On-demand droplet splitting using surface acoustic waves

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1. The S_{11} and S_{12} parameters of the slanted-finger interdigitated transducer (SF-IDT) of the device.

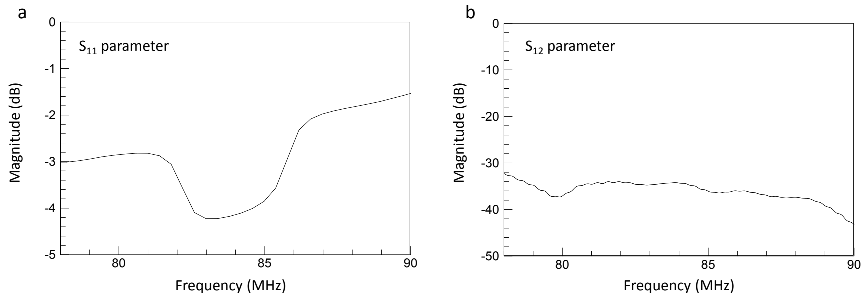

Fig. S1 (a) S_{11} and (b) S_{12} parameters of the SF-IDT measured by the network analyzer (Agilent Technologies, E5071C).

2. Theoretical analysis

The Ray acoustic regime could be applied for the understanding of the acoustic radiation forces acting on a particle by an axially focused transducer when the particle size is much larger than the wavelength of the acoustic wave[1, 2]. The acoustic radiation forces could be estimated by calculating the momentum change of the Gaussian shaped SAW beam as it is deflected and reflected at the liquid-sphere interface (see Fig. S2). For a sphere with an acoustic impedance higher than the fluid medium, the acoustic beam is mainly deflected back from the surface of the sphere towards the center of the acoustic beam, and vice versa. The acoustic radiation forces exerted on the sphere by a deflected acoustic beam are equivalent to the change in momentum (P) of the acoustic beam. As a result, the acoustic scattering force pushed the sphere in the direction of the acoustic beam propagation, whereas the acoustic gradient force is exerted away from the center of the acoustic beam and normal to the wave propagation direction.

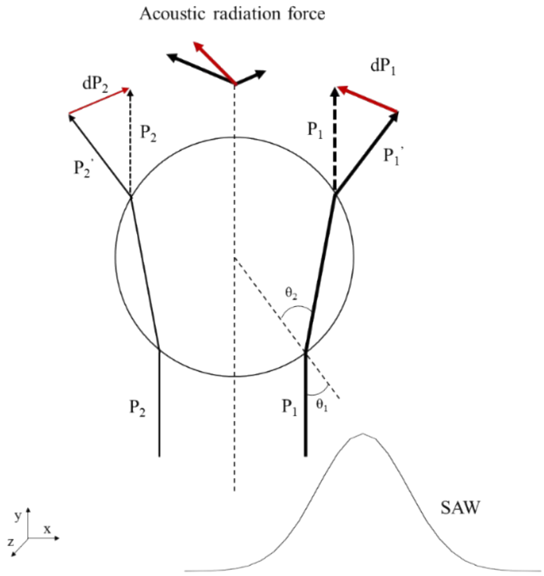

Fig. S2 A schematic of the deflection and reflection of the SAW-based Gaussian acoustic beam at the surface of the sphere. P_1 and P_2 are the momenta of the acoustic rays, and θ_1 and θ_2 are the incident and reflection angles, respectively.

The net force d*F* by an individual acoustic ray can be expressed as $d\vec{F} = d\vec{F}_s + d\vec{F}_g$, where $d\vec{F}_s$ and $d\vec{F}_g$ are the scattering and gradient forces. Each of them can also be defined by: $d\vec{F}_s = \frac{I\,dS}{c_{oil}}\vec{s}$ and $d\vec{F}_g = \frac{I\,dS}{c_{oil}}\vec{g}$, where $dS = r^2 \sin\theta_1 d\theta$. *I* is the acoustic intensity and c_{oil} is the speed of sound in the fluid medium (oil phase in this experiment). θ_1 is the incident angle of the acoustic wave, and *r* is the radius of the sphere.

Considering the infinite number of reflection and transmission of the acoustic ray at the sphere interface[3], the following expressions for the scattering and gradient forces are obtained:

$$d\vec{F}_s = \frac{1}{c_{oil}} \int_0^{2\pi}\int_0^{\pi/2} I(r,\theta_1,\varphi)\left[1 + R\cos 2\theta_1 - T^2 \frac{\cos 2(\theta_1 - \theta_2) + R\cos 2\theta_1}{1 + R^2 + 2R\cos 2\theta_1}\right] dS, \tag{1}$$

$$d\vec{F}_g = \frac{1}{c_{oil}} \int_0^{2\pi}\int_0^{\pi/2} I(r,\theta_1,\varphi)\left[R\sin 2\theta_1 - T^2 \frac{\sin 2(\theta_1 - \theta_2) + R\sin 2\theta_1}{1 + R^2 + 2R\cos 2\theta_2}\right] dS, \tag{2}$$

where *R* and *T* are the Rayleigh reflection and transmission coefficients, respectively.

