Electronic Supplementary Material (ESI) for Lab on a Chip.

Confinement and viscosity ratio effect on droplet break-up in a concentrated emulsion flowing through a narrow constriction

Ya Gai[†], Jian Wei Khor[†], and Sindy K.Y. Tang*

Department of Mechanical Engineering, Stanford University, Stanford, CA 94305, USA

*sindy@stanford.edu

[†]Equal contributions

ELECTRONIC SUPPLEMENTARY INFORMATION

Note S1. Tabulating the values for f^*, a^*, b^* in Eq. (2) in the main text.

Son (2007) described the detailed procedures to calculate the shear rate of a non-Newtonian fluid in a rectangular channel with a height-to-width ratio (H/W_c) close to one.[1] We rewrite the equations here for convenience:

$$\dot{\gamma}_a = \left(\frac{6Q}{W_c H^2}\right)\left(1 + \frac{H}{W_c}\right) f^* \qquad \text{Eq. (S1)}$$

$$\dot{\gamma} = \frac{2}{3}\dot{\gamma}_a \left(\frac{b^*}{a^*} + \frac{a^*}{n f^*}\right) \qquad \text{Eq. (S2)}$$

Equation S1 calculates the apparent shear rate of a Newtonian fluid in a rectangular channel. Equation S2 calculates the shear rate of a non-Newtonian fluid in a rectangular channel assuming a power-law fluid. The values of a^*, b^*, and f^* are functions of H/W_c only. To find the corresponding values of the three parameters in our experiments, we fitted a^*, b^*, and f^* respectively as a function of H/W_c using the data provided by Son,[1] as shown in Note S1-Figure 1. We fitted each curve using a 5th degree polynomial. The equation for the fitted curve is shown in Eq. (S3-S5):

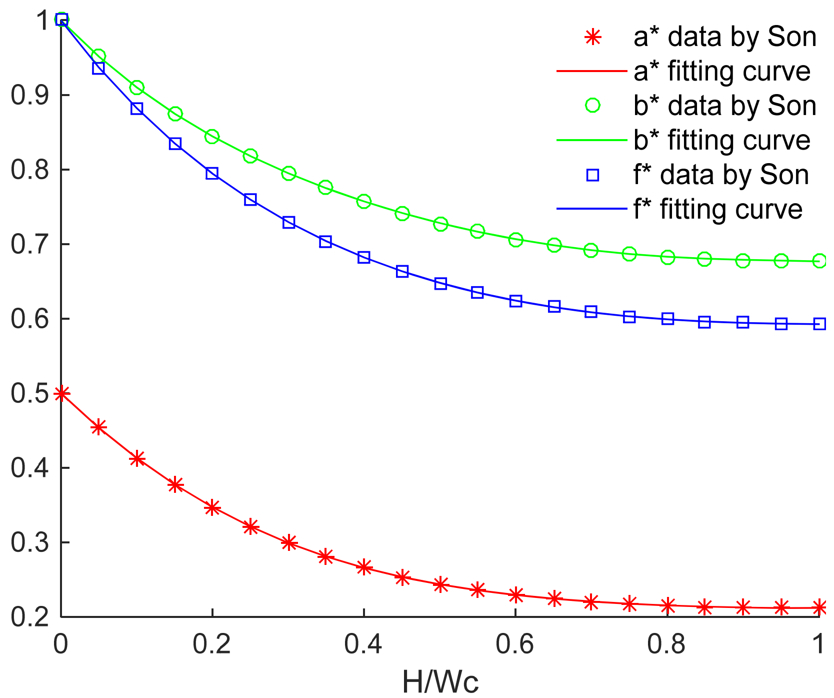
$$a^* = 0.1965\left(\frac{H}{W_c}\right)^5 - 0.3034\left(\frac{H}{W_c}\right)^4 - 0.3776\left(\frac{H}{W_c}\right)^3 + 1.1782\left(\frac{H}{W_c}\right)^2 - 0.9814\left(\frac{H}{W_c}\right) + 0.4999 \qquad \text{Eq. (S3)}$$

