Supplemental Information:

Using an Analytical Solution Approach to Permit High Volume Groundwater Withdrawals

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$$dQ_s = \sqrt{\left(\frac{\partial Q_s}{\partial S} dS\right)^2 + \left(\frac{\partial Q_s}{\partial T} dT\right)^2 + \left(\frac{\partial Q_s}{\partial \lambda} d\lambda\right)^2}$$

Equation SI-1. Uncertainty in stream depletion Q_s, due to errors in the storage coefficient, S, transmissivity, T and streambed conductance, λ.

$$\frac{\partial Q_S}{\partial S} = \frac{Q_w \exp\left(-\frac{Sd^2}{4Tt}\right)\sqrt{\frac{Sd^2}{Tt}}}{2S\sqrt{\pi}} + \frac{Q_w \lambda^2 t \exp\left(\frac{\lambda^2 t}{4ST} + \frac{\lambda d}{2T}\right) \operatorname{erfc}\left(\sqrt{\frac{\lambda^2 t}{4ST}} + \sqrt{\frac{Sd^2}{4Tt}}\right)}{4S^2 T}$$
$$+ \frac{2Q_w \left(\frac{d^2}{4Tt\sqrt{\frac{Sd^2}{Tt}}} - \frac{\lambda^2 t}{4S^2 T\sqrt{\frac{\lambda^2 t}{ST}}}\right) \exp\left[-\left(\frac{1}{2}\sqrt{\frac{Sd^2}{Tt}} + \frac{1}{2}\sqrt{\frac{\lambda^2 t}{ST}}\right)^2 + \frac{\lambda d}{2T} + \frac{\lambda^2 t}{4ST}\right]}{\sqrt{\pi}}$$

Equation SI-2. Partial derivative of Eqn. SI-1 with respect to the storage coefficient, S.

$$\frac{\partial Q_S}{\partial T} = \frac{Q_w \exp\left(-\frac{Sd^2}{4Tt}\right)\sqrt{\frac{Sd^2}{Tt}}}{2T\sqrt{\pi}} + \frac{2Q_w \left(-\frac{Sd^2}{4T^2 t\sqrt{\frac{Sd^2}{Tt}}} - \frac{\lambda^2 t}{4ST^2\sqrt{\frac{\lambda^2 t}{ST}}}\right) \exp\left(-\left(\frac{1}{2}\sqrt{\frac{Sd^2}{Tt}} + \frac{1}{2}\sqrt{\frac{\lambda^2 t}{ST}}\right)^2 + \frac{\lambda^2 t}{4ST} + \frac{\lambda d}{2T}\right)}{\sqrt{\pi}}$$
$$-Q_w \exp\left(\frac{\lambda^2 t}{4ST} + \frac{\lambda d}{2T}\right)\left(-\frac{\lambda d}{2T^2} - \frac{\lambda^2 t}{4ST^2}\right) \operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{\lambda^2 t}{ST}} + \frac{1}{2}\sqrt{\frac{Sd^2}{Tt}}\right)$$

Equation SI-3. Partial derivative of Eqn. SI-1 with respect to the transmissivity, T.

$$\frac{\partial Q_S}{\partial \lambda} = \frac{Q_w \lambda t \exp\left(-\left(\frac{1}{2}\sqrt{\frac{Sd^2}{Tt}} + \frac{1}{2}\sqrt{\frac{\lambda^2 t}{ST}}\right)^2 + \frac{\lambda^2 t}{4ST} + \frac{\lambda d}{2T}\right)}{ST\sqrt{\pi}\sqrt{\frac{\lambda^2 t}{ST}}}$$
$$-Q_w \exp\left(\frac{\lambda^2 t}{4ST} + \frac{\lambda d}{2T}\right)\left(\frac{d}{2T} + \frac{\lambda t}{2ST}\right) \operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{\lambda^2 t}{ST}} + \frac{1}{2}\sqrt{\frac{Sd^2}{Tt}}\right)$$

Equation SI-4. Partial derivative of Eqn. SI-1 with respect to the streambed conductance, λ.