$$R = \frac{Z_2/\cos\theta_2 \; - \; Z_1/\cos\theta_1}{Z_2/\cos\theta_2 \; + \; Z_1/\cos\theta_1}, \; T = 1 - R, \tag{3}$$

where Z_1 and Z_2 are the oil (fluid medium) and the water (particle/droplet) acoustic impedance, respectively. From the Snell's law, $\sin\theta_2 = \frac{c_{water}}{c_{oil}} \sin\theta_1$.

A schematic for the calculation of the acoustic gradient force exerted on the sphere inside a microchannel is shown in Fig. S3.

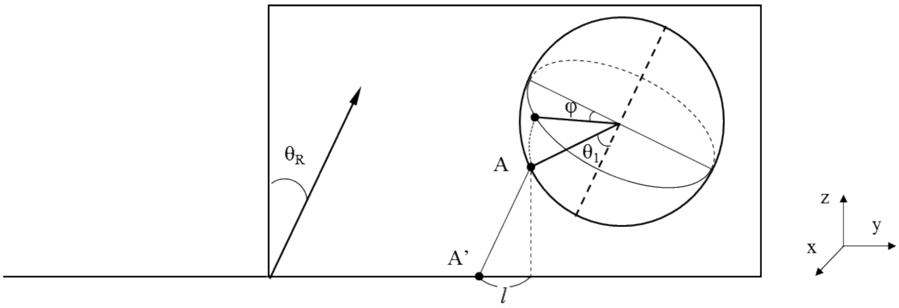

Fig. S3 A schematic for the calculation of the acoustic gradient force exerted on the sphere inside a microchannel.

On the other hand, the acoustic intensity can be expressed as:

$$I(A) = I(r, \theta_1, \varphi) = I(x, y, z), \quad (4)$$

where $x = r\sin\theta_1\cos\varphi$, $y = r\sin\theta_1\sin\varphi\cos\theta_R + r\cos\theta_1\sin\theta_R$, and $z = r\cos\theta_1\cos\theta_R - r\sin\theta_1\sin\varphi\sin\theta_R$. The acoustic intensity can be estimated by considering the particle velocity inside the microchannel. During the propagation of the longitudinal wave in the fluid medium, the fluid's particle displacement due to the acoustic wave can be indicated as[4, 5]:

$$u_y = Ae^{j\omega t}e^{-jk_Ly}e^{\alpha k_Lz}, \quad (5)$$

$$u_z = -j\alpha Ae^{j\omega t}e^{-jk_Ly}e^{\alpha k_Lz}, \quad (6)$$

where $\alpha = j\alpha_1 = \sqrt{1 - \left(\frac{c_s}{c_{oil}}\right)^2}$, k_L is the leaky SAW wave number ($k_L = k_r + jk_i$). k_L is a complex number with an imaginary part representing the SAW energy dissipation within the liquid medium. It can be estimated by extending the method of Campbell and Jones to the solid-liquid structures assuming both stress and displacement to be continuous boundary conditions at z = 0. The particle velocity is defined as v = du/dt, and the amplitude of the particle velocity is expressed by:

$$|v_y + v_z| = \sqrt{1 + \alpha_1^2}\, 2\pi f A\, e^{k_i(y + \alpha_1 z)}. \quad (7)$$

For water as a working fluid and the $LiNbO_3$ substrate, we can find α_1 = 2.47 by using c_{water} = 1500 m/s, c_s = 3994 m/s, and k_i = 2768m^{-1}. The particle displacement (A) at the substrate surface is usually ranged from 0.35 to 0.5Å. As the acoustic sound pressure level is proportional to the particle velocity, the acoustic intensity profile could be expressed as:

$$I(A) = \left(1+\alpha_1^2\right)(2\pi f)^2 A^2 Z_o\, e^{2k_i\left(y+\alpha_1 z\right)} e^{-\left(\frac{x}{\omega_0}\right)^2} \qquad (8)$$

As the acoustic gradient force pushes the droplet away from the acoustic beam center, the drag force is directed against the acoustic gradient force. The drag force usually has been estimated by the Stokes' drag force. However, in the current scenario, as the droplets are squeezed between the microchannel walls from the bottom and top, it dramatically increases the drag force that could no longer be estimated by the Stokes' law. Therefore, it is more appropriate to estimate the drag force on the droplets as done in a Hele-Shaw cell by using the following formula:

$$F_d = 24\pi\mu\frac{r^2}{h}u, \qquad (9)$$

where μ is the fluid viscosity, h is the microchannel height, and u is the mean flow velocity.

References

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