

1 SUPPLEMENTARY MATERIAL

2 *Theory*

3 *MCR-ALS*

4 Multivariate curve-resolution (MCR) coupled to alternating least-squares (ALS) is capable of
5 handling data matrices with varying component profiles in one of the data dimensions. It can be
6 thought that this makes it especially suitable for the convenient processing of kinetic-
7 spectrophotometric matrix data; typically many examples of successful application of MCR-ALS for
8 resolving this kind of matrix data have been published in scientific literature.¹⁻³

9 In MCR-ALS, an augmented data matrix is created from a group of data matrices for several
10 samples. We consider matrices of size $J \times K$, where J is the number of data points in the spectral
11 dimension it means the number of wavelengths and K is the number of data points in the kinetic or
12 temporal dimension it means the number of reaction times. This mathematical resource allows this
13 algorithm to resolve linear dependence data. The augmented matrix could be constructed assembling
14 data matrices following columns direction, rows direction or both directions simultaneously. In general,
15 augmentation can be performed in either direction, depending on the type of experiment being
16 analyzed.

17 However, usually the matrices are augmented in the mode suspected to lose the linearity. In the
18 present case, the mode of augmentation should be the spectral one, since identical data profiles were
19 obtained in spectral dimension correspond to unreacted HCF. An augmented matrix \mathbf{D} of size $J(I+1) \times$
20 K was developed by joining calibration matrices $\mathbf{X}_{c,i}$ and the unknown matrix \mathbf{X}_u .^{4,5}

21 Therefore, the bilinear decomposition of the augmented matrix \mathbf{D} was performed according to
22 the expression:

$$23 \quad \mathbf{D} = \mathbf{S}_{\text{aug}} \mathbf{G}^T + \mathbf{E} \quad (1)$$

24 where the rows of \mathbf{D} contain the spectra measured for different samples at several values of the
 25 temporal dimension, \mathbf{S}_{aug} contains spectra for intervenient species, \mathbf{G} contains the temporal or kinetic
 26 profiles and \mathbf{E} is a matrix of residual not fitted by the model. The property dimensions for \mathbf{D} , \mathbf{S} , \mathbf{G} and
 27 \mathbf{E} are $(I + 1)J \times K$, $(I+1)J \times N$, $K \times N$, and $(I + 1)J \times K$ respectively (being I the number of calibration
 28 matrices , J the number of wavelengths , K number of temporal data and N is the number of
 29 responsive components). As can be seen, \mathbf{D} contains data for the I different samples.

30 The iterative ALS procedure aims at minimizing the Frobenius norm of $\|\mathbf{E}\|$, and was
 31 initialized using an initial estimation of the kinetic profiles of pure components used for calculating $\hat{\mathbf{S}}$
 32 ($\hat{\mathbf{S}}$ means the estimation of \mathbf{S}) as :

$$33 \quad \hat{\mathbf{S}} = \mathbf{D} \mathbf{G}^T+ \quad (2)$$

34 where '+' means the pseudoinverse of the matrix $\mathbf{G}^T = [\mathbf{G}(\mathbf{G}_T\mathbf{G})^{-1}]$.

35 Using the matrix $\hat{\mathbf{S}}$ (equation 2) and the original matrix \mathbf{D} , the matrix \mathbf{G} was re-estimated by
 36 least squares as :

$$37 \quad \hat{\mathbf{G}} = (\hat{\mathbf{S}}^+ \mathbf{D})^T \quad (3)$$

38 The generalized inverse of \mathbf{G} can be obtained only if the kinetic profiles of the sample
 39 components are different, as in the present case. The generalized inverse of \mathbf{S} could be obtained
 40 because the matrix augmentation broke the linear dependence of the individual spectral profiles.