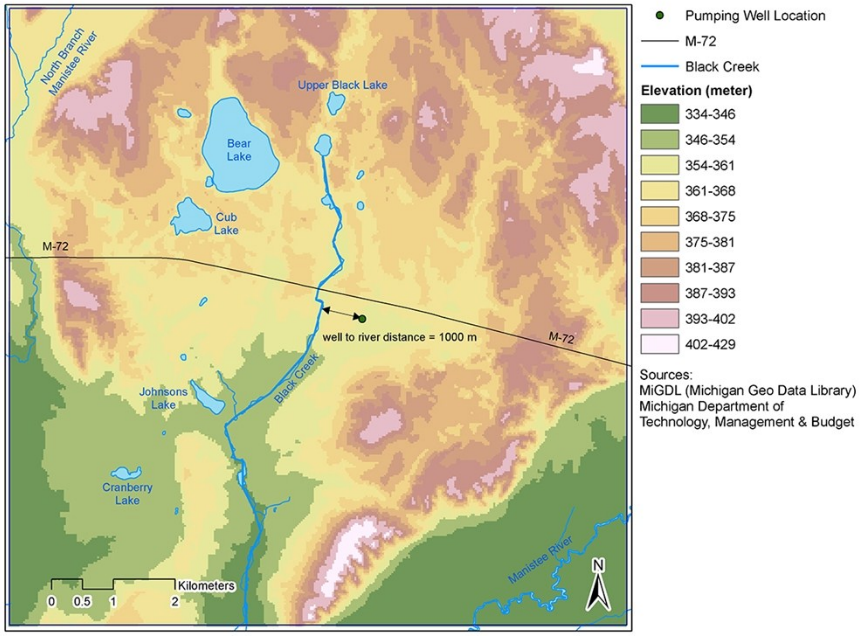

Figure SI-1. Map of the study site in Kalkaska County in the northern Lower Peninsula of Michigan.

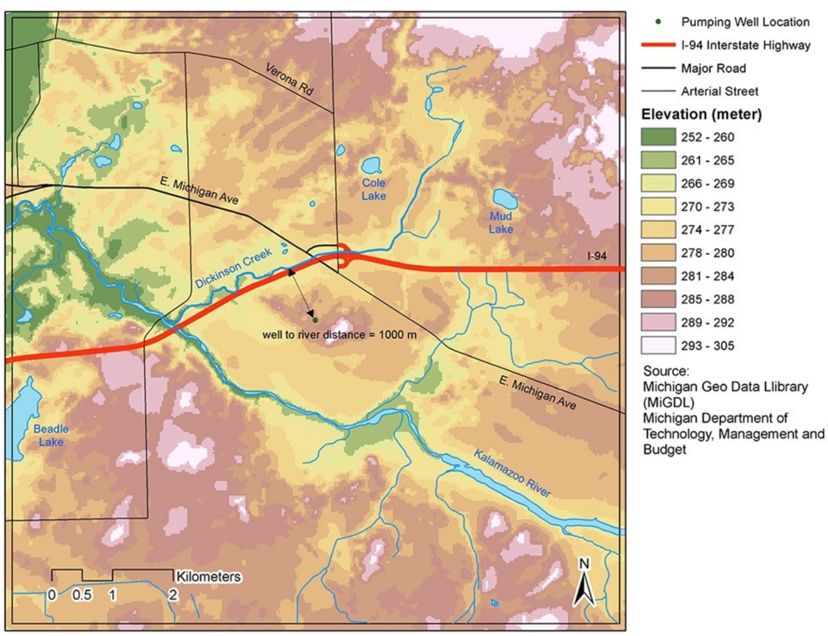

Figure SI-2. Map of the study site in Calhoun County in the southwestern Lower Peninsula of Michigan.

Stream Type		Water Withdrawal Management Zone			
		Zone A	Zone B	Zone C	Zone D
Cold	Stream	< 14%	NA	14 to < 20%	≥ 20%
	Small River	< 10.5%		10.5 to <21%	≥ 21%
Cold Transitional	Stream	NA	< 4%	NA	≥ 4%
	Small River		< 2%		≥ 2%
	Large River		< 3%		≥ 3%
Cool	Stream	< 6%	6 to < 15%	15 to < 25%	≥ 25%
	Small River	< 15%	15 to < 19%	19 to < 25%	≥ 25%
	Large River	< 14%	14 to < 19%	19 to < 25%	≥ 25%
Warm	Stream	< 10%	10 to < 18%	18 to < 24%	≥ 24%
	Small River	< 8%	8 to < 13%	13 to < 17%	≥ 17%
	Large River	< 10%	10 to < 16%	16 to < 22%	≥ 22%

Table SI-1. Water withdrawal management zones for different stream types[1] based on maximum values of streamflow depletion as a percentage of stream index flow ($Q_{s,max}/Q_{index} \times 100$). The WWAT automatically requires a site specific review for any HVGW proposal that falls into either Zone C or Zone D. NA: not applicable. Cold stream types do not have a Zone B; cold transitional stream types do not have Zones A or D.

