Supplementary information

Correlation plots of distributions of different degree of skewness

The distribution of cell properties, such as cell volume and Mg contents, are approximately log-normal (Figures 3a and 3b). It can be readily deduced that the correlation plot of the corresponding percentiles of two normal distributions is linear (Figure 4a). However, the distributions become asymmetric for algal cells under Cr(VI) stresses (Figures 3c-3h). The correlation plots for asymmetric distributions with different degree of skewness are nonlinear, but the shape and curvature of the plot are not easily deduced. The following simulation is to explore the relationship between the symmetry of the distributions and the shape of the correlation plots. Skew-normal distributions are used to approximate the asymmetric cell-properties distributions. The probability density function of skew-normal distribution $f(z,\alpha)$ is given by the following equation.[37, 38]

$$f(z,\alpha) = 2\phi(z)\Phi(\alpha z), \quad -\infty < z < \infty$$

where $\phi(z)$ and $\Phi(z)$ are the probability density function (PDF) and cumulative distribution function (CDF) of the standard normal distribution, respectively; α is the shape parameter. The skewness of a distribution increases with α (Table S1). In this study, distributions of α from -8 to +8 have been generated; the corresponding skewness ranges from -0.93 to 0.93. It is noted that the basewidth of the simulated distribution decreases as $|\alpha|$ increases (Table S1). Since the experimental basewidths in logarithmic scale (*e.g.*, Δ*log*(intensity) and Δ*log*(volume)) are approximately the same for a cell population (Figure 3), the basewidths of all simulated distributions are set to the same value. The basewidths are the dynamic range of cell properties. Since the cell properties in this study (Mg and Cr contents and cell volume) are proportional to each other, the dynamic ranges are the same for a cell population. A further consideration of the simulation is the magnitude of the basewidth. The basewidths of the experimental distributions are approximately 1.2 to 1.5 (coefficient of variation CV = 0.5 – 0.6). The basewidth of the simulated distributions is set to 1.5 to match the experimental results (Table S1). The peak width and peak position of the skew-normal distributions can be adjusted by introducing scale (ω) and location (ξ) parameters to the equation above.[38]

$$f(z,\alpha,\xi,\omega) = \frac{2}{\omega}\phi\left(\frac{z-\xi}{\omega}\right)\Phi\left(\frac{\alpha(z-\xi)}{\omega}\right)$$

For each α, a scale parameter (ω) is applied to obtain basewidth of 1.5 (Table S1). The position of the distribution is not adjusted because the relative position of the peaks has no effect on the shape of the correlation plot. The location parameter ξ is set to zero.

Figure S1 shows representative pairs of distributions of different combination of skewness and their respective correlation plots. For distribution pair of zero skewness (Figure S1(a)), the correlation plot is linear (Figure S1(b)). The correlation plots are also linear for pairs of asymmetric distributions of the same skewness (figures not shown).

As one of the distributions becomes asymmetric while the other remains normal (Figures S1(c) and (e)), the correlation plot becomes non-linear (Figures S1(d) and (f)). The curvature increases as the difference in skewness increases (Figure S1(d) *versus* Figure S1(f)). In addition, the sign of the curvature of the correlation plot depends on the sign of the difference in skewness. In Figure S1(d), the difference in α is +2 (skewness difference = +0.454) and the curvature is positive (concave upward). In Figure S1(h), the difference in α is -2 (skewness difference = -0.454) and the curvature is negative (concave downward).

The correlation plots of two asymmetric distributions of different degree of skewness are expected to be nonlinear. However, for small difference in skewness, the curvature of the correlation plots is relatively small. For example, the difference in skewness is 0.33 for the distributions in Figure S1(i) ($\alpha = 2$ and 4) and their correlation plot in Figure S1(j) is fairly linear. The curvature of the correlation plots is large (Figure S1(l)) for distributions of opposite sign of skewness (Figure S1(k)). In general, nonlinear correlation plots indicate that the two distributions are different in shape. The larger the difference in the shape of the distributions, the larger the curvature of the correlation plot. The above general trend of the correlation plots can be extrapolated to distributions that are composed of more than one distinct population (*e.g.*, mixture of 2 distinct cell populations). Such distributions will give correlation plot with abrupt changes in slope at the boundary of the populations.