41 Finally, \mathbf{E} was calculated applying equation (1) using matrix \mathbf{D} and the estimation of \mathbf{G} and $\hat{\mathbf{G}}$.
 42 These steps could be implemented in an alternating least squares cycles , so that in each iteration new
 43 \mathbf{S} and \mathbf{G} matrices were obtained. During the iterative recalculation of \mathbf{S} and \mathbf{G} a series of constraints
 44 were applied to improve these solutions, to give them a physical meaning and to limit their possible
 45 number for the same data fitting, such as 1) non-negativity for spectral and kinetic profiles, 2)
 46 stoichiometric relations among different chemical species in equilibrium or in kinetics. Iterations

47 continued until an optimal solution was obtained that fulfills the postulated constraints and the
48 established convergence criteria.

49 After MCR-ALS decomposition of matrix \mathbf{D} , concentration information contained in \mathbf{S} can be
50 used for quantitative predictions, by first defining the analyte concentration score as the area under the
51 profile for the i th. sample.

52 Calibration samples are always within those employed to build the augmented matrix \mathbf{D} . Their
53 associated scores can be used to build a pseudo-univariate calibration graph against the nominal analyte
54 concentrations. Prediction of analyte concentration in unknowns then proceeds by interpolation of the
55 corresponding analyte scores in the calibration graph .⁴

56

57 *U-PLS*

58 Unfolded partial least squares (U-PLS) operates in a similar way to partial least squares-1 (PLS-
59 1), except that second-order data are first vectorized or unfolded along one of the data dimensions, and
60 then a conventional partial least-squares (PLS) model is built using these unfolded data and the
61 nominal analyte concentrations. ^{6,7}

62 The \mathbf{I} calibration data matrices are first vectorized into $JK \times 1$ vectors, and then a usual PLS
63 model is built using these data together with the vector of calibration concentrations y (size $I \times 1$). This
64 provides a set of loadings \mathbf{P} and weight loadings \mathbf{W} (both of size $JK \times A$, where A is the number of
65 latent factors), as well as regression coefficients \mathbf{v} (size $A \times 1$)

66 The parameter A can be selected by techniques such as leave-one-out cross-validation .⁸

67 Notice that PLS is a latent variable method, and hence no prior information as to the spectral or
68 time evolution of the analyte are in principle required for its successful operation.

69 If no unexpected components are present in the test sample, \mathbf{v} could be used to estimate the
70 analyte concentration according to

71
$$\mathbf{y}_u = \mathbf{t}_u^T \mathbf{v} \quad (4)$$

72 where \mathbf{t}_u is the test sample score, obtained by projecting the vectorized data for the test sample
 73 $\text{vec}(\mathbf{X}_u)$ onto the space of the A latent factors:

74
$$\mathbf{t}_u = (\mathbf{W}^T \mathbf{P})^{-1} \mathbf{W}^T \text{vec}(\mathbf{X}_u) \quad (5)$$

75 where $\text{vec}(\cdot)$ implies the vectorization operator.

76

77 ***N-PLS***

78 Multiway regression methods such as N-PLS extend the tradicional PLS algorithm to higher orders,
 79 using the multidimensional structure of the data for model building and prediction .⁶

80 The tridimensional matrix \mathbf{X} ($I \times J \times K$) is decomposed in a series of triads. In the case of
 81 three-way data, the model is given by the following equation:

82
$$x_{ijk} = \sum_{f=1}^N t_i w_j^f w_k^f + e_{ijk} \quad (6)$$

83 where x_{ijk} is the variation of absorbance intensity for sample i at wavelength wavelength j and
 84 time k , N is the number of components, t is an element of the score matrix \mathbf{T} and, two w are
 85 elements of the two loading matrices \mathbf{W} , one in spectral dimension \mathbf{w}^J ($J \times 1$) and the other for
 86 temporal dimension \mathbf{w}^K ($K \times 1$), and e_{ijk} is a residue not fitted by the model. The model finds the scores
 87 yielding maximum covariance with analyte concentrations as the dependent variable. The advantage of
 88 using N-PLS over bidimensional regression is a stabilization of the decomposition involved in Eq. (6),
 89 which potentially gives increased interpretability and better predictions

