

## Supplementary Information

# Prediction of proton affinities of organic molecules with the Any-Particle Molecular-Orbital second-order proton propagator approach

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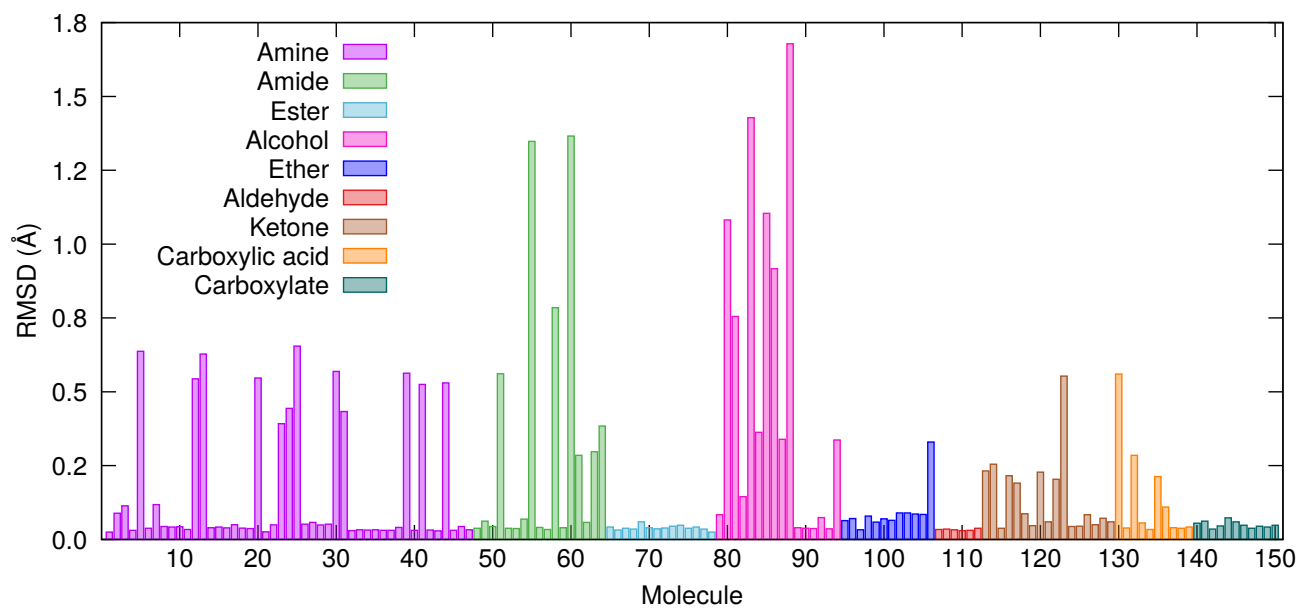


Figure S1: Root-mean-square deviation (RMSD) between protonated ( $AH^+$ ) and deprotonated (A) structures for the 150 molecules.

Table S1: Experimental ( $\text{PA}_{\text{exp}}$ ) and calculated ( $\text{PA}_{\text{PP2}}$ ) proton affinities, difference between  $\text{PA}_{\text{exp}}$  and  $\text{PA}_{\text{PP2}}$  values ( $\Delta\text{PA}_{\text{PP2}}^{\text{exp}}$ ), and Root-mean-square deviation (RMSD) between protonated ( $\text{AH}^+$ ) and deprotonated (A) amines.