The basewidths of the simulation in Figure S1 are set to 1.5, similar in magnitude to the experimental basewidths. It is noted that the linear dynamic range of the cell properties is relatively small (approximately 30) and the spacing between the data points over the entire dynamic range is relatively even. For log-normal distributions of larger basewidths, the high-percentile data points may dominate the correlation plot. Analysis of such correlation plots should be mindful of possible bias on the high percentiles of the population.

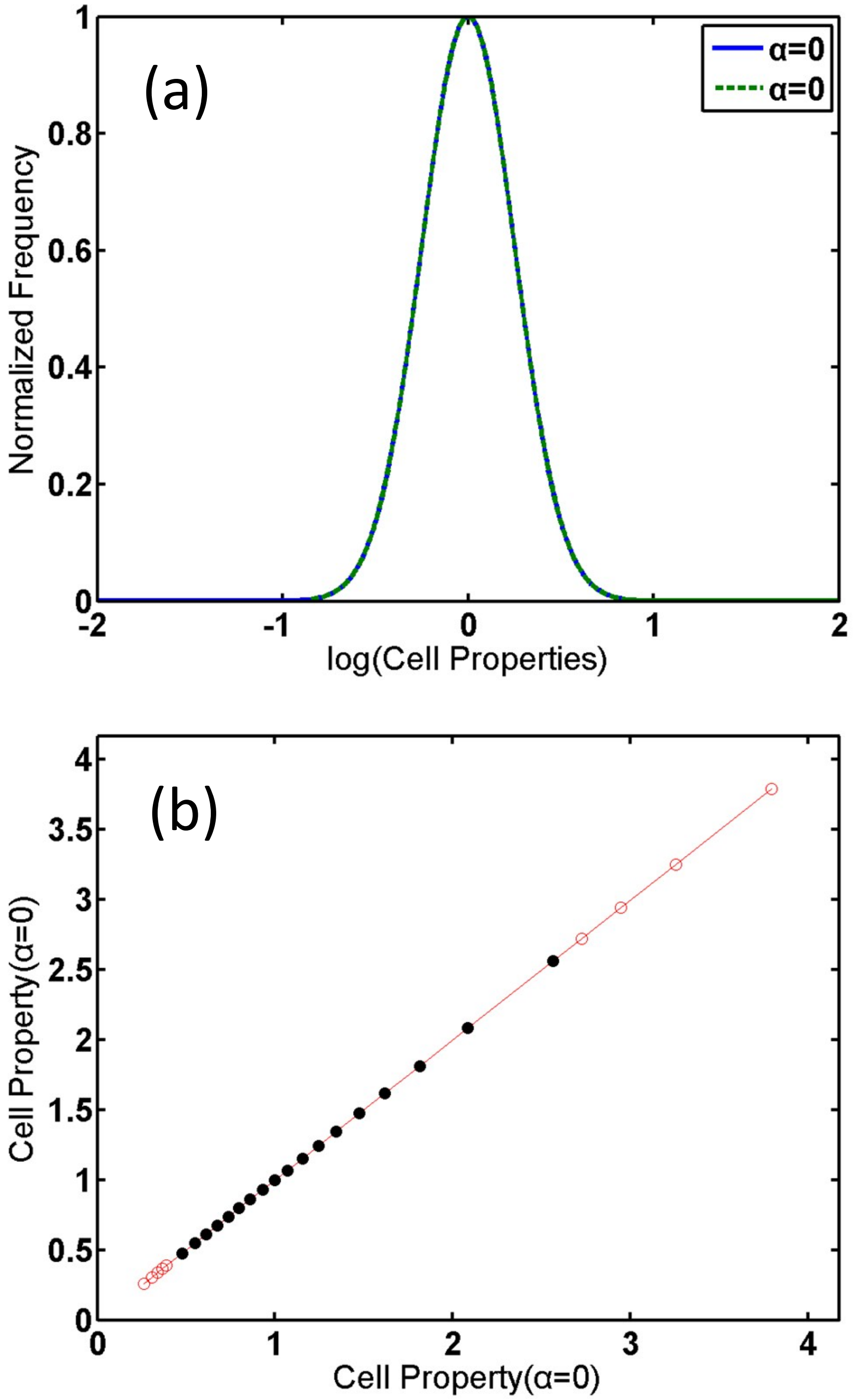
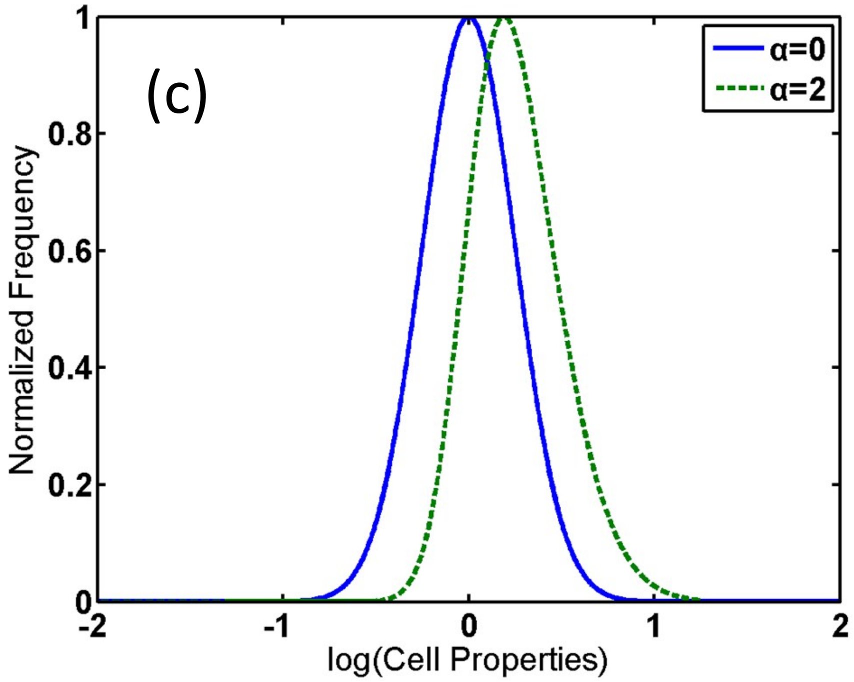
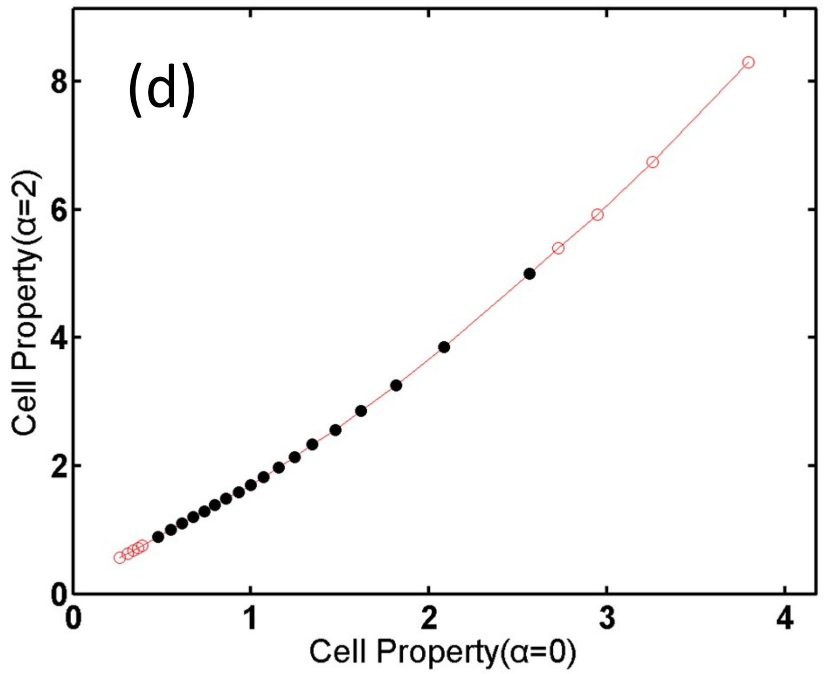


Fig S1. Simulated distributions of logarithm of cell properties and their correlation plots. (a, b) $\alpha = 0$ for both distributions, (c, d) $\alpha = 0$ and 2, (e, f) $\alpha = 0$ and 4, (g, h) $\alpha = 0$ and -2, (i, j) $\alpha =$ 2 and 4, (k, l) $\alpha = 2$ and -2. ● 5^{th} to 95^{th} percentiles of the distribution with increment of 5 percentile. ○ 1^{st} to 4^{th} and 96^{th} to 99^{th} percentiles with increment of 1 percentile.