90 The algorithm used the data matrix of the I calibration samples within the concentration vector
 91 \mathbf{Y} ($I \times 1$) for obtaining loading and the regression coefficient \mathbf{v} (size $A \times 1$). As U-PLS, if no unexpected

92 components are present in the test sample, \mathbf{v} could be used to estimate the analyte concentration
 93 according to

$$94 \quad \mathbf{y}_u = \mathbf{t}_u^T \mathbf{v} \quad (7)$$

95 Cross-validation can also be employed to estimate the number of calibration latent variables.⁸
 96

97 *N-PLS/RBL and U-PLS/RBL*

98 If unexpected constituents occur in a test sample, neither the U-PLS nor N-PLS scores for the
 99 latter sample can be used for analyte prediction using the trained model. In this case, it is necessary to
 100 resort to a technique which is able to: (1) detect the new sample as an outlier, indicating that further
 101 actions are necessary before prediction, and (2) isolate the contribution of the unexpected component
 102 from that of the calibrated analytes, in order to recalculate appropriate scores for the test sample. U-
 103 PLS and N-PLS will consider a sample as an outlier if the residuals of the test data reconstruction (s_p)
 104 are abnormally large in comparison with the typical instrumental noise.

$$105 \quad s_p = \|\mathbf{e}_p\| / (JK-A)^{1/2} = \|\text{vec}(\mathbf{X}_u) - \mathbf{P} (\mathbf{W}^T \mathbf{P})^{-1} \mathbf{W}^T \text{vec}(\mathbf{X}_u)\| / (JK-A)^{1/2} =$$

$$106 \quad = \|\text{vec}(\mathbf{X}_u) - \mathbf{P} \mathbf{t}_u\| / (JK-A)^{1/2} \quad (8)$$

107 $\|\cdot\|$ indicates the Euclidean norm

108 In such a case, residual bilinearization can be employed to model the presence of unexpected
 109 sample components using principal component analysis (PCA) or singular value decomposition (SVD),
 110 which allows one to estimate profiles for the unexpected components in the three data dimensions.⁷

111 For a single unexpected component the expression is :

$$112 \quad \text{vec}(\mathbf{X}_u) = \mathbf{P} \mathbf{t}_u + \text{vec}[\mathbf{g}_{\text{unx}} \mathbf{b}_{\text{unx}} (\mathbf{c}_{\text{unx}})^T] + \mathbf{e}_u \quad (9)$$

113 Where \mathbf{b}_{unx} and \mathbf{c}_{unx} are the left and right eigenvectors of \mathbf{E}_p and g_{unx} is a scaling factor
114 appropriate for SVD analysis:

$$115 \quad (\mathbf{g}_{\text{unx}}, \mathbf{b}_{\text{unx}}, \mathbf{c}_{\text{unx}}) = \text{SVD}_1(\mathbf{E}_p) \quad (10)$$

116 where \mathbf{E}_p is the $J \times K$ matrix obtained after reshaping the $JK \times 1$ \mathbf{e}_p vector of eq (8) and SVD_1
117 indicates taking the first principal component.

118 The RBL procedure consists in keeping constant the matrix of calibration loadings (P), and
119 varying the test sample scores (\mathbf{t}_u) until the norm of the residual vector $\|\mathbf{e}_u\|$ is minimized in eq (9)
120 using a Gauss-Newton procedure, so that a final value of \mathbf{t}_u vector is obtained and applied for
121 calculating the analyte concentration using eq.(8). So, the number of unexpected components (N_{unx}) can
122 be determined by comparing the final residuals \mathbf{su} with the instrumental noise level, with \mathbf{su} given by:

$$123 \quad S_u = \frac{\|\mathbf{e}_u\|}{[JK - (A + N_{\text{uns}})]^{1/2}} \quad (11)$$

124 where \mathbf{e}_u is calculated from Eq. (9). Typically, a plot of s_u computed for trial values of
125 N_{unx} will show decreasing values, starting at s_p when $N_{\text{unx}} = 0$, until it stabilizes at a value compatible
126 with the experimental noise, allowing to locate the correct number of unexpected components.^{7,9}
127 However, some reports that have been recently published in scientific literature suggest that the number
128 of unexpected components could be determined by compare value of the property of interest (sugar
129 concentration in the present case) with those obtained applying a referente method.¹⁰

130 Once the RBL step is finished, and the correct test sample scores have been found, they are
131 employed to provide the analyte concentration as is regularly done in all PLS models.