| Molecule (A)     |                                                                                     | Proton affinity (kcal mol <sup>-1</sup> ) |                   |                                   | RMSD   |
|------------------|-------------------------------------------------------------------------------------|-------------------------------------------|-------------------|-----------------------------------|--------|
|                  |                                                                                     | PA <sub>exp</sub>                         | PA <sub>PP2</sub> | ΔPA <sub>PP2</sub> <sup>exp</sup> |        |
| Primary amines   |                                                                                     |                                           |                   |                                   |        |
| 1                | NH <sub>3</sub>                                                                     | 204.0150                                  | 203.0074          | 1.0076                            | 0.0250 |
| 2                | CHF <sub>2</sub> CH <sub>2</sub> NH <sub>2</sub>                                    | 208.0545                                  | 207.9932          | 0.0613                            | 0.0890 |
| 3                | CH <sub>2</sub> FCH <sub>2</sub> NH <sub>2</sub>                                    | 213.1931                                  | 213.2649          | -0.0718                           | 0.1140 |
| 4                | CH <sub>3</sub> NH <sub>2</sub>                                                     | 214.8660                                  | 214.3026          | 0.5634                            | 0.0310 |
| 5                | (c-C <sub>3</sub> H <sub>5</sub> )NH <sub>2</sub>                                   | 216.2000                                  | 216.4450          | -0.2450                           | 0.6370 |
| 6                | CH <sub>3</sub> CH <sub>2</sub> NH <sub>2</sub>                                     | 217.9732                                  | 217.9324          | 0.0408                            | 0.0382 |
| 7                | CH <sub>2</sub> =(CH <sub>3</sub> )CH <sub>2</sub> NH <sub>2</sub>                  | 219.3000                                  | 218.7165          | 0.5835                            | 0.1180 |
| 8                | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>2</sub> NH <sub>2</sub>                     | 219.3595                                  | 219.6320          | -0.2725                           | 0.0438 |
| 9                | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>3</sub> NH <sub>2</sub>                     | 220.2438                                  | 220.6167          | -0.3729                           | 0.0420 |
| 10               | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>4</sub> NH <sub>2</sub>                     | 220.7218                                  | 221.1563          | -0.4345                           | 0.0430 |
| 11               | (CH <sub>3</sub> ) <sub>2</sub> CHNH <sub>2</sub>                                   | 220.7935                                  | 220.6698          | 0.1237                            | 0.0339 |
| 12               | (CH <sub>3</sub> ) <sub>2</sub> CHCH <sub>2</sub> NH <sub>2</sub>                   | 221.0325                                  | 221.4031          | -0.3706                           | 0.5440 |
| 13               | (c-C <sub>6</sub> H <sub>11</sub> )CH <sub>2</sub> NH <sub>2</sub>                  | 221.5000                                  | 222.5515          | -1.0515                           | 0.6280 |
| 14               | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>6</sub> NH <sub>2</sub>                     | 221.5583                                  | 221.6314          | -0.0731                           | 0.0400 |
| 15               | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>5</sub> NH <sub>2</sub>                     | 221.6778                                  | 221.4423          | 0.2355                            | 0.0420 |
| 16               | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>7</sub> NH <sub>2</sub>                     | 222.0124                                  | 221.7490          | 0.2634                            | 0.0395 |
| 17               | CH <sub>3</sub> CH <sub>2</sub> (CH <sub>3</sub> )CHNH <sub>2</sub>                 | 222.2036                                  | 222.3947          | -0.1911                           | 0.0500 |
| 18               | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>8</sub> NH <sub>2</sub>                     | 222.2753                                  | 221.8274          | 0.4479                            | 0.0385 |
| 19               | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>9</sub> NH <sub>2</sub>                     | 222.4000                                  | 221.6729          | 0.7271                            | 0.0372 |
| 20               | (CH <sub>3</sub> ) <sub>3</sub> CNH <sub>2</sub>                                    | 223.2553                                  | 224.3664          | -1.1112                           | 0.5465 |
| 21               | (C <sub>6</sub> H <sub>11</sub> )NH <sub>2</sub>                                    | 223.3270                                  | 224.1381          | -0.8112                           | 0.0260 |
| 22               | CH <sub>3</sub> CH <sub>2</sub> C(CH <sub>3</sub> ) <sub>2</sub> NH <sub>2</sub>    | 224.1396                                  | 224.6616          | -0.5220                           | 0.0494 |
| Secondary amines |                                                                                     |                                           |                   |                                   |        |
| 23               | (CH <sub>3</sub> ) <sub>2</sub> NH                                                  | 222.1560                                  | 221.9750          | 0.1810                            | 0.3920 |
| 24               | CH <sub>3</sub> CH <sub>2</sub> NHCH <sub>3</sub>                                   | 225.1912                                  | 224.9844          | 0.2068                            | 0.4440 |
| 25               | (CH <sub>3</sub> ) <sub>2</sub> CHNHCH <sub>3</sub>                                 | 227.6052                                  | 227.2951          | 0.3100                            | 0.6546 |
| 26               | (CH <sub>3</sub> CH <sub>2</sub> ) <sub>2</sub> NH                                  | 227.6291                                  | 227.7126          | -0.0835                           | 0.0517 |
| 27               | CH(CH <sub>3</sub> ) <sub>2</sub> NHCH <sub>2</sub> CH <sub>3</sub>                 | 229.4455                                  | 229.8365          | -0.3910                           | 0.0580 |
| 28               | (C <sub>3</sub> H <sub>7</sub> ) <sub>2</sub> NH                                    | 229.9952                                  | 230.4776          | -0.4823                           | 0.0490 |
| 29               | (C <sub>4</sub> H <sub>9</sub> ) <sub>2</sub> NH                                    | 231.4771                                  | 232.0688          | -0.5917                           | 0.0520 |
| 30               | (s-C <sub>4</sub> H <sub>9</sub> ) <sub>2</sub> NH                                  | 234.3929                                  | 235.0229          | -0.6299                           | 0.5690 |
| 31               | (t-C <sub>4</sub> H <sub>9</sub> ) <sub>2</sub> NH                                  | 236.1138                                  | 236.3189          | -0.2051                           | 0.4330 |
| Tertiary amines  |                                                                                     |                                           |                   |                                   |        |
| 32               | (CH <sub>3</sub> ) <sub>3</sub> N                                                   | 226.7925                                  | 226.6356          | 0.1569                            | 0.0300 |
| 33               | CH <sub>3</sub> CH <sub>2</sub> N(CH <sub>3</sub> ) <sub>2</sub>                    | 229.5000                                  | 229.5505          | -0.0505                           | 0.0330 |
| 34               | (CH <sub>3</sub> ) <sub>2</sub> N(n-C <sub>3</sub> H <sub>7</sub> )                 | 230.1000                                  | 230.8581          | -0.7581                           | 0.0320 |
| 35               | (CH <sub>3</sub> ) <sub>2</sub> CHCH <sub>2</sub> N(CH <sub>3</sub> ) <sub>2</sub>  | 231.5249                                  | 231.3838          | 0.1410                            | 0.0330 |
| 36               | (C <sub>4</sub> H <sub>7</sub> )N(CH <sub>3</sub> ) <sub>2</sub>                    | 231.6444                                  | 231.6490          | -0.0047                           | 0.0310 |
| 37               | (CH <sub>3</sub> ) <sub>2</sub> CHN(CH <sub>3</sub> ) <sub>2</sub>                  | 231.9790                                  | 231.7782          | 0.2008                            | 0.0310 |
| 38               | (C <sub>2</sub> H <sub>5</sub> ) <sub>2</sub> NCH <sub>3</sub>                      | 232.1000                                  | 232.1103          | -0.0103                           | 0.0410 |
| 39               | (c-C <sub>6</sub> H <sub>11</sub> )CH <sub>2</sub> N(CH <sub>3</sub> ) <sub>2</sub> | 233.1740                                  | 232.3432          | 0.8308                            | 0.5630 |
| 40               | (s-C <sub>4</sub> H <sub>9</sub> )(CH <sub>3</sub> ) <sub>2</sub> N                 | 233.2457                                  | 233.2979          | -0.0522                           | 0.0310 |
| 41               | (C <sub>2</sub> H <sub>5</sub> ) <sub>2</sub> N(C <sub>3</sub> H <sub>7</sub> )     | 233.9388                                  | 233.4109          | 0.5279                            | 0.5250 |
| 42               | (t-C <sub>4</sub> H <sub>9</sub> )(CH <sub>3</sub> ) <sub>2</sub> N                 | 234.1300                                  | 234.1788          | -0.0488                           | 0.0320 |
| 43               | (C <sub>2</sub> H <sub>5</sub> ) <sub>3</sub> N                                     | 234.6558                                  | 234.0935          | 0.5623                            | 0.0290 |
| 44               | (t-C <sub>5</sub> H <sub>11</sub> )(CH <sub>3</sub> ) <sub>2</sub> N                | 234.8231                                  | 235.7562          | -0.9331                           | 0.5300 |
| 45               | (c-C <sub>6</sub> H <sub>11</sub> )N(CH <sub>3</sub> ) <sub>2</sub>                 | 235.0860                                  | 234.7254          | 0.3607                            | 0.0310 |
| 46               | (C <sub>3</sub> H <sub>7</sub> ) <sub>3</sub> N                                     | 236.9001                                  | 237.1191          | -0.2190                           | 0.0440 |
| 47               | (n-C <sub>4</sub> H <sub>9</sub> ) <sub>3</sub> N                                   | 238.6472                                  | 239.1323          | -0.4851                           | 0.0330 |

Table S2: Experimental ( $PA_{\text{exp}}$ ) and calculated ( $PA_{\text{PP2}}$ ) proton affinities, difference between  $PA_{\text{exp}}$  and  $PA_{\text{PP2}}$  values ( $\Delta PA_{\text{PP2}}^{\text{exp}}$ ), and Root-mean-square deviation (RMSD) between protonated ( $AH^+$ ) and deprotonated (A) amides.

