## **Supplement Information**

# 1. Energy band structures of BP calculated using different exchange and/or correlation functionals



**Figure S1** Energy band structures of BP calculated using different exchange and/or correlation functionals: (a) DFT-D2 and mBJ, (b) optB86b-vdW and mBJ. The corresponding band gaps are indicated.

#### 2. The carrier relaxation time of BP

The relaxation time  $\tau$  can be derived from the experimentally measured carrier mobility ( $\mu$ ) and effective mass ( $m^*$ ) in the temperature range of 50 ~ 200 K [1, 2]. The obtained  $\tau$  can be generalized to higher temperatures by using an exponential fitting [3]. Table S1 lists all the fitted relaxation time of BP.

 Table S1 The fitted relaxation time of BP along three directions at different temperature.

carrier type	direction	300K	400K	500K	600K	700K	800K
		$(10^{-13}s)$	$(10^{-13}s)$	$(10^{-13}s)$	$(10^{-13}s)$	$(10^{-13}s)$	$(10^{-13}s)$
<i>p</i> -type	x	5.83	5.78	5.78	5.78	5.78	5.78
	У	0.90	0.89	0.89	0.89	0.89	0.89
	Ζ	1.57	1.55	1.55	1.55	1.55	1.55

<i>n</i> -type	x	2.38	2.35	2.35	2.35	2.35	2.35
	У	0.27	0.27	0.27	0.27	0.27	0.27
	Z	1.14	1.13	1.13	1.13	1.13	1.13

#### 3. The Lorenz number of BP as a function of carrier concentration



Figure S2 The calculated Lorenz number of BP as a function of carrier concentration at (a) 300 K and (b) 800 K. Negative and positive carrier concentrations represent n- and p- type carriers, respectively.

### 4. The phonon dispersion relations of Sb-substituted BP



**Figure S3** The phonon dispersion relations of  $P_{0.75}Sb_{0.25}$ .

## Reference

- [1] Y. Akahama, S. Endo, and S. Narita, J. Phys. Soc. Jpn. 52, 2148 (1983).
- [2] A. Morita, Appl. Phys. A **39**, 227 (1986).
- [3] J. Zhang, H. J. Liu, L. Cheng, J. Wei, J. Shi, X. F. Tang, and C. Uher, J. Appl.

Phys. 116, 023706 (2014).