132

133

134 ***PLS/RBL-LD***

135 However, this classical RBL procedure is not appropriate when the unexpected components
136 have profiles that are identical to the analyte profile in one of the data dimensions.^{11,12}
137 In these cases a new RBL procedure for linear dependency (RBL-LD) is proposed. The underlying
138 idea is similar to that of the classical RBL method: to minimize the norm of the residual vector e_u ,
139 computed by fitting the test data to the sum of the relevant contributions (i.e., the part that is modeled
140 by the current calibration and the contribution from the interfering agents) but taking into account the
141 identical profiles in one of the data dimensions. This can be done either by modeling the residuals with
142 MCR-ALS or with PARAFAC with linear dependency (PARALIND) instead of using SVD or PCA.¹²

143

144 ***Software***

145 All routines employed to carry out the calculations described in this paper were written in
146 MATLAB 7.0.¹⁴ A Second-order multivariate calibration toolbox January 2013 For assistance read
147 the document 'mvc2_gui_manual.pdf' and Chemom. Intell. Lab. Syst. 96 (2009) 246 -251¹⁵ as well as
148 an MCR-ALS Multivariate curve resolution- alternating least-squares written by Alejandro Olivieri
149 Department of Analytical Chemistry University of Rosario Argentina were used in the present work
150 Moreover, the N-PLS code is available on the internet at <http://www.models.life.ku.dk/source/> PLS/RBL
151 is available from the authors on request, including a useful graphical user interface for data input and
152 parameter setting, of the type already described for first-order multivariate calibration^{22, 29, 56}, and
153 which also works for PARAFAC.

154 U-PLS/RBL is available at www.chemometry.com, including a graphical user interface.^{6,7, 15}
155 MCR-ALS was implemented using the graphical interface provided by Prof. Romá Tauler in his web
156 page <http://www.ub.edu/mcr/welcome.html>.¹⁶

157

158 **3.4 Figures of merit**

159 Figures of merit such as sensitivity (SEN), analytical sensitivity (γ_n) and limit of detection
 160 (LOD_n) are regularly employed for method comparison.

161 **MCR-ALS:** Sensitivity can be calculated as:

$$162 \quad SEN = S_n \left\{ \delta_n^T \left[Z_{Cal}^T \left(I - Z_{unx} Z_{unx}^+ \right) Z_{cal} \right]^{-1} \delta_n \right\}^{-\frac{1}{2}}$$

163 Being: S_n the slope of MCR-ALS pseudo-univariate plot divided by the number of data point
 164 in each individual data matrix in the augmented mode, δ_n column vector (size $N_{cal} \times 1$) where N_{cal} is
 165 the number of MCR-ALS components present in the calibration set, Z_{cal} profiles in the nonaugmented
 166 data mode for the components present in the calibration set, Z_{unk} profiles in the nonaugmented data
 167 mode for the unexpected sample components present in the tests samples.⁷ Limit of detection and
 168 quantitation can be computed using the traditional univariate approach.¹⁷

169

170 **U-PLS and N-PLS**

171 Sensitivity can be calculated in cases when the second order advantage operates as follows:

$$172 \quad SEN = S_n \left\{ \left(\mathbf{B}_{sus}^T \mathbf{P}_{b,uns} \mathbf{B}_{sus} \right) \left(\mathbf{C}_{sus}^T \mathbf{P}_{c,uns} \mathbf{C}_{sus} \right) \right\}^{-\frac{1}{2}}$$