| Molecule (A)                                                                           | Proton affinity (kcal mol <sup>-1</sup> ) |                   |                                   | RMSD   |
|----------------------------------------------------------------------------------------|-------------------------------------------|-------------------|-----------------------------------|--------|
|                                                                                        | PA <sub>exp</sub>                         | PA <sub>PP2</sub> | ΔPA <sub>PP2</sub> <sup>exp</sup> |        |
| Amides                                                                                 |                                           |                   |                                   |        |
| 48 HCONH <sub>2</sub>                                                                  | 196.5000                                  | 195.4319          | 1.0681                            | 0.0380 |
| 49 HCONHCH <sub>3</sub>                                                                | 203.4656                                  | 202.5231          | 0.9425                            | 0.0620 |
| 50 CH <sub>3</sub> CONH <sub>2</sub>                                                   | 206.4054                                  | 206.1921          | 0.2132                            | 0.0440 |
| 51 CH <sub>3</sub> CH <sub>2</sub> CONH <sub>2</sub>                                   | 209.4168                                  | 208.8441          | 0.5727                            | 0.5610 |
| 52 n-C <sub>3</sub> H <sub>7</sub> NHCHO                                               | 209.9426                                  | 210.1194          | -0.1768                           | 0.0380 |
| 53 (CH <sub>3</sub> ) <sub>2</sub> CHCNH <sub>2</sub> O                                | 209.9904                                  | 210.6936          | -0.7032                           | 0.0370 |
| 54 (CH <sub>3</sub> ) <sub>2</sub> CHCONH <sub>2</sub>                                 | 210.4000                                  | 211.1502          | -0.7502                           | 0.0690 |
| 55 (CH <sub>3</sub> ) <sub>2</sub> NCHO                                                | 212.1176                                  | 210.4077          | 1.7099                            | 1.3480 |
| 56 CH <sub>3</sub> CONHCH <sub>2</sub>                                                 | 212.3566                                  | 211.4569          | 0.8997                            | 0.0410 |
| 57 (CH <sub>3</sub> ) <sub>3</sub> CONH <sub>2</sub>                                   | 212.5000                                  | 213.4125          | -0.9125                           | 0.0340 |
| 58 CH <sub>3</sub> CONHC <sub>2</sub> H <sub>5</sub>                                   | 214.6272                                  | 214.3096          | 0.3176                            | 0.7850 |
| 59 CH <sub>3</sub> CON(CH <sub>3</sub> ) <sub>2</sub>                                  | 217.0172                                  | 218.1054          | -1.0882                           | 0.0400 |
| 60 CH <sub>3</sub> CH <sub>2</sub> CH <sub>2</sub> CON(CH <sub>3</sub> ) <sub>2</sub>  | 220.2916                                  | 219.9572          | 0.3344                            | 1.3660 |
| 61 C <sub>3</sub> H <sub>7</sub> CON(CH <sub>3</sub> ) <sub>2</sub>                    | 220.7696                                  | 221.0226          | -0.2530                           | 0.2850 |
| 62 CH <sub>3</sub> CON(C <sub>2</sub> H <sub>5</sub> ) <sub>2</sub>                    | 221.1759                                  | 220.9626          | 0.2133                            | 0.0580 |
| 63 (CH <sub>3</sub> ) <sub>3</sub> CCON(CH <sub>3</sub> ) <sub>2</sub>                 | 221.5822                                  | 221.6706          | -0.0884                           | 0.2970 |
| 64 (CH <sub>3</sub> ) <sub>3</sub> CCH <sub>2</sub> CON(CH <sub>3</sub> ) <sub>2</sub> | 221.7256                                  | 222.6253          | -0.8997                           | 0.3840 |

Table S3: Experimental ( $PA_{\text{exp}}$ ) and calculated ( $PA_{\text{PP2}}$ ) proton affinities, difference between  $PA_{\text{exp}}$  and  $PA_{\text{PP2}}$  values ( $\Delta PA_{\text{PP2}}^{\text{exp}}$ ), and Root-mean-square deviation (RMSD) between protonated ( $AH^+$ ) and deprotonated (A) esters.

| Molecule (A) |                                                      | Proton affinity (kcal mol <sup>-1</sup> ) |                   |                                   | RMSD   |
|--------------|------------------------------------------------------|-------------------------------------------|-------------------|-----------------------------------|--------|
|              |                                                      | PA <sub>exp</sub>                         | PA <sub>PP2</sub> | ΔPA <sub>PP2</sub> <sup>exp</sup> |        |
| Esters       |                                                      |                                           |                   |                                   |        |
| 65           | HCOOCH <sub>3</sub>                                  | 187.0219                                  | 186.5950          | 0.4270                            | 0.0420 |
| 66           | HCO <sub>2</sub> C <sub>2</sub> H <sub>5</sub>       | 191.0611                                  | 191.4931          | -0.4319                           | 0.0320 |
| 67           | HCO <sub>2</sub> (n-C <sub>3</sub> H <sub>7</sub> )  | 192.3757                                  | 193.0105          | -0.6348                           | 0.0380 |
| 68           | HCO <sub>2</sub> (n-C <sub>4</sub> H <sub>9</sub> )  | 192.6386                                  | 193.3933          | -0.7547                           | 0.0350 |
| 69           | HCOOCH(CH <sub>3</sub> ) <sub>2</sub>                | 193.9053                                  | 195.7294          | -1.8240                           | 0.0600 |
| 70           | CH <sub>3</sub> CO <sub>2</sub> CH <sub>3</sub>      | 196.3671                                  | 196.5088          | -0.1417                           | 0.0400 |
| 71           | C <sub>2</sub> H <sub>5</sub> COOCH <sub>3</sub>     | 198.4225                                  | 197.6158          | 0.8068                            | 0.0360 |
| 72           | CH <sub>3</sub> COOCH <sub>2</sub> CH <sub>3</sub>   | 199.7370                                  | 200.6667          | -0.9296                           | 0.0390 |
| 73           | C <sub>3</sub> H <sub>7</sub> COOCH <sub>3</sub>     | 199.9043                                  | 198.9694          | 0.9350                            | 0.0450 |
| 74           | i-C <sub>3</sub> H <sub>7</sub> COOCH <sub>3</sub>   | 199.9521                                  | 198.9671          | 0.9851                            | 0.0480 |
| 75           | C <sub>3</sub> H <sub>7</sub> COOCH <sub>3</sub>     | 199.9521                                  | 201.8105          | -1.8583                           | 0.0380 |
| 76           | (c-C <sub>3</sub> H <sub>5</sub> )COOCH <sub>3</sub> | 201.2906                                  | 200.8166          | 0.4740                            | 0.0420 |
| 77           | t-C <sub>4</sub> H <sub>9</sub> COOCH <sub>3</sub>   | 202.0076                                  | 201.6238          | 0.3839                            | 0.0350 |
| 78           | CH <sub>3</sub> CH=CHCOOCH <sub>3</sub>              | 203.4894                                  | 203.7523          | -0.2628                           | 0.0250 |

Table S4: Experimental ( $PA_{\text{exp}}$ ) and calculated ( $PA_{\text{PP2}}$ ) proton affinities, difference between  $PA_{\text{exp}}$  and  $PA_{\text{PP2}}$  values ( $\Delta PA_{\text{PP2}}^{\text{exp}}$ ), and Root-mean-square deviation (RMSD) between protonated ( $AH^+$ ) and deprotonated (A) alcohols.