$$b^* = -0.5574\left(\frac{H}{W_c}\right)^5 + 1.6848\left(\frac{H}{W_c}\right)^4 - 2.1243\left(\frac{H}{W_c}\right)^3 + 1.7234\left(\frac{H}{W_c}\right)^2 - 1.0496\left(\frac{H}{W_c}\right) + 0.9998 \qquad \text{Eq. (S4)}$$

$$f^* = -0.2418\left(\frac{H}{W_c}\right)^5 + 0.9235\left(\frac{H}{W_c}\right)^4 - 1.6689\left(\frac{H}{W_c}\right)^3 + 1.9338\left(\frac{H}{W_c}\right)^2 - 1.3537\left(\frac{H}{W_c}\right) + 0.9998 \qquad \text{Eq. (S5)}$$

The values of a^*, b^*, and f^* of all our experiments are listed in Note S1-Table 1.

Note S1-Figure 1



Note S1-Table 1

Experiment #	H/W_c	a*	b*	f*
A1	0.925	0.212	0.678	0.594
A2	0.833	0.214	0.681	0.597
A3	0.925	0.212	0.678	0.594
A4	0.800	0.216	0.683	0.599
A5	0.617	0.228	0.704	0.621
A6	0.625	0.227	0.702	0.620
A7	0.833	0.214	0.681	0.597
A8	0.625	0.227	0.702	0.620
B1-B5	0.833	0.214	0.681	0.597

Figure S1. Data does not collapse when break-up fraction is plotted against a capillary number $Ca_\varepsilon = \frac{\mu_c \dot{\varepsilon} r}{\sigma}$ where the strain rate is defined as the extension rate $\dot{\varepsilon} = \frac{2v(x)\tan(\alpha)}{w(x)}$. $v(x)$ and $w(x)$ are the local velocity of the drop and channel width at position x respectively, and α is the half-angle of the taper. See ESI of our previous work for details.[2]

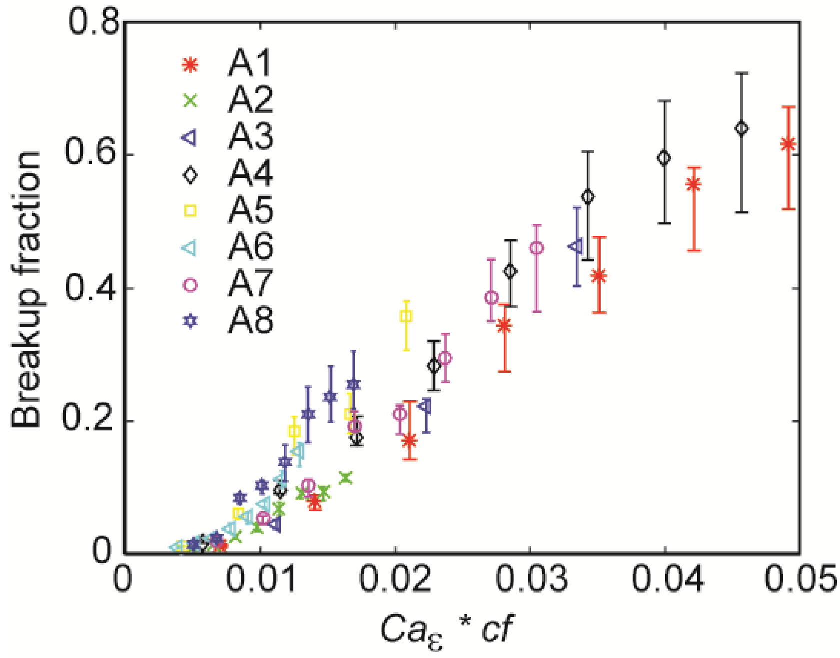


Figure S2. Data does not collapse when break-up fraction is plotted against the product of capillary number and a confinement factor defined as r/W_c, where r is the un-deformed droplet radius and W_c is the constriction width.

