $\alpha < 0$. Since the tensions are given approximately by $\lambda_i\left(\lambda_0\right)/\beta$ for large $\lambda_i\left(\lambda_0\right)$, the tensions in this regime go to zero like $1/\sqrt{\beta}$ as $\beta \to \infty$.

Note that $0 \leq \bar{\rho}_c \leq a$, since $\lambda_i = \frac{\bar{c}\theta_i}{\bar{\rho}_c \sin\theta_i}, \lambda_0 = \frac{\bar{a} - \bar{c}}{\bar{a} - \bar{\rho}_c}$

$$\bar{\rho}_c \to 0 \Rightarrow \lambda_i \to \infty, \lambda_0 = \frac{\bar{a} - \bar{c}}{\bar{a}}, \qquad \text{S20a}$$

$$\bar{\rho}_c \to a \Rightarrow \lambda_0 \to \infty, \lambda_i = \frac{\bar{c}\theta_i}{a\sin\theta_i} \qquad \text{S20b}$$

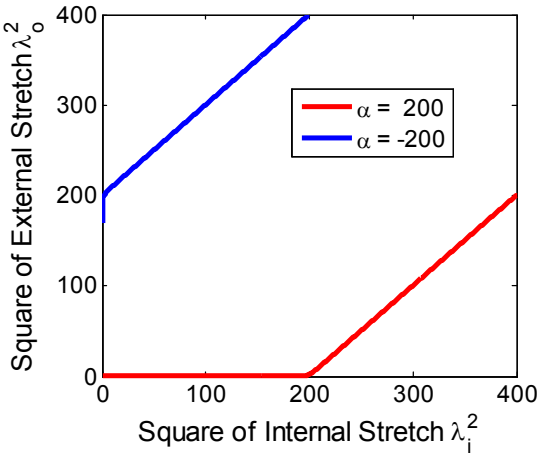

Figure S2 Relationship between stretches for a large absolute value of α.

4. Numerical implementation

We solved the normalized governing equations. These are the volume conservation equation (13), force balance equations (14, 15), and the normalized version of the energy balance equation (11)

$$\overline{T}_i\left[\cos\theta_i - \lambda_i/\lambda_o\right] + \frac{\overline{W}(\lambda_i) - \overline{W}(\lambda_0)}{\lambda_0} + \overline{\gamma}_{LV}\cos\theta_c = 0 \qquad \text{(S21)}$$

These four equations are completed by the relationship between tension and stretch (4a,b)

$$\overline{T}_i = \overline{T}_{Ri} + \mu h\left(\lambda_i - \lambda_i^{-3}\right) \quad \overline{T}_o = 1 + \mu h\left(\lambda_o - \lambda_o^{-3}\right) \qquad \text{(S22)}$$

and the relationships between stretch and geometrical parameters (2a,b)

$$\lambda_i = \frac{\overline{c}\,\theta_i}{\overline{\rho}_c \sin\theta_i}; \quad \lambda_o = \frac{\overline{a} - \overline{c}}{\overline{a} - \overline{\rho}_{c\ i}} \qquad \text{(S23)}$$

Together, these form eight coupled nonlinear equations for the eight unknowns $\theta_c, \theta_i, \overline{c}, \overline{\rho}_c, \lambda_i, \lambda_o, \overline{T}_i, \overline{T}_o$. This system of equations was solved using the nonlinear solution routines *"fsolve"* or "*lsqnonlin*" in the *Matlab®* computing environment. There are a number of alternative ways in which to approach and solve these equations numerically. For example it is possible to eliminate some variables to reduce the number of equations and unknowns. We implemented several combinations of solution methods, number of equations solved, and sets of unknowns. When the algorithms converged, results agreed across the different methods used. However, not all methods were equally robust; we used the one just described as it most reliably produced a converged solution.