| Molecule (A) |                                                       | Proton affinity (kcal mol <sup>-1</sup> ) |                   |                                   | RMSD   |
|--------------|-------------------------------------------------------|-------------------------------------------|-------------------|-----------------------------------|--------|
|              |                                                       | PA <sub>exp</sub>                         | PA <sub>PP2</sub> | ΔPA <sub>PP2</sub> <sup>exp</sup> |        |
| Alcohols     |                                                       |                                           |                   |                                   |        |
| 79           | CH <sub>2</sub> FCH <sub>2</sub> OH                   | 171.0325                                  | 170.8744          | 0.1582                            | 0.0840 |
| 80           | CHF <sub>2</sub> CH <sub>2</sub> OH                   | 173.8528                                  | 172.4725          | 1.3803                            | 1.0820 |
| 81           | CH <sub>3</sub> OH                                    | 180.2820                                  | 180.5668          | -0.2848                           | 0.7550 |
| 82           | CH <sub>3</sub> CH <sub>2</sub> OH                    | 185.5641                                  | 185.2321          | 0.3320                            | 0.1450 |
| 83           | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>2</sub> OH    | 187.9780                                  | 187.4090          | 0.5690                            | 1.4280 |
| 84           | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>3</sub> OH    | 188.6233                                  | 188.8250          | -0.2016                           | 0.3630 |
| 85           | (CH <sub>3</sub> ) <sub>2</sub> CH <sub>2</sub> OH    | 189.6988                                  | 190.0610          | -0.3622                           | 1.1040 |
| 86           | i-CH <sub>3</sub> (CH <sub>2</sub> ) <sub>2</sub> OH  | 189.7705                                  | 191.4539          | -1.6833                           | 0.9170 |
| 87           | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>4</sub> OH    | 190.0096                                  | 189.6252          | 0.3844                            | 0.3390 |
| 88           | (CH <sub>3</sub> ) <sub>3</sub> CCH <sub>2</sub> OH   | 190.1291                                  | 189.5860          | 0.5431                            | 1.6780 |
| 89           | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>5</sub> OH    | 190.9656                                  | 190.0034          | 0.9622                            | 0.0400 |
| 90           | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>6</sub> OH    | 190.9656                                  | 190.2340          | 0.7316                            | 0.0380 |
| 91           | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>7</sub> OH    | 190.9656                                  | 190.3770          | 0.5886                            | 0.0370 |
| 92           | (c-C <sub>5</sub> H <sub>9</sub> )OH                  | 191.0000                                  | 191.4147          | -0.4147                           | 0.0740 |
| 93           | (c-C <sub>6</sub> H <sub>11</sub> )CH <sub>2</sub> OH | 191.7065                                  | 191.5946          | 0.1119                            | 0.0360 |
| 94           | CH <sub>3</sub> CH <sub>2</sub> CH(OH)CH <sub>3</sub> | 194.7897                                  | 194.8554          | -0.0657                           | 0.3370 |

Table S5: Experimental ( $PA_{\text{exp}}$ ) and calculated ( $PA_{\text{PP2}}$ ) proton affinities, difference between  $PA_{\text{exp}}$  and  $PA_{\text{PP2}}$  values ( $\Delta PA_{\text{PP2}}^{\text{exp}}$ ), and Root-mean-square deviation (RMSD) between protonated ( $AH^+$ ) and deprotonated (A) ethers.

| Molecule (A) |                                                                     | Proton affinity (kcal mol <sup>-1</sup> ) |                   |                                   | RMSD   |
|--------------|---------------------------------------------------------------------|-------------------------------------------|-------------------|-----------------------------------|--------|
|              |                                                                     | PA <sub>exp</sub>                         | PA <sub>PP2</sub> | ΔPA <sub>PP2</sub> <sup>exp</sup> |        |
| Ethers       |                                                                     |                                           |                   |                                   |        |
| 95           | CH <sub>3</sub> OCH <sub>3</sub>                                    | 189.2925                                  | 189.1709          | 0.1217                            | 0.0640 |
| 96           | CH <sub>3</sub> CH <sub>2</sub> OCH <sub>3</sub>                    | 193.2600                                  | 193.9537          | -0.6937                           | 0.0710 |
| 97           | n-C <sub>3</sub> H <sub>7</sub> OCH <sub>3</sub>                    | 194.7658                                  | 194.7401          | 0.0257                            | 0.0330 |
| 98           | (C <sub>4</sub> H <sub>9</sub> )OCH <sub>3</sub>                    | 196.0564                                  | 196.7902          | -0.7338                           | 0.0790 |
| 99           | (CH <sub>3</sub> ) <sub>2</sub> CHOCH <sub>3</sub>                  | 197.4904                                  | 199.3061          | -1.8157                           | 0.0590 |
| 100          | (C <sub>2</sub> H <sub>5</sub> ) <sub>2</sub> O                     | 197.9924                                  | 198.0447          | -0.0524                           | 0.0700 |
| 101          | (c-C <sub>6</sub> H <sub>11</sub> )CH <sub>2</sub> OCH <sub>3</sub> | 199.2113                                  | 198.8910          | 0.3202                            | 0.0650 |
| 102          | (C <sub>3</sub> H <sub>7</sub> ) <sub>2</sub> O                     | 200.2629                                  | 201.0242          | -0.7613                           | 0.0900 |
| 103          | (c-C <sub>6</sub> H <sub>11</sub> )OCH <sub>3</sub>                 | 200.8843                                  | 201.5569          | -0.6726                           | 0.0900 |
| 104          | (CH <sub>3</sub> ) <sub>2</sub> CHOCH <sub>2</sub> CH <sub>3</sub>  | 201.4101                                  | 202.5946          | -1.1845                           | 0.0860 |
| 105          | (C <sub>4</sub> H <sub>9</sub> ) <sub>2</sub> O                     | 203.6329                                  | 202.9359          | 0.6970                            | 0.0850 |
| 106          | (n-C <sub>5</sub> H <sub>11</sub> ) <sub>2</sub> O                  | 203.8002                                  | 204.8730          | -1.0728                           | 0.3300 |

Table S6: Experimental ( $\text{PA}_{\text{exp}}$ ) and calculated ( $\text{PA}_{\text{PP2}}$ ) proton affinities, difference between  $\text{PA}_{\text{exp}}$  and  $\text{PA}_{\text{PP2}}$  values ( $\Delta\text{PA}_{\text{PP2}}^{\text{exp}}$ ), and Root-mean-square deviation (RMSD) between protonated ( $\text{AH}^+$ ) and deprotonated (A) aldehydes.

| Molecule (A) |                                       | Proton affinity (kcal mol <sup>-1</sup> ) |                   |                                   | RMSD   |
|--------------|---------------------------------------|-------------------------------------------|-------------------|-----------------------------------|--------|
|              |                                       | PA <sub>exp</sub>                         | PA <sub>PP2</sub> | ΔPA <sub>PP2</sub> <sup>exp</sup> |        |
| Aldehydes    |                                       |                                           |                   |                                   |        |
| 107          | CH <sub>2</sub> O                     | 170.3872                                  | 167.0601          | 3.3271                            | 0.0340 |
| 108          | CH <sub>3</sub> CHO                   | 183.6759                                  | 183.9891          | -0.3132                           | 0.0350 |
| 109          | CH <sub>3</sub> CH <sub>2</sub> CHO   | 187.8585                                  | 187.7296          | 0.1289                            | 0.0330 |
| 110          | n-C <sub>3</sub> H <sub>7</sub> CHO   | 189.4598                                  | 189.3784          | 0.0814                            | 0.0300 |
| 111          | n-C <sub>4</sub> H <sub>9</sub> CHO   | 190.3920                                  | 190.2593          | 0.1326                            | 0.0310 |
| 112          | (CH <sub>3</sub> ) <sub>2</sub> CHCOH | 190.5593                                  | 191.2533          | -0.6940                           | 0.0380 |

Table S7: Experimental ( $\text{PA}_{\text{exp}}$ ) and calculated ( $\text{PA}_{\text{PP2}}$ ) proton affinities, difference between  $\text{PA}_{\text{exp}}$  and  $\text{PA}_{\text{PP2}}$  values ( $\Delta\text{PA}_{\text{PP2}}^{\text{exp}}$ ), and Root-mean-square deviation (RMSD) between protonated ( $\text{AH}^+$ ) and deprotonated (A) ketones.