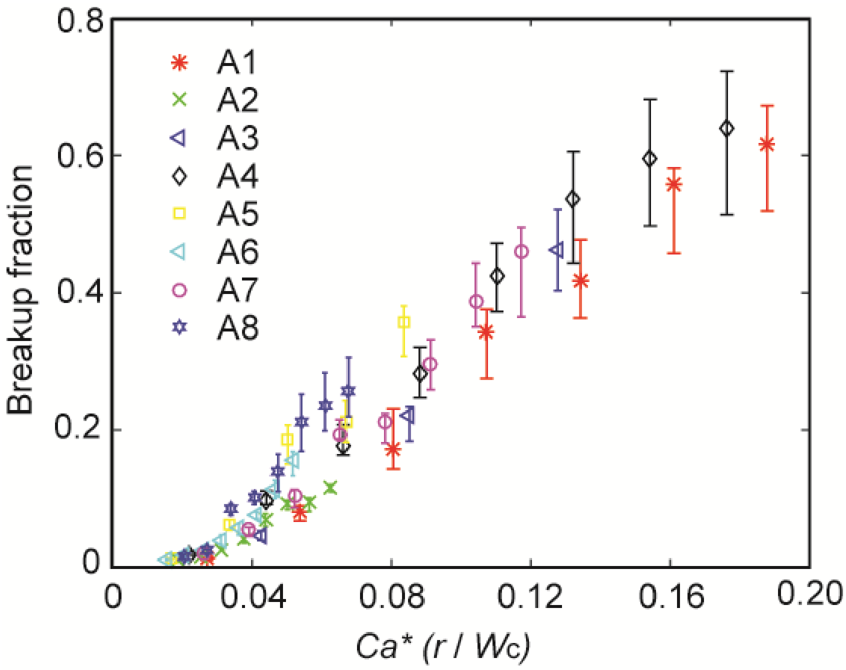


Figure S3. Break-up fraction as a function of surfactant concentration at 2% w/w, 4% w/w, and 6% w/w respectively, for Ca = 0.0261 (red), Ca = 0.0522 (green), Ca = 0.0783 (blue), and Ca = 0.1044 (black). The measured interfacial tensions between the water and oil phases are: 26.25 ± 0.28 mN/m, 26.11 ± 0.27 mN/m, and 26.17 ± 0.35 mN/m for 2% w/w, 4% w/w, and 6% w/w surfactant concentrations respectively. Dashed lines are for visual guides only.