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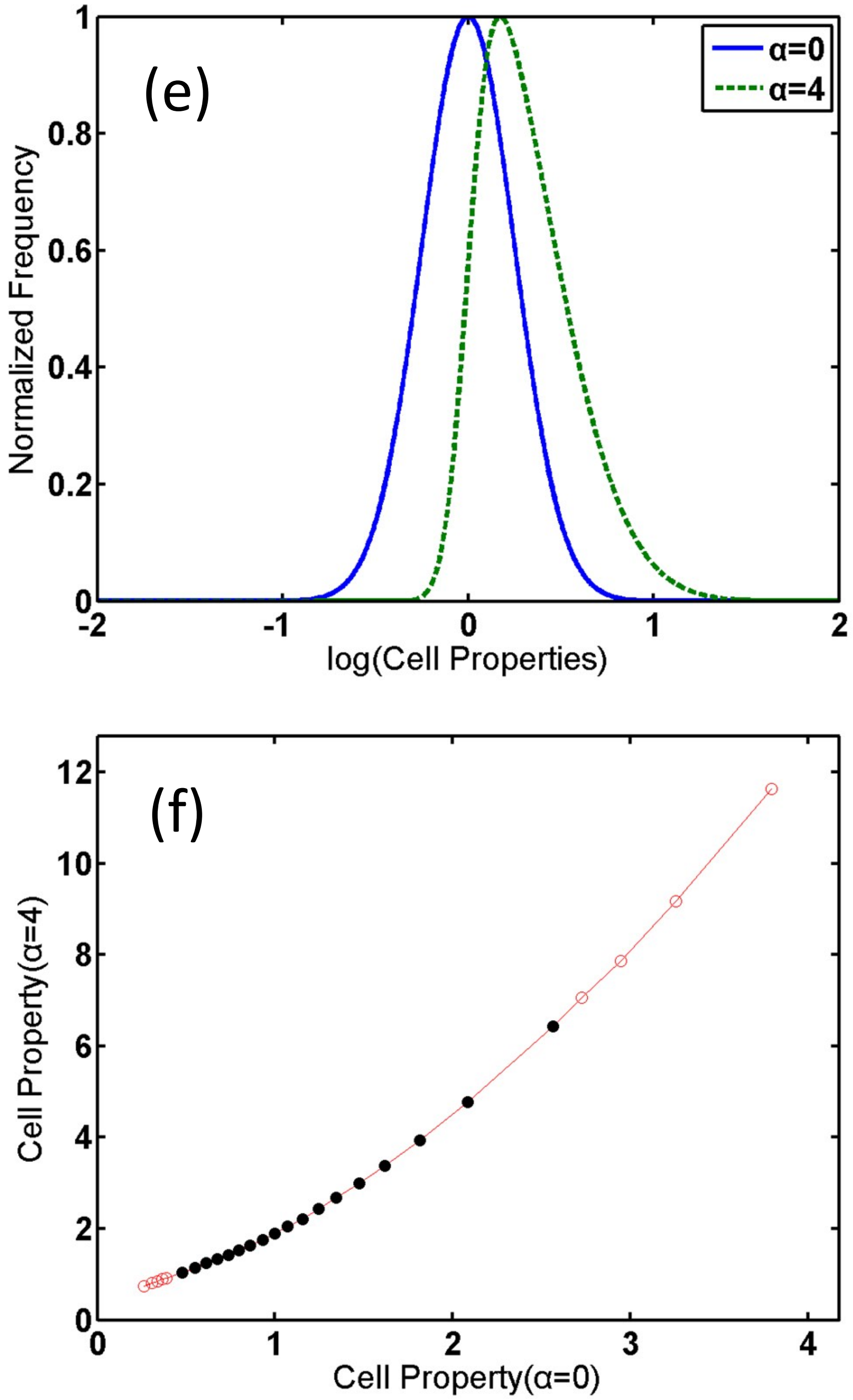