| Molecule (A) |                                                                                   | Proton affinity (kcal mol <sup>-1</sup> ) |                   |                                   | RMSD   |
|--------------|-----------------------------------------------------------------------------------|-------------------------------------------|-------------------|-----------------------------------|--------|
|              |                                                                                   | PA <sub>exp</sub>                         | PA <sub>PP2</sub> | ΔPA <sub>PP2</sub> <sup>exp</sup> |        |
| Ketones      |                                                                                   |                                           |                   |                                   |        |
| 113          | (CH <sub>3</sub> ) <sub>2</sub> CO                                                | 194.0727                                  | 193.9791          | 0.0936                            | 0.2320 |
| 114          | c-(C <sub>4</sub> H <sub>6</sub> )=O                                              | 194.6000                                  | 195.9000          | -1.3000                           | 0.2550 |
| 115          | c-(C <sub>5</sub> H <sub>8</sub> )=O                                              | 196.8690                                  | 197.7311          | -0.8621                           | 0.0380 |
| 116          | CH <sub>3</sub> COC <sub>2</sub> H <sub>5</sub>                                   | 197.7294                                  | 196.6057          | 1.1237                            | 0.2160 |
| 117          | n-C <sub>3</sub> H <sub>7</sub> COCH <sub>3</sub>                                 | 199.0201                                  | 198.0424          | 0.9777                            | 0.1910 |
| 118          | (CH <sub>3</sub> ) <sub>2</sub> CHCOCH <sub>3</sub>                               | 199.8805                                  | 200.9757          | -1.0952                           | 0.0870 |
| 119          | (C <sub>2</sub> H <sub>5</sub> ) <sub>2</sub> CO                                  | 200.0000                                  | 198.2684          | 1.7316                            | 0.0470 |
| 120          | (CH <sub>3</sub> ) <sub>3</sub> COCH <sub>3</sub>                                 | 200.7887                                  | 199.4099          | 1.3788                            | 0.2280 |
| 121          | CH <sub>3</sub> CH <sub>2</sub> CH <sub>2</sub> COCH <sub>2</sub> CH <sub>3</sub> | 201.5296                                  | 200.4569          | 1.0728                            | 0.0600 |
| 122          | 4-(CH <sub>3</sub> )C <sub>6</sub> H <sub>9</sub> =O                              | 201.9359                                  | 203.9967          | -2.0608                           | 0.2040 |
| 123          | (C <sub>3</sub> H <sub>7</sub> ) <sub>2</sub> CO                                  | 201.9598                                  | 199.6128          | 2.3470                            | 0.5530 |
| 124          | c-(C <sub>8</sub> H <sub>14</sub> )=O                                             | 203.0115                                  | 203.9829          | -0.9714                           | 0.0440 |
| 125          | (i-C <sub>3</sub> H <sub>7</sub> ) <sub>2</sub> CO                                | 203.2266                                  | 204.2181          | -0.9915                           | 0.0450 |
| 126          | c-(C <sub>6</sub> H <sub>10</sub> )=O                                             | 203.6000                                  | 203.0812          | 0.5188                            | 0.0840 |
| 127          | (c-C <sub>3</sub> H <sub>5</sub> )COCH <sub>3</sub>                               | 204.3260                                  | 203.4433          | 0.8827                            | 0.0500 |
| 128          | c-(C <sub>7</sub> H <sub>12</sub> )=O                                             | 204.5000                                  | 203.9414          | 0.5586                            | 0.0720 |
| 129          | (t-C <sub>4</sub> H <sub>9</sub> ) <sub>2</sub> CO                                | 205.8556                                  | 205.4818          | 0.3738                            | 0.0600 |

Table S8: Experimental ( $\text{PA}_{\text{exp}}$ ) and calculated ( $\text{PA}_{\text{PP2}}$ ) proton affinities, difference between  $\text{PA}_{\text{exp}}$  and  $\text{PA}_{\text{PP2}}$  values ( $\Delta\text{PA}_{\text{PP2}}^{\text{exp}}$ ), and Root-mean-square deviation (RMSD) between protonated ( $\text{AH}^+$ ) and deprotonated (A) carboxylic acids.

| Molecule (A)       |                                                                  | Proton affinity (kcal mol <sup>-1</sup> ) |                   |                                   | RMSD   |
|--------------------|------------------------------------------------------------------|-------------------------------------------|-------------------|-----------------------------------|--------|
|                    |                                                                  | PA <sub>exp</sub>                         | PA <sub>PP2</sub> | ΔPA <sub>PP2</sub> <sup>exp</sup> |        |
| Carboxylic acids   |                                                                  |                                           |                   |                                   |        |
| 130                | HCOOH                                                            | 177.3423                                  | 174.8708          | 2.4715                            | 0.5600 |
| 131                | CH <sub>2</sub> FCOOH                                            | 182.9350                                  | 184.0744          | -1.1394                           | 0.0390 |
| 132                | CH <sub>3</sub> COOH                                             | 187.3088                                  | 186.9824          | 0.3264                            | 0.2850 |
| 133                | C <sub>2</sub> H <sub>5</sub> COOH                               | 190.5354                                  | 191.4908          | -0.9554                           | 0.0560 |
| 134                | (CH <sub>3</sub> ) <sub>2</sub> CCOOH                            | 195.1960                                  | 195.3420          | -0.1460                           | 0.0340 |
| 135                | c(C <sub>4</sub> H <sub>7</sub> )COOH                            | 195.4111                                  | 195.6487          | -0.2376                           | 0.2130 |
| 136                | c(C <sub>5</sub> H <sub>9</sub> )COOH                            | 195.4111                                  | 196.3013          | -0.8902                           | 0.1100 |
| 137                | (CH <sub>3</sub> ) <sub>2</sub> CCHCOOH                          | 196.7017                                  | 197.3875          | -0.6857                           | 0.0400 |
| 138                | CH <sub>3</sub> (CH) <sub>2</sub> COOH                           | 196.8929                                  | 196.2483          | 0.6447                            | 0.0380 |
| 139                | (c-C <sub>6</sub> H <sub>11</sub> )COOH                          | 196.8929                                  | 195.9369          | 0.9560                            | 0.0420 |
| Carboxylate anions |                                                                  |                                           |                   |                                   |        |
| 140                | CF <sub>3</sub> COO <sup>-</sup>                                 | 322.6220                                  | 321.8195          | 0.8025                            | 0.0550 |
| 141                | CHF <sub>2</sub> COO <sup>-</sup>                                | 330.2321                                  | 330.3197          | -0.0876                           | 0.0620 |
| 142                | CH <sub>3</sub> COCOO <sup>-</sup>                               | 333.4606                                  | 334.6183          | -1.1577                           | 0.0350 |
| 143                | ClCH <sub>2</sub> COO <sup>-</sup>                               | 336.2279                                  | 338.1673          | -1.9394                           | 0.0460 |
| 144                | CH <sub>2</sub> FCOO <sup>-</sup>                                | 339.2258                                  | 340.0145          | -0.7887                           | 0.0730 |
| 145                | Cl(CH <sub>2</sub> ) <sub>2</sub> COO <sup>-</sup>               | 340.8401                                  | 341.6334          | -0.7933                           | 0.0600 |
| 146                | HCOO <sup>-</sup>                                                | 345.2217                                  | 345.0049          | 0.2168                            | 0.0480 |
| 147                | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>3</sub> COO <sup>-</sup> | 346.1441                                  | 345.1271          | 1.0170                            | 0.0380 |
| 148                | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>2</sub> COO <sup>-</sup> | 346.6053                                  | 344.4722          | 2.1331                            | 0.0450 |
| 149                | CH <sub>3</sub> CH <sub>2</sub> COO <sup>-</sup>                 | 347.5278                                  | 345.1525          | 2.3753                            | 0.0420 |
| 150                | CH <sub>3</sub> COO <sup>-</sup>                                 | 348.4502                                  | 350.7056          | -2.2554                           | 0.0480 |