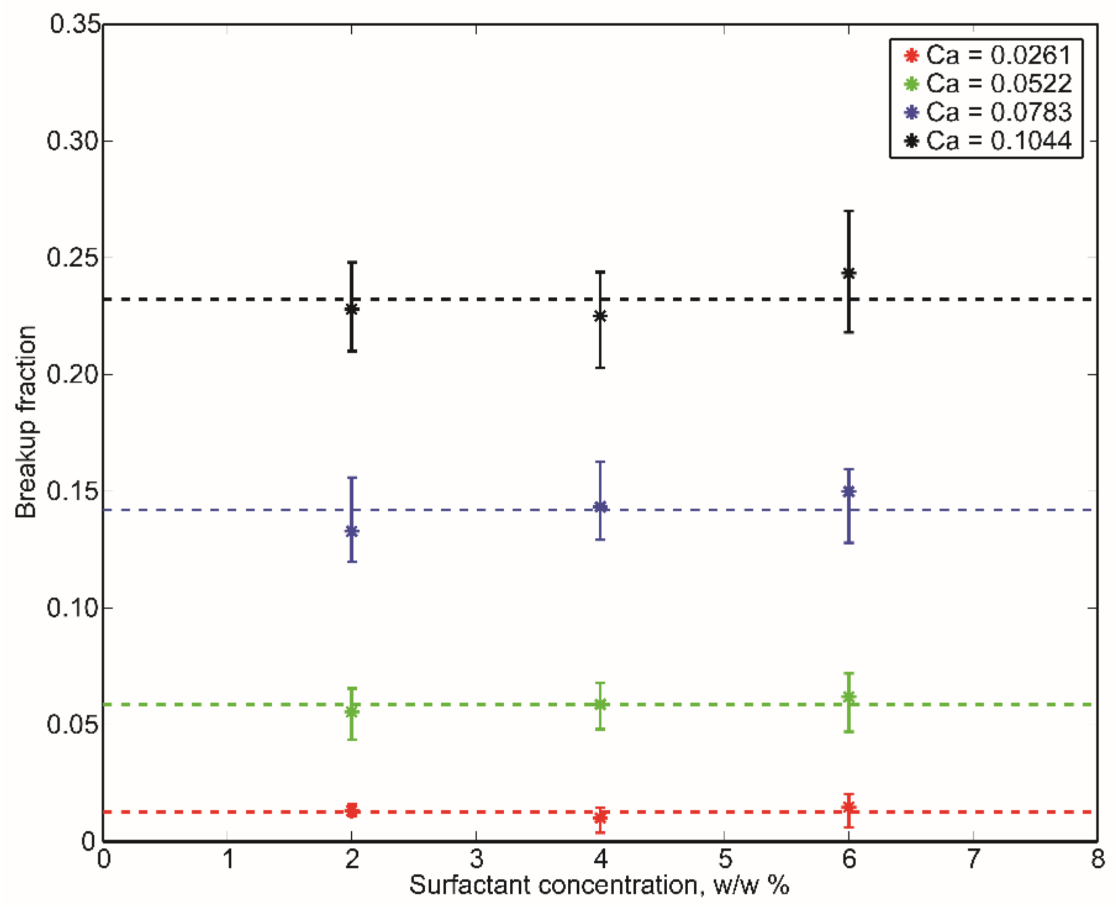


Table S1. Summary of all break-up results presented in Figures 2 and 4 in the main text.

Experiment #	Ca	Break-up fraction	Upper limit of error bar	Lower limit of error bar	Total number of drops counted upstream to calculate break-up fraction
A1	0.0296	0.0138	0.0152	0.0105	5157
	0.0592	0.0816	0.0894	0.0674	5151
	0.0888	0.1717	0.2302	0.1426	5382
	0.1184	0.3435	0.3757	0.2749	5076
	0.1480	0.4178	0.4767	0.3630	5156
	0.1776	0.5570	0.5812	0.4569	5015
	0.2072	0.6169	0.6721	0.5191	5011
A2	0.0311	0.0104	0.0125	0.0076	5284
	0.0414	0.0145	0.0166	0.0122	6235
	0.0518	0.0257	0.0282	0.0237	6006
	0.0621	0.0411	0.0478	0.0351	6083
	0.0725	0.0687	0.0780	0.0589	5581
	0.0828	0.0930	0.0974	0.0839	5156
	0.0932	0.0947	0.1012	0.0811	5042
	0.1036	0.1159	0.1203	0.1100	5457
A3	0.0373	0.0455	0.0522	0.0386	5495
	0.0746	0.2221	0.2333	0.1834	5323
	0.1119	0.4631	0.5212	0.4033	5803
A4	0.0241	0.0186	0.0222	0.0173	5490
	0.0482	0.0962	0.1114	0.0915	5182

	0.0733	0.1761	0.2075	0.1634	5385
	0.0964	0.2825	0.3206	0.2467	5169
	0.1206	0.4251	0.4720	0.3722	5110
	0.1447	0.5376	0.6053	0.4430	5367
	0.1688	0.5956	0.6814	0.4973	5031
	0.1929	0.6391	0.7229	0.5136	5338
A5	0.0219	0.0120	0.0130	0.0110	5212
	0.0439	0.0621	0.0691	0.0535	5127
	0.0658	0.1859	0.2069	0.1498	5036
	0.0878	0.2120	0.2423	0.1820	5144
	0.1097	0.3574	0.3806	0.3072	5216
A6	0.0271	0.0113	0.0121	0.0095	5149
	0.0361	0.0181	0.0216	0.0151	5306
	0.0452	0.0235	0.0250	0.0195	5047
	0.0542	0.0394	0.0470	0.0329	5562
	0.0633	0.0568	0.0633	0.0462	5562
	0.0723	0.0762	0.0802	0.0703	5892
	0.0813	0.1131	0.1261	0.1003	5713
	0.0904	0.1547	0.1679	0.1327	5726
A7	0.0299	0.0197	0.0204	0.0181	5231
	0.0448	0.0540	0.0580	0.0450	5160
	0.0597	0.1047	0.1128	0.0867	5568
	0.0742	0.1929	0.2149	0.1866	5799
	0.0896	0.2110	0.2241	0.1810	5875