Description	Layer #1	Layer #2	Layer #3
Boundary Conditions			
North Boundary	Constant Head 361.8 m (N.W.) to 346.4 m (N.E.)	Constant Head 361.8 m (N.W.) to 346.4 m (N.E.)	Constant Head 361.8 m (N.W.) to 346.4 m (N.E.)
East Boundary	Constant Head 346.4 m (N.E.) to 337 m (S.E.)	No Flow	No Flow
West Boundary	Constant Head 361.8 m (N.W.) to 331 m (S.W.)	No Flow	No Flow
South Boundary	Constant Head 331 m (S.W.) to 337 m (S.E.)	Constant Head 331 m (S.W.) to 337 m (S.E.)	Constant Head 331 m (S.W.) to 337 m (S.E.)
Thickness [m]	20 – 120	1 – 20	200 – 230
Horizontal Hydraulic Conductivity[¥], $K_x = K_y$ [m/d]	3.97	8.64×10^{-6}	1.10×10^{-2}
Vertical Hydraulic Conductivity[±], K_z [m/day]	3.97×10^{-1}	8.64×10^{-7}	1.10×10^{-3}
Storage Coefficient[§], S [-]	0.16	0.10	0.16
Elevation of Bottom of Layer #3	100 m		
Spatial Discretization	200 m × 200 m (largest cell) to 25 m × 25 m (smallest cell)		
Specific Discharge of Screened Aquifer in Absence of Pumping [m/d]	0.18		
Elevation of Stream Bottom (North – South) [m]	359 – 339		
Streambed Slope [-]	2.6×10^{-3}		
Manning's Roughness Coefficient for Stream Bed[‡] [-]	0.025		

Table SI-2. Additional MODFLOW parameter values for the study site in Kalkaska County. [§]The values of S were based on typical values presented in Morris and Johnson.[2] [¥]The horizontal hydraulic conductivities were calibrated values. [±]K_z was assumed to be 1/10 of the horizontal hydraulic conductivity. [‡]Manning's roughness coefficient was based on typical values presented in Arcement and Schneider.[4]

Description	Layer #1	Layer #2	Layer #3
Boundary Conditions			
North Boundary	Constant Head 253.0 m (N.W.) to 279.3 m (N.E.)	No Flow	Constant Head 253.0 m (N.W.) to 276.0 m (N.E.)
East Boundary	Constant Head 279.3 m (N.E.) to 281.0 m (S.E.)	No Flow	Constant Head 276.0 m (N.E.) to 273.2 m (S.E.)
West Boundary	Constant Head 253.0 m (N.W.) to 274.5 m (S.W.)	No Flow	Constant Head 253.0 m (N.W.) to 281.0 m (S.W.)
South Boundary	Constant Head 274.5 m (S.W.) to 281.0 m (S.E.)	No Flow	Constant Head 281.0 m (S.W.) to 273.2 m (S.E.)
Thickness [m]	7 – 40	1 – 5	90 – 105
Horizontal Hydraulic Conductivity[¥], $K_x = K_y$ [m/day]	9.50×10^{2}	8.64×10^{-6}	4.32×10^{-3}
Vertical Hydraulic Conductivity[±], K_z [m/day]	95.0	8.64×10^{-7}	4.32×10^{-4}
Storage Coefficient[§], S [-]	0.16	0.10	10^{-5}
Elevation of Bottom of Layer #3	150 m		
Spatial Discretization	400 m × 400 m (largest cell) to 25 m × 25 m (smallest cell)		
Specific Discharge of Screened Aquifer in Absence of Pumping [m/day]	0.26		
Elevation of Stream Bottom (North – South) [m]	269 – 265 m		
Streambed Slope [-]	7.8×10^{-4}		
Manning's Roughness Coefficient[‡] for Stream Bed [-]	0.025		

Table SI-3. Additional MODFLOW parameter values for the study site in Calhoun County. [§]The values of *S* were based on typical values presented in Morris and Johnson.[2] [¥]The horizontal hydraulic conductivities were calibrated values. [±] K_z was assumed to be 1/10 of the horizontal hydraulic conductivity. [‡]Manning's roughness coefficient was based on typical values presented in Arcement and Schneider.[4]

References

1 D. A. Hamilton and P. W. Seelbach, *Michigan's Water Withdrawal Assessment Process and Internet Screening Tool*, Fisheries Special Rpt. 55, Michigan Department of Natural Resources, Lansing, MI, 2011.

2 D. A. Morris and A. I. Johnson, *Summary of Hydrologic and Physical Properties of Rock and Soil Materials, as Analyzed by the Hydrologic Laboratory of the U.S. Geological Survey, 1948-60*, Geological Survey Water-Supply Paper 1839-D, U.S. Government Printing Office, Washington, D.C., 1967.

3 Michigan Department of Technology, Management and Budget, *Michigan Geographic Data Library*, https://www.mcgi.state.mi.us/mgdl/ (accessed May 2015).

4 G. J. Arcement and V. R. Schneider. *Guide for Selecting Manning's Roughness Coefficients for Natural Channels and Flood Plains*, U.S. Geological Water-Supply Paper 2339, U.S. Government Printing Office, Washington, D.C., 1989.