Table S9: Mean absolute error ( $|\Delta|$ ), mean absolute deviation (MAD) and maximum absolute deviation ( $\text{AD}_{\text{max}}$ ). Values in  $\text{kcal mol}^{-1}$ .

| Functional group     | $ \Delta $ | MAD                 | $\text{AD}_{\text{max}}$ |
|----------------------|------------|---------------------|--------------------------|
| Amine (47)           | 0.3831     | 0.2464              | 1.1112                   |
| Alcohol (16)         | 0.5484     | 0.3281              | 1.6833                   |
| Amide (17)           | 0.6555     | 0.3617              | 1.7099                   |
| Ether (12)           | 0.6793     | 0.3673              | 1.8157                   |
| Ester (14)           | 0.7750     | 0.3841              | 1.8583                   |
| Aldehyde (6)         | 0.7795     | 0.8492              | 3.3271                   |
| Ketone (17)          | 1.0788     | 0.4100              | 2.3470                   |
| Carboxylic acid (10) | 0.8453     | 0.4372              | 2.4715                   |
| Carboxylate (11)     | 1.2333     | 0.6854              | 2.3753                   |
| Total                | 0.6797     | 0.4522 <sup>a</sup> | 3.3271                   |

<sup>a</sup> Average mean absolute deviation.

Table S10: Adiabatic gas phase proton affinities calculated for selected molecules.

|             | Molecule                                                                         | Energy (hartree)         |                     | Proton affinity (kcal mol <sup>-1</sup> ) |                          |                          | Absolute error (kcal mol <sup>-1</sup> )   |                                             | $\Delta\text{PA}_{\text{PP2}}^{\text{ad}}$ | RMSD (Å) |
|-------------|----------------------------------------------------------------------------------|--------------------------|---------------------|-------------------------------------------|--------------------------|--------------------------|--------------------------------------------|---------------------------------------------|--------------------------------------------|----------|
|             |                                                                                  | $\Delta E_{\text{el}}^a$ | $\Delta ZPE^a$      | $\text{PA}_{\text{ad}}^b$                 | $\text{PA}_{\text{PP2}}$ | $\text{PA}_{\text{exp}}$ | $\Delta\text{PA}_{\text{ad}}^{\text{exp}}$ | $\Delta\text{PA}_{\text{PP2}}^{\text{exp}}$ |                                            |          |
| 1           | NH <sub>3</sub>                                                                  | -0.3371                  | 0.0156              | 203.2209                                  | 203.0074                 | 204.0150                 | 0.7941                                     | 1.0076                                      | 0.2135                                     | 0.0250   |
| 2           | CHF <sub>2</sub> CH <sub>2</sub> NH <sub>2</sub>                                 | -0.3441                  | 0.0149              | 208.0182                                  | 207.9932                 | 208.0545                 | 0.0363                                     | 0.0613                                      | 0.0250                                     | 0.0890   |
| 3           | CH <sub>2</sub> FCH <sub>2</sub> NH <sub>2</sub>                                 | -0.3520                  | 0.0152              | 212.8169                                  | 213.2649                 | 213.1931                 | 0.3762                                     | -0.0718                                     | -0.3044                                    | 0.1140   |
| 4           | CH <sub>3</sub> NH <sub>2</sub>                                                  | -0.3548                  | 0.0155              | 214.4214                                  | 214.3026                 | 214.8660                 | 0.4446                                     | 0.5634                                      | 0.1188                                     | 0.0310   |
| 5           | c-(C <sub>3</sub> H <sub>5</sub> )NH <sub>2</sub>                                | -0.3534                  | 0.0149              | 213.8938                                  | 216.4450                 | 216.2000                 | 2.3062                                     | -0.2450                                     | -2.0612                                    | 0.6370   |
| 6           | CH <sub>3</sub> CH <sub>2</sub> NH <sub>2</sub>                                  | -0.3597                  | 0.0153              | 217.5695                                  | 217.9324                 | 217.9732                 | 0.4037                                     | 0.0408                                      | -0.3629                                    | 0.0382   |
| 7           | CH <sub>2</sub> =(CH <sub>3</sub> )CH <sub>2</sub> NH <sub>2</sub>               | -0.3605                  | 0.0150              | 218.2802                                  | 218.7165                 | 219.3000                 | 1.0198                                     | 0.5835                                      | -0.4363                                    | 0.1180   |
| 8           | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>2</sub> NH <sub>2</sub>                  | -0.3616                  | 0.0154              | 218.7127                                  | 219.6320                 | 219.3595                 | 0.6468                                     | -0.2725                                     | -0.3743                                    | 0.0438   |
| 9           | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>3</sub> NH <sub>2</sub>                  | -0.3628                  | 0.0154              | 219.4754                                  | 220.6167                 | 220.2438                 | 0.7684                                     | -0.3729                                     | -0.3955                                    | 0.0420   |
| 10          | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>4</sub> NH <sub>2</sub>                  | -0.3634                  | 0.0154              | 219.8625                                  | 221.1563                 | 220.7218                 | 0.8593                                     | -0.4345                                     | -0.4248                                    | 0.0430   |
| 11          | (CH <sub>3</sub> ) <sub>2</sub> CHNH <sub>2</sub>                                | -0.3627                  | 0.0150              | 219.6342                                  | 220.6698                 | 220.7935                 | 1.1593                                     | 0.1237                                      | -1.0356                                    | 0.0339   |
| 12          | (CH <sub>3</sub> ) <sub>2</sub> CHCH <sub>2</sub> NH <sub>2</sub>                | -0.3633                  | 0.0153              | 219.8806                                  | 221.4031                 | 221.0325                 | 1.1519                                     | -0.3706                                     | -0.7813                                    | 0.5440   |
| 13          | c-(C <sub>6</sub> H <sub>11</sub> )CH <sub>2</sub> NH <sub>2</sub>               | -0.3672                  | 0.0152              | 222.3699                                  | 222.5515                 | 221.5000                 | -0.8699                                    | -1.0515                                     | 0.1816                                     | 0.6280   |
| 14          | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>6</sub> NH <sub>2</sub>                  | -0.3640                  | 0.0154              | 220.2225                                  | 221.6314                 | 221.5583                 | 1.3358                                     | -0.0731                                     | -1.2627                                    | 0.0400   |
| 15          | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>5</sub> NH <sub>2</sub>                  | -0.3637                  | 0.0154              | 220.0834                                  | 221.4423                 | 221.6778                 | 1.5944                                     | 0.2355                                      | -1.3589                                    | 0.0420   |
| 16          | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>7</sub> NH <sub>2</sub>                  | -0.3641                  | 0.0154              | 220.3109                                  | 221.7490                 | 222.0124                 | 1.7015                                     | 0.2634                                      | -1.4381                                    | 0.0395   |
| 17          | CH <sub>3</sub> CH <sub>2</sub> (CH <sub>3</sub> )CHNH <sub>2</sub>              | -0.3649                  | 0.0149              | 221.1175                                  | 222.3947                 | 222.2036                 | 1.0861                                     | -0.1911                                     | -0.8950                                    | 0.0500   |
| 18          | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>8</sub> NH <sub>2</sub>                  | -0.3653                  | 0.0152              | 221.1821                                  | 221.8274                 | 222.4000                 | 1.2179                                     | 0.5726                                      | -0.6453                                    | 0.0372   |
| 20          | (CH <sub>3</sub> ) <sub>3</sub> CNH <sub>2</sub>                                 | -0.3667                  | 0.0149              | 222.1975                                  | 224.3664                 | 223.2553                 | 1.0577                                     | -1.1112                                     | 0.0534                                     | 0.5465   |
| 21          | (C <sub>6</sub> H <sub>11</sub> )NH <sub>2</sub>                                 | -0.3673                  | 0.0152              | 222.4564                                  | 224.1381                 | 223.3270                 | 0.8705                                     | -0.8112                                     | -0.0594                                    | 0.0260   |
| 22          | CH <sub>3</sub> CH <sub>2</sub> C(CH <sub>3</sub> ) <sub>2</sub> NH <sub>2</sub> | -0.3688                  | 0.0148              | 223.5898                                  | 224.6616                 | 224.1396                 | 0.5497                                     | -0.5220                                     | -0.0277                                    | 0.0494   |
| 88          | (CH <sub>3</sub> ) <sub>3</sub> CCH <sub>2</sub> OH                              | -0.3109                  | 0.0129              | 188.5206                                  | 189.5860                 | 190.1291                 | 1.6085                                     | 0.5431                                      | -1.0654                                    | 1.6780   |
| 107         | CH <sub>2</sub> O                                                                | -0.2797                  | 0.0144              | 167.9753                                  | 167.0601                 | 170.3872                 | 2.4119                                     | 3.3271                                      | 0.9152                                     | 0.0340   |
| 107 CCSD(T) | CH <sub>2</sub> O                                                                | -0.2839 <sup>c</sup>     | 0.0144 <sup>c</sup> | 170.6268 <sup>c</sup>                     | 167.0601                 | 170.3872                 | <b>-0.2396</b>                             | 3.3271                                      | 3.0875                                     | 0.0340   |
| 122         | 4-(CH <sub>3</sub> )C <sub>6</sub> H <sub>9</sub> (=O)                           | -0.3280                  | 0.0134              | 198.9387                                  | 203.9967                 | 201.9359                 | 2.9973                                     | -2.0608                                     | -0.9365                                    | 0.2040   |
| 123         | (C <sub>3</sub> H <sub>7</sub> ) <sub>2</sub> CO                                 | -0.3284                  | 0.0132              | 199.2876                                  | 199.6128                 | 201.9598                 | 2.6723                                     | 2.3470                                      | -0.3253                                    | 0.5530   |
| 130         | HCOOH                                                                            | -0.2901                  | 0.0134              | 175.1430                                  | 174.8708                 | 177.3423                 | 2.1992                                     | 2.4715                                      | 0.2722                                     | 0.5600   |
| 130 CCSD(T) | HCOOH                                                                            | -0.2945 <sup>c</sup>     | 0.0134 <sup>c</sup> | 177.8705 <sup>c</sup>                     | 174.8708                 | 177.3423                 | <b>-0.5283</b>                             | 2.4715                                      | 1.9432                                     | 0.5600   |
| 148         | CH <sub>3</sub> (CH <sub>2</sub> ) <sub>2</sub> COO <sup>-</sup>                 | -0.5621                  | 0.0140              | 345.4585                                  | 344.4722                 | 346.6053                 | 1.1468                                     | 2.1331                                      | 0.9863                                     | 0.0450   |
| 149         | CH <sub>3</sub> CH <sub>2</sub> COO <sup>-</sup>                                 | -0.5629                  | 0.0139              | 345.9894                                  | 345.1525                 | 347.5278                 | 1.5383                                     | 2.3753                                      | 0.8369                                     | 0.0420   |
| 150         | CH <sub>3</sub> COO <sup>-</sup>                                                 | -0.5637                  | 0.0139              | 346.4993                                  | 350.7056                 | 348.4502                 | 1.9509                                     | -2.2554                                     | 0.3045                                     | 0.0480   |
| 150 CCSD(T) | CH <sub>3</sub> COO <sup>-</sup>                                                 | -0.5683 <sup>c</sup>     | 0.0139 <sup>c</sup> | 349.3800 <sup>c</sup>                     | 350.7056                 | 348.4502                 | <b>-0.9298</b>                             | -2.2554                                     | 1.3256                                     | 0.5600   |

