

Supplementary Materials

Micro-mechanics of electrostatically stabilized suspensions of cellulose nanofibrils under steady state shear flow

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Appendix S1

For a prolate ellipsoid with an aspect ratio $r' = a/b$ (where a and b are the major and minor axes of the prolate ellipsoid, respectively), the shape factor writes $\lambda = (r'^2 - 1)/(r'^2 + 1)$ ¹. For more complicated particles such as the studied NFCs, the estimate of λ is not straightforward². For simplicity sake, as shown in Fig. 8, a first order estimation of λ was used. Hence, the

coefficient λ was calculated from the prolate ellipsoid that displays the same inertia axes and moments as NFC i .

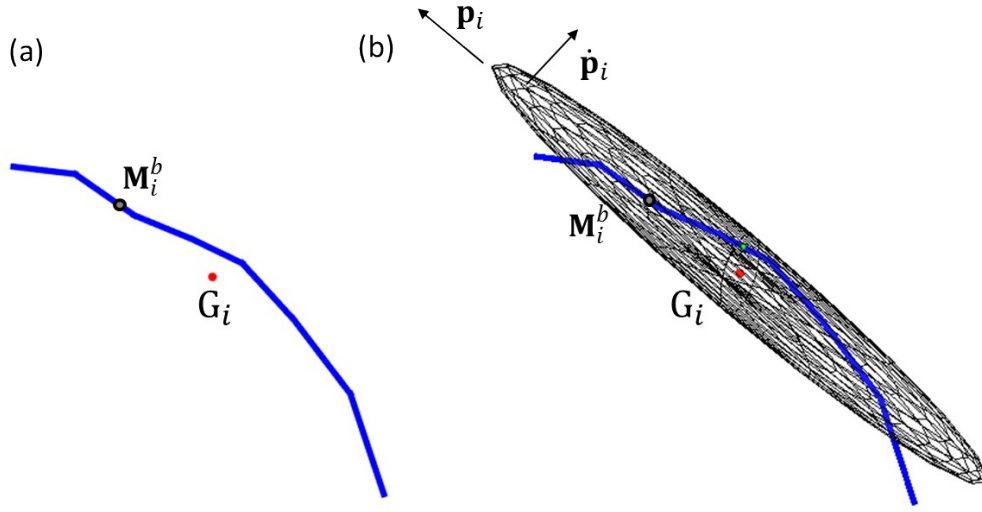


Figure 8. Sketch that shows (a) the NFC i and (b) its associated equivalent prolate ellipsoid with the same inertia axes and moments.

Appendix S2

Each tortuous nanofiber i of diameter d and length l contained n_{seg} straight segments of length l_{seg} that had a random relative misorientation θ_{seg} . Their center of mass G_i was randomly placed in the elementary volume Ω . Following this procedure, various fibrous networks with various nanofiber contents ϕ were generated with random orientation of nanofibers, *i.e.*, the generated microstructures exhibited isotropic fiber orientation with second order orientation

tensors $A = \frac{1}{3}e_i \otimes e_i$ (Fig. 5a and Fig. 5c for the corresponding pole figure of orientation). This type of microstructures was chosen because it constituted a reference state, potentially close to that of NFC suspensions at rest (where Brownian motion presumably induces particle dispersion^{3,4}).

To mimic microstructures with a pronounced NFC orientation along the flow direction³ under shear flow, *i.e.*, for $\nabla v = \dot{\gamma} e_1 \otimes e_2$, the evolution of the mean orientation vector p_i of each NFC i was computed up to the steady state overall orientation of the fibrous networks, solving the Jeffery-based model (eqn (17)) with $D_r = C_I \sqrt{2D:\overline{D}}$ and $C_I = 10^{-3}$ ^{5,6}. For convenience, calculations were done using the expansion in spheric harmonics up to the fourth order⁷ of the orientation distribution . Hence, the following approximation was used⁸:

$$\frac{1}{\psi} \frac{\partial \psi(p)}{\partial p} \approx 4 \frac{-21A \cdot p + 63p \cdot A : p \otimes p}{3 - 42A : p \otimes p + 63p \otimes p : A : p \otimes p}, \quad (22)$$

All the simulations were run using Matlab and a suitable time step $\Delta t = \Delta\gamma/\dot{\gamma}$ such that $\Delta\gamma = 0.01$ until reaching a steady state, which was close to a shear strain $\gamma = 100$. Then, the simulation procedure led to orientated fibrous microstructures with $A \approx 0.9e_1 \otimes e_1 + 0.05(e_2 \otimes e_2 + e_3 \otimes e_3)$ (Fig. 5b and Fig. 5d for the corresponding pole figures of orientation). In accordance with the Jeffery-based models, it is worth to note that the resulting fiber orientation only depended on initial orientation state and on the applied shear strain, but not on the shear rate $\dot{\gamma}$.

References

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