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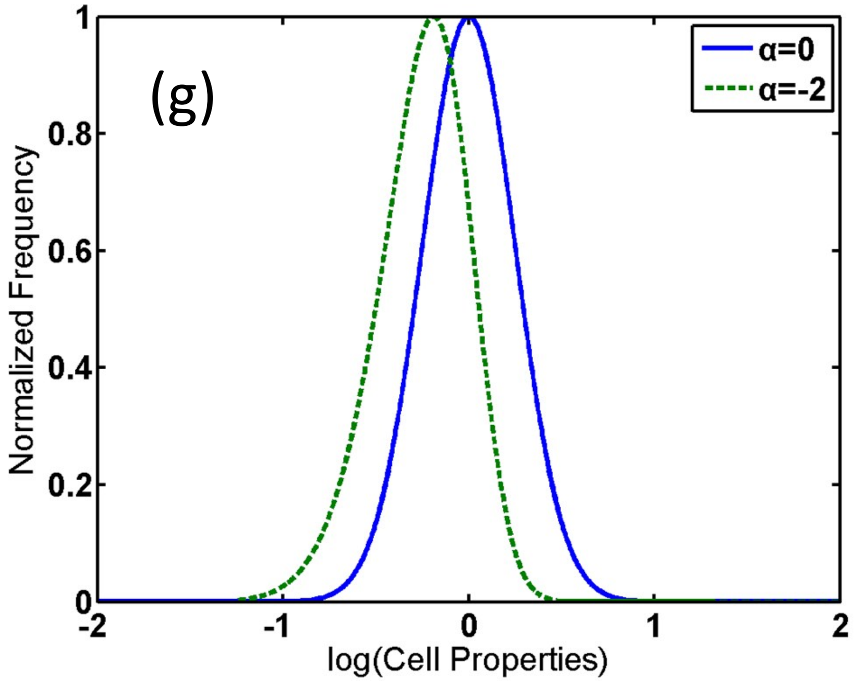

