Electronic Supplementary Material (ESI) for Soft Matter.

Supporting Information

Title: Bistable self-assembly in homogeneous colloidal systems for flexible modular architectures

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Movie:

Field-induced transformation between staggered and compact three-particle cluster. External field pulses with field intensity of 1.2mT and duration of 1ms are applied.
The playback speed of the movie is accelerated by two with respect to the recording time.

Analytic expressions for the energies of three 3sd-particles

For a linear chain of three particles (meaning, that all three particle centers are on the y-axis, so $\varphi = \psi = 0$ and equal to the polar angle of the third particle central dipole), we obtain the distance between the dipoles belonging to the nearest neighboring particles in the chain

$$r^{ch-nn}_{ij} = 1 + s_i^2 + s_j^2 - 2s_i s_j Z^{ch}_{ij} - 2[s_i \cos(\beta i) - s_j \cos(\beta j)],$$

where $Z^{ch}_{ij} = \cos(\beta(i-j))$.

The distance between the dipoles in the next nearest neighboring (side particles) particles can be obtained as:

$$r^{ch-sp}_{ij} = 4 + s_i^2 + s_j^2 - 2s_i s_j Z^{ch}_{ij} - 4[s_i \cos(\beta i) - s_j \cos(\beta j)].$$

Finally, the dipolar interaction in the linear chain of three particles has the form:

$$U^{ch}_{total} = 2\sum_{i,j=-1}^{1} m_i m_j \left\{ \frac{Z^{ch}_{ij}}{\left(r^{ch-nn}_{ij}\right)^{\frac{3}{2}}} - 3\frac{\left[\cos(\beta i) + s_j Z^{ch}_{ij} - s_i\right]\left[\cos(\beta j) - s_i Z^{ch}_{ij} + s_j\right]}{\left(r^{ch-nn}_{ij}\right)^{\frac{5}{2}}} \right\} + \sum_{i,j=-1}^{1}$$

For a staggered configuration, the energy will also depend on the stagger angle θ between the center-center distances of the side particles.

The distance between the dipoles in the nearest neighboring particles in a staggered chain can be written as:

$$r_{i(j)k}^{nn} = d^2 + s_j^2 + s_k^2 - 2s_k\left[dcos\left(\beta k + (-)\frac{\theta}{2}\right) + s_{i(j)}Z_{i(j)k}\right] + 2ds_{i(j)}sin\left(\alpha + (-)\beta i(j) + \frac{\theta}{2}\right).$$

For the side particles, the distance between the dipoles is

$$r_{ij}^{sp} = 2d^2 + s_i^2 + s_j^2 + 2s_is_jZ_{ij} - 2d^2cos\theta + 4dsin\frac{\theta}{2}[s_i\cos(\alpha + \beta i) + s_j\cos(\alpha - \beta j)],$$

where $Z_{i(j)k} = sin(\alpha + (-)\beta(i(j) - k)),\ Z_{ij} = \cos(2\alpha + \beta(i - j))$.

The energies are

$$U_{i(j)k}^{nn} = m_{i(j)}m_k\left\{\frac{Z_{i(j)k}}{\left(r_{i(j)k}^{12}\right)^{\frac{3}{2}}} + (-)3\frac{\left[s_k - dcos\left(\beta k + (-)\frac{\theta}{2}\right) - s_{i(j)}Z_{i(j)k}\right]\left[s_{i(j)} + dsin\left(\alpha + (-)\beta i(j) + \frac{\theta}{2}\right) - s_kZ_{i(j)k}\right]}{\left(r_{i(j)k}^{12}\right)^{\frac{5}{2}}}\right\},$$

$$U_{ij}^{sp} = -m_im_j\left\{\frac{Z_{ij}}{\left(r_{ij}^{13}\right)^{\frac{3}{2}}} + 3\frac{\left[s_i + 2dsin\frac{\theta}{2}cos(\alpha + \beta i) + s_jZ_{ij}\right]\left[s_j + 2dsin\frac{\theta}{2}cos(\alpha - \beta i) - s_iZ_{ij}\right]}{\left(r_{ij}^{13}\right)^{\frac{5}{2}}}\right\},$$

$$U_{total}^{M} = \sum_{i,j,k=-1}^{1}\left\{U_{ik}^{12} + U_{ij}^{13} + U_{jk}^{23}\right\}.$$

This energy has to be minimized with respect to α, for a given value of θ. The final candidate for the ground state is the ring, in which the central dipoles of three particles assume a triangular alignment. This energy depends on only one angle, characterizing the deviations of central moments from the sides of the triangle based on the dipoles' positions.

$$r_{ij}^{ring} = d^2 + s_i^2 + s_j^2 - 2s_is_j\,Z_{ij}^{ring} + 2d\left[s_i\sin\left(\frac{\pi}{6} + \varphi + \beta i\right) + s_j\cos(\varphi + \beta j)\right],$$

where $Z_{ij}^{ring} = -sin\left(\frac{\pi}{6} + \beta(i - j)\right)$,

$$U_{total}^{ring} = 3\sum_{i,j=-1}^{1} m_im_j\left\{\frac{Z_{ij}^{ring}}{\left(r_{ij}^{ring}\right)^{\frac{3}{2}}} + 3\frac{\left[s_i - s_jZ_{ij}^{ring} + d\sin\left(\frac{\pi}{6} + \varphi + \beta i\right)\right]\left[s_j - s_iZ_{ij}^{ring} + d\cos(\varphi + \beta j)\right]}{\left(r_{ij}^{ring}\right)^{\frac{5}{2}}}\right\}.$$

The three candidates – the chain, the staggered chain and the ring – provide ground states or local energy minima depending on the parameters β, m_i, s_i for i=-1,0,1, as is shown in the main text.