<sup>a</sup>  $\Delta E_{\text{el}} = E(\text{AH}^+) - E(\text{A})$  and  $\Delta ZPE = ZPE(\text{AH}^+) - ZPE(\text{A})$ . MP2/6-311++G(2d,2p)<sup>b</sup>  $\text{PA}^{\text{ad}} = -\Delta E_{\text{el}} - \Delta ZPE + \frac{5}{2}RT$ . ( $\frac{5}{2}RT = 1.4812$  kcal mol<sup>-1</sup>)<sup>c</sup> CCSD(T)/6-311++G(2d,2p)

# 1 Theory appendix

Here we summarize the generalized any particle propagator theory and explain how it is employed to calculate proton binding energies of molecular systems.

## 1.1 APMO/HF theory

The molecular Hamiltonian,  $H^{TOT}$ , includes contributions from classical and quantum particles, such that:

$$H^{TOT} = - \sum_i^{N^Q} \frac{1}{2M_i} \nabla_i^2 + \sum_i^{N^Q} \sum_{j>i}^{N^Q} \frac{Q_i Q_j}{r_{ij}} + \sum_i^{N^Q} \sum_j^{N^C} \frac{Q_i Q_j}{r_{ij}} + \sum_i^{N^C} \sum_{j>i}^{N^C} \frac{Q_i Q_j}{r_{ij}} \quad , \quad (1)$$

where  $Q_i$  is the charge of the particle  $i$ , and  $N^Q$  and  $N^C$  are the number of quantum and classical particles, respectively.

At the APMO/HF level the molecular wavefunction,  $\Psi_0$ , is approximated as a product of single-configurational wavefunctions,  $\Phi^\alpha$ , for different types of quantum species  $\alpha$ :

$$\Psi_0 = \prod_{\alpha}^{N^Q} \Phi^\alpha. \quad (2)$$

Each  $\Phi^\alpha$  is represented in terms of molecular orbitals (MO),  $\psi_i^\alpha$ . These  $\psi_i^\alpha$  are obtained by solving the

equations

$$f^\alpha(i)\psi_i^\alpha = \varepsilon_i^\alpha \psi_i^\alpha, \quad \forall i, \alpha. \quad (3)$$

Each  $f^\alpha(i)$  is an effective one-particle Fock operator for the quantum species  $\alpha$  written as

$$f^\alpha(i) = h^\alpha(i) + Q_\alpha^2 \sum_{j \in \alpha} [J_j^\alpha \mp K_j^\alpha] + \sum_{\beta \neq \alpha}^{NQ} Q_\alpha \sum_{j \in \beta} Q_\beta J_j^\beta. \quad (4)$$

In the above equation  $h^\alpha(i)$  is the one-particle core Hamiltonian,

$$h^\alpha(i) = -\frac{\nabla_i^2}{2M_\alpha} + \sum_j^{N^C} \frac{Q_j^C Q_\alpha}{r_{ij}} \quad (5)$$

and  $J^\alpha$  and  $K^\alpha$  are Coulomb and exchange operators defined as,

$$J_j^\alpha(1)\psi_i^\alpha(1) = \left[ \int d\mathbf{r}_2 \psi_j^{\alpha*}(2) \frac{1}{r_{12}} \psi_j^\alpha(2) \right] \psi_i^\alpha(1), \quad (6)$$

$$K_j^\alpha(1)\psi_i^\alpha(1) = \left[ \int d\mathbf{r}_2 \psi_j^{\alpha*}(2) \frac{1}{r_{12}} \psi_i^\alpha(2) \right] \psi_j^\alpha(1). \quad (7)$$

The sign preceding the exchange operator in Eq. 4 is chosen depending on the bosonic (positive) or fermionic (negative) nature of the  $\alpha$  particles. In practice, all the quantum nuclei species are treated as fermions in the highest spin state.