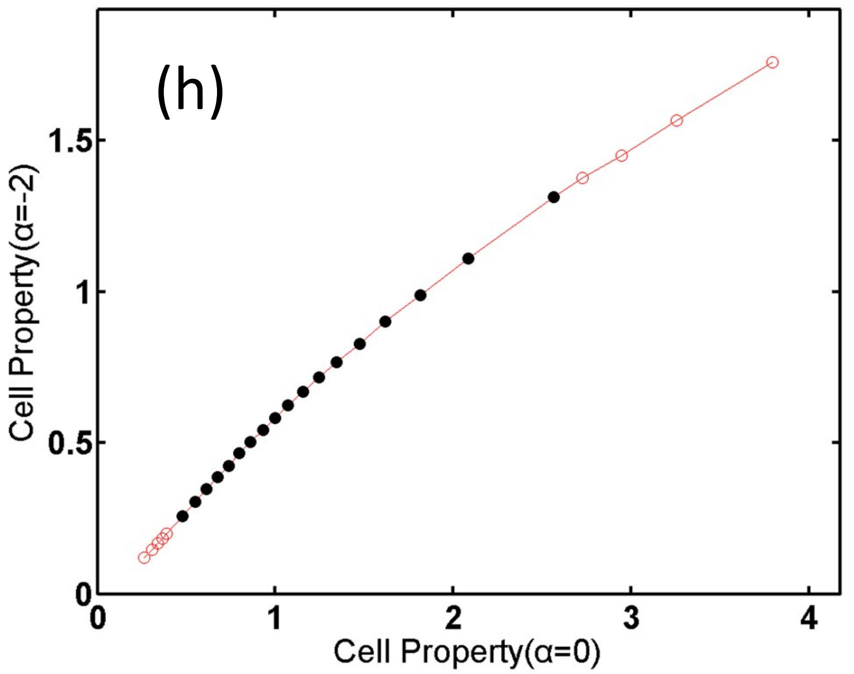

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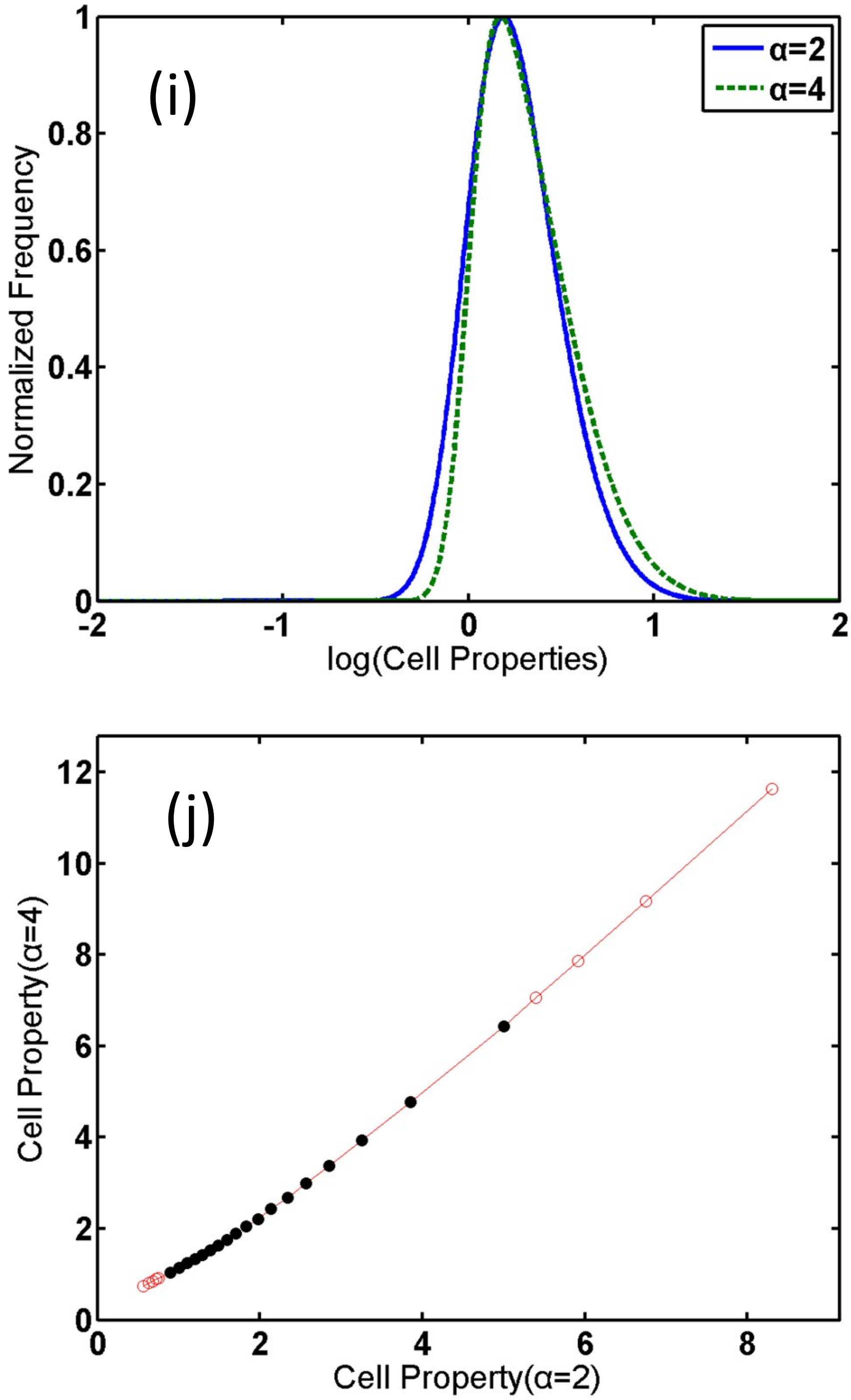

