Supplementary information

Title: Size-Dependent Hardness of Five-fold Twin Structured Ag Nanowires

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Double Contact Model

The details of this model can be found in the work by Feng et al.[22, 29] and only the key ideas will be presented here to help understanding of the results. To apply the model, the required inputs are indentation load P, indentation stiffness S, tip radius R_1(=112nm), nanowire radius R_2, as well as reduced moduli for contacts 1 and 2, i.e., E_{r1}, and E_{r2}, where E_{r1} and E_{r2} can be determined by the moduli of substrate (E_s), indenter (E_i) and nanowire (E_n). The following are the key set of analytical equations for solving the nanowire indentation hardness H_n.

$$H_n = H_1 = \frac{P}{A_{c1}} = \frac{P}{\pi k_1 a_1^2}, \quad k_1 = \frac{b_1}{a_1} = (1+\frac{R_1}{R_2})^{-2/3} \tag{S1}$$

$$\frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2}, \quad S_1 = \lambda_1 2E_{r1} a_1 \sqrt{k_1}, \quad S_2 = \lambda_2 2E_{r2}\sqrt{a_2 b_2} \tag{S2}$$

$$\lambda_1 = \frac{\pi}{2\sqrt{k_1}K'_1}, \quad K'_1 = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-\left(1-k_1^2\right)\sin^2\theta}} d\theta \tag{S3}$$

$$\lambda_2 = \frac{\pi}{2\sqrt{k_2}K'_2}, \quad K'_2 = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-\left(1-k_2^2\right)\sin^2\theta}} d\theta, \quad k_2 = \frac{b_2}{a_2} \tag{S4}$$

$$H_2 = \frac{P}{A_{c2}} = \frac{P}{\pi a_2 b_2}, \quad b_2 = \sqrt{\frac{2PR_2}{\pi a_1 E_{r2}}}, \quad a_2 = 2R_2\left\{\left(\frac{2.2-\alpha}{1+\alpha}\right)^{0.342} + \frac{s}{m2^{m-1}}\left(\frac{a_1}{2R_2}\right)^m\right\} \tag{S5}$$

$$\alpha = \frac{E_{sp} - E_{np}}{E_{sp} + E_{np}}, \quad E_{sp} = \frac{E_s}{1-\nu_s^2}, \quad E_{np} = \frac{E_n}{1-\nu_n^2} \tag{S6}$$

$$s = 0.8756(\alpha + 0.89)^{0.231} - 0.11\alpha, \quad m = 1.672 - 0.45\alpha + 0.066\alpha^2 + 0.111\alpha^3 \tag{S7}$$

$$h_{e1} = 3P/2S_1, \quad h_{e2} = 3P/2S_2 \tag{S8}$$

where S_1 and S_2 are the contact stiffnesses for contacts 1 and 2, respectively; h_{e1} is the elastic displacement of the indenter for contact 1. E_{r1} and E_{r2} are the reduced moduli for contacts 1 and 2, respectively, where the reduced modulus is defined in **Equation S2**. a_1 and b_1 are major and minor radii of contact 1 (shown in **Figure 3**); a_2 and b_2 are major and minor radii of contact 2 (shown in **Figure 3**). R_1 is the tip radius, and R_2 is the nanowire radius.

As aforementioned, the nanowire's hardness H_n can be determined by H_1 at contact 1, and based on **Equation S1**, k_1 and a_1 need to be solved to calculate H_1. Here, k_1 can be easily solved by the known indenter radius R_1 and nanowire radius R_2. The solving of a_1 needs to jointly solve **Equation S2-S7**, which is quite involved as discussed below. First, based on **Equation S3**, λ_1 can be solved only based on R_1 and R_2, and thus, based on **Equation S2**, the contact stiffness for contact 1, i.e., S_1, becomes a function only of a_1. Then, based on **Equation S5** and the required inputs, the contact radii (a_2 and b_2) of elliptical contact 2 are function only of a_1, and thus based on **Equations S2 and S4**, the contact stiffness of contact 2, i.e., S_2, becomes a function only of a_1 in **Equation S2**. Consequently, both S_1 and S_2 are functions only of a_1, and then the stiffness equation ($1/S$=$1/S_1$+$1/S_2$) in **Equation S2** and experimentally measured stiffness total stiffness S can be used to solve a_1. Here, S is fitted with a power law against the measured maximum load at the point of unload (S=AP^n) for each partial unloading. Finally, the nanowire hardness H_n , i.e., the contact pressure in contact 1, can be determined by the solved a_1 and measure load P by **Equation S1**.

Reference

[22] G. Feng, W.D. Nix, Y. Yoon, C.J. Lee. A study of the mechanical properties of nanowires using nanoindentation, J Appl Phys, 2006, 99

[29] D. Askari, G. Feng. Finite element analysis of nanowire indentation on a flat substrate, J Mater Res. 2012, 27, 586-591.