	0.1046	0.2954	0.3314	0.2586	5917
	0.1195	0.3864	0.4435	0.3503	5122
	0.1344	0.4599	0.4950	0.3647	5447
A8	0.0310	0.0158	0.0176	0.0125	5152
	0.0414	0.0236	0.0253	0.0193	5582
	0.0517	0.0862	0.0918	0.0759	5405
	0.0621	0.1028	0.1109	0.0915	5266
	0.0724	0.1383	0.1641	0.1101	5669
	0.0828	0.2114	0.2520	0.1685	5170
	0.0931	0.2359	0.2831	0.1988	6021
	0.1034	0.2550	0.3059	0.2191	5617
B1	0.0261	0.0131	0.0158	0.0109	5642
	0.0391	0.0340	0.0450	0.0240	5385
	0.0522	0.0556	0.0656	0.0436	5431
	0.0652	0.0910	0.1026	0.0801	5582
	0.0783	0.1329	0.1559	0.1197	5385
	0.0913	0.1780	0.1967	0.1623	5642
	0.1044	0.2280	0.2480	0.2100	5176
	0.1174	0.2760	0.2978	0.2448	5138
	0.1305	0.3540	0.3820	0.3210	5331
B2	0.0122	0.0150	0.0175	0.0138	5120
	0.0244	0.0237	0.0265	0.0193	5959
	0.0366	0.0611	0.0687	0.0540	5108
	0.0487	0.0960	0.1053	0.0840	5217

	0.0609	0.1475	0.1636	0.1234	5814
	0.0731	0.2000	0.2229	0.1771	5672
	0.0853	0.2479	0.2714	0.2227	5131
	0.0975	0.3300	0.3612	0.3117	5241
B3	0.0105	0.0160	0.0190	0.0140	5764
	0.0211	0.0393	0.0450	0.0310	5021
	0.0316	0.0800	0.0940	0.0680	5269
	0.0421	0.1630	0.1804	0.1495	5974
	0.0527	0.2200	0.2400	0.1930	5063
	0.0632	0.2800	0.3116	0.2476	5544
	0.0738	0.3715	0.4062	0.3280	5705
	0.0843	0.4530	0.4997	0.3897	5272
	0.0948	0.5099	0.5588	0.4400	5892
B4	0.0098	0.0200	0.0230	0.0170	5507
	0.0197	0.0790	0.0946	0.0607	5317
	0.0295	0.1500	0.1700	0.1300	5429
	0.0394	0.2320	0.2642	0.1884	5695
	0.0492	0.3650	0.4060	0.3210	5823
	0.0590	0.4870	0.5230	0.4320	5950
	0.0689	0.5596	0.6020	0.4980	5034
	0.0787	0.6310	0.6830	0.5610	5039
B5	0.0105	0.0150	0.0180	0.0110	5382
	0.0210	0.0320	0.0380	0.0250	5766
	0.0316	0.0730	0.0830	0.0670	5795

	0.0421	0.1100	0.1216	0.1021	5187
	0.0526	0.1550	0.1770	0.1480	5192
	0.0631	0.2100	0.2287	0.1913	5102
	0.0736	0.2500	0.2800	0.2320	5068
	0.0841	0.3200	0.3518	0.2758	5326
	0.0947	0.3920	0.4281	0.3371	5103
	0.1052	0.4500	0.5003	0.3880	5384

References

1 Son, Y. Determination of shear viscosity and shear rate from pressure drop and flow rate relationship in a rectangular channel. *Polymer* **48**, 632-637 (2007).

2 Rosenfeld, L., Fan, L., Chen, Y., Swoboda, R. & Tang, S. K. Y. Break-up of droplets in a concentrated emulsion flowing through a narrow constriction. *Soft Matter* **10**, 421-430 (2014).