173

174 where SEN is the sensitivity for component n, S_n is the integrated total signal for component n at
 175 unit concentration, \mathbf{B}_{sus} and \mathbf{C}_{sus} are the matrices containing the profiles for all suspected components
 176 (i.e., those present in the training set of samples) in each data dimension, ‘nn’ implies selecting the

177 (n,n) element corresponding to the nth. analyte of interest, ‘*’ implies the Hadamard matrix product,
 178 and the projection matrices $\mathbf{P}_{b,uns}$ and $\mathbf{P}_{c,uns}$ are given by:

179
$$\mathbf{P}_{b,uns} = \mathbf{I} - \mathbf{B}_{uns}\mathbf{B}_{u+ns}$$

180
$$\mathbf{P}_{c,uns} = \mathbf{I} - \mathbf{C}_{uns}\mathbf{C}_{u+ns}$$

181 where \mathbf{B}_{uns} and \mathbf{C}_{uns} contain the profiles for the unsuspected components as columns.

182 Notice that when the second-order advantage is employed, this equation implies that SEN for
 183 component n is sample-specific and cannot be defined for the multivariate method as a whole. In such
 184 cases an average value for a set of samples can be estimated and reported .¹⁸

185 In the case of PLS/RBL, the appropriate expression for the estimation of sensitivity is : ^{7, 18}

186
$$SEN_n = 1 / \| (\mathbf{P}_{eff}^+)^T \mathbf{v} \|$$

187 where \mathbf{v} is the ($A \times 1$) latent vector of regression coefficients for the PLS model, and \mathbf{P}_{eff} is a
 188 matrix given by:

189

190
$$\mathbf{P}_{eff} = (\mathbf{P}_{c,uns} \otimes \mathbf{P}_{b,uns})^T \mathbf{P}$$

191 where \mathbf{P} is the ($JK \times A$) loading matrix provided by the PLS model, $\mathbf{P}_{c,uns}$ and $\mathbf{P}_{b,uns}$ have the
 192 same meaning as above, and \otimes implies the Kronecker product.

193 More useful than SEN seems to be the analytical sensitivity γ_n , defined, in analogy with
 194 univariate calibration, as the quotient between SEN and the instrumental noise level. Its inverse
 195 establishes the minimum difference of concentration which can be appreciated across the lineal range,
 196 and is independent on instrument or scale.⁷ So that, the analytical sensitivity is suitable for comparing
 197 analytical methods based on different response nature.

198 Moreover, the limit of detection (LOD) can be calculated as a interval obtaining the lower and
 199 an upper limits of it , propopsed by Olivieri et al . in a recently publication ¹⁹

200
$$\text{LOD}_{\min} = 3.3 [\text{SEN}^{-2} \text{var}(x) + h_{0\min} \text{SEN}^{-2} \text{var}(x) + h_{0\min} \text{var}(y_{\text{cal}})]^{1/2}$$

201
$$\text{LOD}_{\max} = 3.3 [\text{SEN}^{-2} \text{var}(x) + h_{0\max} \text{SEN}^{-2} \text{var}(x) + h_{0\max} \text{var}(y_{\text{cal}})]^{1/2}$$

202 Being,
$$h_{0\min} = \frac{\bar{y}_{\text{cal}}^2}{\sum_{i=1}^I y_i^2}$$
 where y_i is the centered concentration for the i th calibration sample ,

203 meanwhile the upper limit can be estimated as $h_{0\max} = \max(h_{0\text{cal}})$, in which

204
$$h_{0\text{cal}} = h_{\text{cal}} + h_{0\min} \left[1 - \left(\frac{y_{\text{cal}}}{\bar{y}_{\text{cal}}} \right)^2 \right]$$
 where h_{cal} and y_{cal} are the leverage and (centered) analyte concentration

205 of a generic calibration sample. LOD_{\min} and LOD_{\max} depend on the leverage, which is a function of the

206 calibration score matrix \mathbf{T} so that the limits of the LOD will depend on the calibration design and the

207 number of calibration latent variables.¹⁹

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