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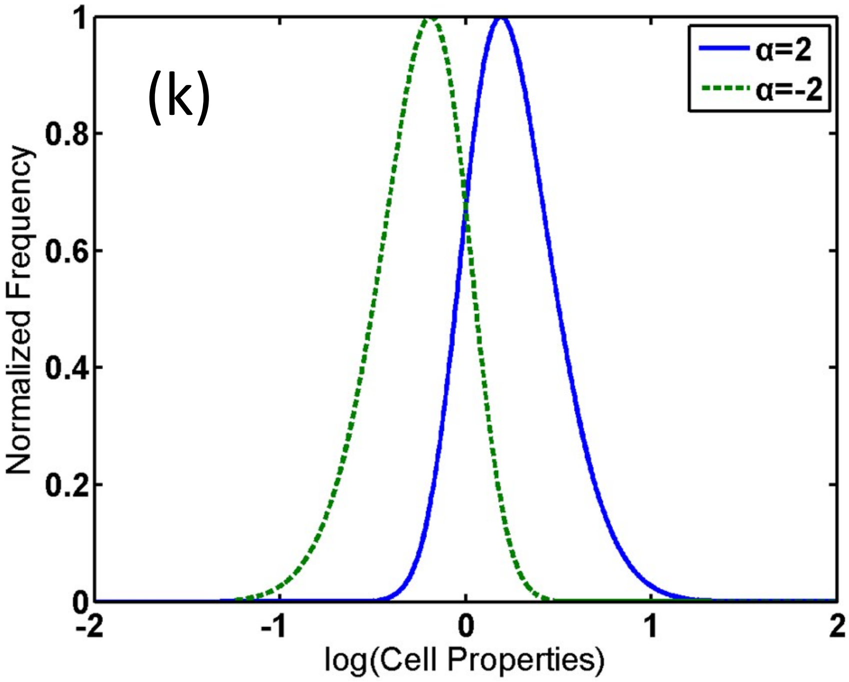

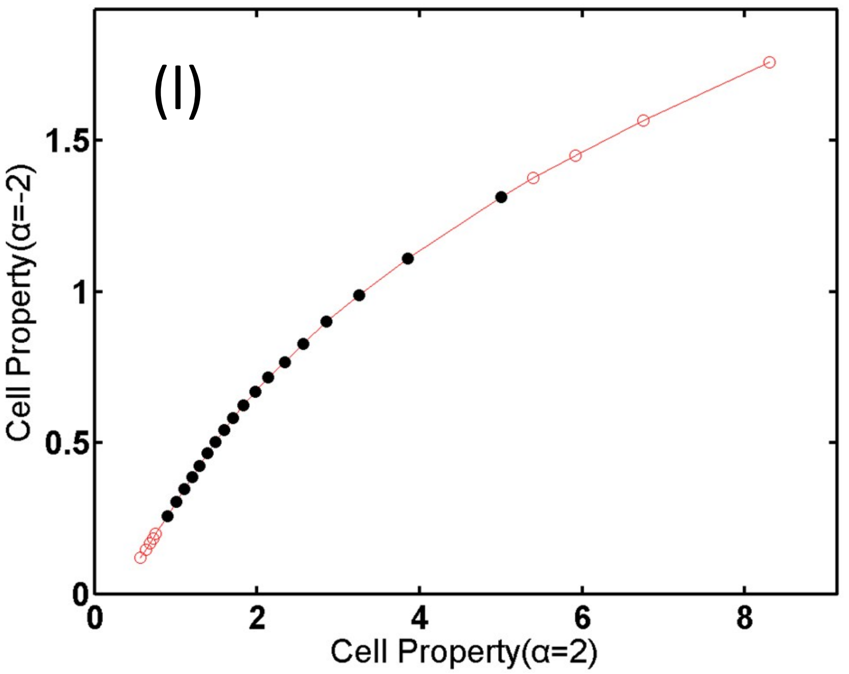

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Table S1. Shape parameters *versus* skewness and basewidths of the simulated distribution.

shape parameter, α	**skewness**	**relative basewidth**	**scale parameter (ω) to produce basewidth of 1.5**
-8	-0.934	0.55	0.45
-4	-0.784	0.60	0.41
-2	-0.454	0.70	0.36
0	0	1.00	0.25
2	0.454	0.70	0.36
4	0.784	0.60	0.41
8	0.934	0.55	0.45

References

37. A. Azzalini, *Scand J Stat*, 1985, **12**, 171-178.
38. A. Azzalini, *Scand J Stat*, 2005, **32**, 159-188.