## 1.2 APMO propagator theory

Here we summarize the APMO propagator theory. Details can be found in References<sup>1,2</sup>. For a system comprising  $N^Q$  fermionic species  $\{\alpha, \beta, \dots\}$ , the one-particle Green function for species  $\alpha$  can be written as:

$$\mathbf{G}^\alpha(\omega^\alpha) = (\mathbf{a}^\alpha | (\omega^\alpha \hat{I} - \hat{H})^{-1} \mathbf{a}^\alpha) \quad (8)$$

where  $\mathbf{a}^\alpha$  contains all the single annihilation operators for the  $\alpha$  species,  $\{a_i^\alpha, a_a^\alpha\}$ . The parameter  $\omega^\alpha$  has energy units and take the values of the binding energies for particles of the  $\alpha$  species. The superoperators,  $\hat{I}$  and  $\hat{H}$ , are defined as follows:

$$\hat{I}A = A \quad (9)$$

$$\hat{H}A = [A, H^{TOT}]_- = AH^{TOT} - H^{TOT}A \quad (10)$$

where  $H^{TOT}$  stands for the APMO Hamiltonian in its second quantized form. By applying Löwdin's inner projection technique with an appropriate superoperator space  $\mathbf{h}^\alpha$ , Eq(10) can be rewritten as<sup>1</sup>

follows:

$$\mathbf{G}^\alpha(\omega^\alpha) = (\mathbf{a}^\alpha | \mathbf{h}^\alpha) (\mathbf{h}^\alpha | (\omega^\alpha \hat{I} - \hat{H}) \mathbf{h}^\alpha)^{-1} (\mathbf{h}^\alpha | \mathbf{a}^\alpha) \quad (11)$$

The super-operator space,  $\mathbf{h}^\alpha$ , is defined in such a way that it changes the number of  $\alpha$  particles by one,

while conserving the number of particles of the other species:

$$\mathbf{h}^\alpha = \{\mathbf{a}^\alpha\} \cup \{\mathbf{f}_3^\alpha\} \cup \{\mathbf{f}_5^\alpha\} \cup \dots \quad (12)$$

$$\mathbf{h}^\alpha = \{\mathbf{a}^\alpha\} \cup \{\mathbf{f}_3^{\alpha\alpha\alpha}, \mathbf{f}_3^{\beta\alpha\beta}, \mathbf{f}_3^{\gamma\alpha\gamma} \dots\} \cup \{\mathbf{f}_5^\alpha\} \cup \dots$$

$$\mathbf{h}^\alpha = \{a_a, a_i\} \cup \{a_i^\dagger a_a a_b, a_a^\dagger a_i a_j, a_I^\dagger a_a a_A, a_A^\dagger a_i a_I, a_\Gamma^\dagger a_i a_\Lambda, a_\Lambda^\dagger a_a a_\Gamma, \dots\} \cup \{\mathbf{f}_5^\alpha\} \cup \dots$$

The projection space,  $\mathbf{h}^\alpha$ , is partitioned into a primary space,  $\mathbf{a}^{\alpha\dagger} = \{a_a^{\alpha\dagger}, a_i^{\alpha\dagger}\}$  and a complementary

space,  $\mathbf{f}^\alpha$ . The latter space contains operators associated to ionizations of an  $\alpha$  particle coupled to

excitations of any type of particle in the system. Using this partition, the propagator matrix can be

rearranged and subsequently transformed into the expression

$$\mathbf{G}^{\alpha^{-1}}(\omega^\alpha) = (\mathbf{a}^\alpha | (\omega^\alpha \hat{I} - \hat{H}) | \mathbf{a}^\alpha) - (\mathbf{a}^\alpha | \hat{H} | \mathbf{f}^\alpha) (\mathbf{f}^\alpha | (\omega^\alpha \hat{I} - \hat{H}) | \mathbf{f}^\alpha)^{-1} (\mathbf{f}^\alpha | \hat{H} | \mathbf{a}^\alpha) \quad (13)$$

which can also be presented as a Dyson-like equation:

$$\mathbf{G}^{\alpha-1}(\omega^\alpha) = \mathbf{G}_0^{\alpha-1}(\omega^\alpha) - \Sigma^\alpha(\omega^\alpha) \quad (14)$$

with

$$\mathbf{G}_0^\alpha(\omega^\alpha)_{pq} = \frac{\delta_{pq}}{\omega^\alpha - \epsilon_p^\alpha}. \quad (15)$$

where  $\epsilon_p^\alpha$  is the energy of the  $p$ -th canonical Hartree-Fock orbital of the  $\alpha$  species. The self-energy matrix for the  $\alpha$  species,  $\Sigma^\alpha(\omega^\alpha)$ , is defined by Eq.(14). To obtain working equations for computing the values of  $\omega^\alpha$ , we apply two approximations. First, we approximate the APMO wavefunction using perturbation theory with the Hartree-Fock wavefunction as the reference wavefunction. Second, we truncate the operator space (Eq. (12)). Approximations to second and third order can be obtained by truncating  $\mathbf{f}^\alpha$  to  $\mathbf{f}_3^\alpha$ .

In addition, we can also neglect the off-diagonal terms of the self-matrix. Within this approximation, the Dyson equation takes the form

$$\omega_p^\alpha = \epsilon_p^\alpha + \Sigma_{pp}^\alpha(\omega_p^\alpha) \quad (16)$$

where  $\varepsilon_p^\alpha$  is the  $p^{th}$  canonical orbital energy for the  $\alpha$  species. The relaxation and correlation effects missing in the Koopmans's approximation ( $\omega_p^\alpha = \varepsilon_p^\alpha$ ) are included via the energy dependent self-energy term,  $\Sigma_{pp}^\alpha(\omega_p^\alpha)$ .  $\Sigma_{pp}^\alpha(\omega_p^\alpha)$  can be further decomposed into intraspecies and interspecies contributions:

$$\Sigma_{pp}^\alpha(\omega^\alpha) = \Sigma_{pp}^{\alpha,\alpha}(\omega^\alpha) + \sum_{\beta \neq \alpha}^{N^Q} \Sigma_{pp}^{\alpha,\beta}(\omega^\alpha) \quad (17)$$

In a system comprised of quantum protons and electrons, the self-energy for protons contains a proton-proton ( $\alpha - \alpha$ ) term and proton-electron ( $\alpha - \beta$ ) term.

## References

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