Electronic Supplementary Information

Detection and quantification of subtle changes in red blood cell density using a cell phone

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Fig. S1 Magnetic field profile inside the capillary for a plane perpendicular to the capillary's length (end view) and the plane though the center of the capillary along its length (side view). Arrows indicate field direction, with their lengths corresponding to magnitude.



Fig. S2 Intensity profile (red trace) of a 10X microscope image of levitating RBCs with 1.05 and 1.2 g/mL beads. Gaussian fits of the intensity peaks (blue trace) are performed to quantify their positions. Interpeak distances are then used to calculate the sample's levitation ratio.



Fig. S3 (A) Levitation height as a function of Gd(3+) concentration for 1.05 g/mL beads (top), RBCs (middle), and 1.2 g/mL beads (bottom). Fits to the experimental data (black lines) indicate the data are in excellent agreement with the predicted relationship. Goodness of fit was determined with the standard error of the regression and found to be 2.0 μ m (1.05 g/mL beads), 3.3 μ m (cells), and 2.9 μ m (1.2 g/mL beads). (B) Density resolution (ratio of density difference to levitation height difference of the reference beads) versus Gd(3+) concentration. The fit to experimental data (black line) has a standard error of the regression of 0.02×10^{-3} g/mL/ μ m.

S1. Derivation of the equation for levitation height as a function of gadolinium concentration

The formula for levitation height is given by 1

$$h = \frac{g\mu_o d^2}{4B_o^2} \frac{(\rho_{\rm cell} - \rho_{\rm m})}{(\chi_{\rm cell} - \chi_{\rm m})} + \frac{d}{2}$$
(1)

in which g is acceleration due to gravity, μ_o is the magnetic permeability of free space, d is the separation distance of the magnets, B_o is the magnitude of the magnetic field at the surface of the magnets, ρ_{cell} and ρ_m are the densities of the cell and suspension medium, respectively, and χ_{cell} and χ_m are the magnetic susceptibilities of the cell and suspension medium, respectively.

Both the density and magnetic susceptibility of the suspension medium are functions of the Gd concentration, [Gd]. The density is given by

$$\rho_{\rm m} = \frac{\rho_{\rm Gd} V_{\rm Gd} + \rho_{\rm water} V_{\rm water}}{V_{\rm total}} = \frac{V_{\rm Gd}}{V_{\rm total}} \rho_{\rm Gd} + \frac{V_{\rm water}}{V_{\rm total}} \rho_{\rm water}$$
(2)

where $\frac{V_{\text{Gd}}}{V_{\text{total}}}$ and $\frac{V_{\text{water}}}{V_{\text{total}}}$ are the volume fractions of the Gd molecules and water, respectively, and ρ_{Gd} and ρ_{water} are their densities. The volume fraction of Gd molecules is given by $[\text{Gd}]V_m$, where V_m , the molar volume, is the molecular weight of the Gd molecules divided by their mass density, both of which are known quantities. The volume fraction of water in the solution is therefore $1 - [\text{Gd}]V_m$.

The density of the suspension medium can then be written as

$$\rho_{\rm m} = [\mathrm{Gd}] V_m \rho_{\mathrm{Gd}} + (1 - [\mathrm{Gd}] V_m) \rho_{\mathrm{water}} \tag{3}$$

$$= [\mathrm{Gd}]V_m(\rho_{\mathrm{Gd}} - \rho_{\mathrm{water}}) + \rho_{\mathrm{water}} \tag{4}$$

Similarly, the magnetic susceptibility of the suspension medium is

$$\chi_{\rm m} = [{\rm Gd}] V_m (\chi_{\rm Gd} - \chi_{\rm water}) + \chi_{\rm water}$$
⁽⁵⁾

By substituting these expressions for $\rho_{\rm m}$ and $\chi_{\rm m}$ into the formula for levitation height we obtain the expression for levitation height as a function of [Gd]

$$h = \frac{g\mu_o d^2}{4B_o^2} \left(\frac{\rho_{\text{cell}} - \rho_{\text{water}} - [\text{Gd}] V_m (\rho_{\text{Gd}} - \rho_{\text{water}})}{\chi_{\text{cell}} - \chi_{\text{water}} - [\text{Gd}] V_m (\chi_{\text{Gd}} - \chi_{\text{water}})} \right) + \frac{d}{2}$$
(6)

¹S. Tasoglu, J. A. Khoory, H. C. Tekin, C. Thomas, A. E. Karnoub, I. C. Ghiran and U. Demirci, *Adv. Mater.*, 2015, **27**, 3901-3908.

S2. Derivation of the expression for density resolution as a function of gadolinium concentration.

We have defined the density resolution as the ratio of the density difference to levitation height difference of the reference beads,

$$\frac{\Delta\rho}{\Delta h} = \frac{\rho_{\rm pmma} - \rho_{\rm ps}}{h_{\rm ps} - h_{\rm pmma}} \tag{7}$$

where the subscripts refer to the PS (polystyrene) and PMMA beads. We substitute the expression for levitation height as a function of gadolinium concentration (Eq. 6 above) for $h_{\rm ps}$ and $h_{\rm pmma}$ to obtain

$$\frac{\Delta\rho}{\Delta h} = \frac{\rho_{\rm pmma} - \rho_{\rm ps}}{\frac{g\mu_o d^2}{4B_o^2} \left(\frac{\rho_{\rm ps} - \rho_{\rm water} - V_m (\rho_{\rm Gd} - \rho_{\rm water})[{\rm Gd}]}{\chi_{\rm ps} - \chi_{\rm water} - V_m (\chi_{\rm Gd} - \chi_{\rm water})[{\rm Gd}]} - \frac{\rho_{\rm pmma} - \rho_{\rm water} - V_m (\rho_{\rm Gd} - \rho_{\rm water})[{\rm Gd}]}{\chi_{\rm pmma} - \chi_{\rm water} - V_m (\chi_{\rm Gd} - \chi_{\rm water})[{\rm Gd}]}\right)}$$
(8)

After algebraic rearrangement this can be written as

$$\frac{\Delta\rho}{\Delta h} = \left(\frac{4B_o^2 \left(\rho_{\rm pmma} - \rho_{\rm ps}\right)}{g\mu_o d^2}\right) \times \left(\frac{\left[\mathrm{Gd}\right]^2 - \left(\frac{\chi_{\rm ps} + \chi_{\rm pmma} - 2\chi_{\rm water}}{V_m (\chi_{\rm Gd} - \chi_{\rm water})}\right) \left[\mathrm{Gd}\right] + \frac{(\chi_{\rm ps} - \chi_{\rm water})(\chi_{\rm pmma} - \chi_{\rm water})}{V_m^2 (\chi_{\rm Gd} - \chi_{\rm water})^2}\right)} \left(\frac{\left(\frac{(\rho_{\rm Gd} - \rho_{\rm water})(\chi_{\rm ps} - \chi_{\rm pmma})}{V_m (\chi_{\rm Gd} - \chi_{\rm water})^2} - \frac{(\rho_{\rm ps} - \rho_{\rm pmma})}{V_m (\chi_{\rm Gd} - \chi_{\rm water})}\right) \left[\mathrm{Gd}\right] + \frac{(\chi_{\rm pmma} - \chi_{\rm water})(\rho_{\rm ps} - \rho_{\rm water})(\rho_{\rm pmma} - \rho_{\rm water})}{V_m^2 (\chi_{\rm Gd} - \chi_{\rm water})^2}}\right) \tag{9}$$