Electronic Supplementary Material (ESI) for Lab on a Chip.

Supporting information

On-Chip Micromagnet Frictionometer Based on Magnetically Driven Colloids for Nano-Bio Interface

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Modeling of the Phase-Locked Angle

As the trapped colloid synchronously moved along with the rotating field H_{app} around the perimeter of a micro-magnet with a phase lag in the H_{app} direction, the steady rotation was governed by the balance between the magnetic ($F_{mag}^{\emptyset}$) and drag (F_D) forces, which is the sum of frictional (F_f) and viscous (F_{vis}) forces. Newton's equations of motion for a rotating colloid in cylindrical coordinates are given by[1]

$$\sum \vec{F} = m\vec{a} \tag{s1}$$

$$\sum F_{net}^{\phi} = F_{mag}^{\phi} - (\mu_k N + F_{vis}) = 0 \tag{s1a}$$

$$\sum F_{net}^{z} = F_{mag}^{z} + (F_{buoy} - F_{grav}) + N = 0 \tag{s1b}$$

Because the forces on the colloid are derived from a magnet field distribution, the magnetic forces are correlated with the phase-locked angle $\emptyset$, which consists of tangential-$F_{mag}^{\emptyset}$ and vertical-F_{mag}^{z} components. F_{grav} and F_{buoy} are the gravitational and buoyance forces, respectively, and *N* is the normal vertical force that causes the F_f with a friction coefficient of μ_k. Assuming a rotating field with angular velocity ω_{app}, colloid angle θ, and phase angle $\emptyset$ from a reference axis, where $\emptyset = \omega_{app}t - \theta$, the balancing force equations of Eqs. (s1a) and (s1b) can be rewritten as:

$$G(\phi, H_{app}, \mu_k, \omega_{app}) + F_{vis}^{o} \cdot \frac{d\phi}{dt} = 0 \tag{s2}$$

$$G = F_{mag}^{\phi} - \left(F_f + F_{vis}\right) \tag{s2a}$$

$$F_f = -\mu_k \left(F_{mag}^{z} + (F_{buoy} - F_{grav})\right) \tag{s2b}$$

where F_{vis}^{o} is the coefficient of viscous force and independent to the angular velocity.

Although there are instantaneous variations in the phase-locked angle $\emptyset(t)$ due to imperfections on the colloid and substrate surfaces (even in the phase-locked regime), its time-averaged value is fixed for a balanced and steady rotation,[2] i.e., $\langle d\phi/dt \rangle = 0$. The net tangential force is balanced by the frictional and viscous forces, and the relationship between μ_k and the phase angle ϕ from Eq. (s2) is given as following equations:

$$\phi_{lock} = \phi(t)\big|_{average} = G^{-1}(\phi), \quad \text{with } G(\phi, H_{app}, \mu_k, \omega_{app}) = 0 \tag{s3}$$

$$\left.\frac{-\mu_{\mathrm{k}} F^{z}_{mag}(\phi, H_{app})}{F^{\phi}_{mag}(\phi, H_{app})}\right|_{\phi_{lock}} + \left.\frac{F_{vis}}{F^{\phi}_{mag}(\phi, H_{app})}\right|_{\phi_{lock}} = 1 \tag{s3a}$$

$$F_f = F^{\phi}_{mag}(\phi_{lock}, H_{app}) - F_{vis} \cong \mu_{\mathrm{k}} \left| F^{z}_{mag}(\phi_{lock}, H_{app}) \right| \tag{s3b}$$

The derived phase-locked model of Eq. (s3) is correlated with the balancing forces. Eq. (s3a) is testified by the relationship between the remotely controlled magnetic and viscous forces through the parameters of magnetic field H_{app} and rotating field frequency ω_{app} and the simulated trigonometric functions of magnetic forces in Eqs. (s3a) and (s3b). The frictional force is quantified by Eq. (s3b) using the tangential and vertical magnetic forces of the measured phase-locked angle and the velocity-dependent viscous force.

References

1. S. T. Thornton and J. B. Marion, Classical Dynamics of Particles and Systems, 5th ed.; Brooks/Cole: Pacific Grove, CA, 2003.
2. X. Hu, R. Abedini-Nassab, B. Lim, Y. Yang, M. Howdyshell, R. Sooryakumar, B. B. Yellen and C. G. Kim, *J. Appl. Phys.*, 2015, **118**, 203904.

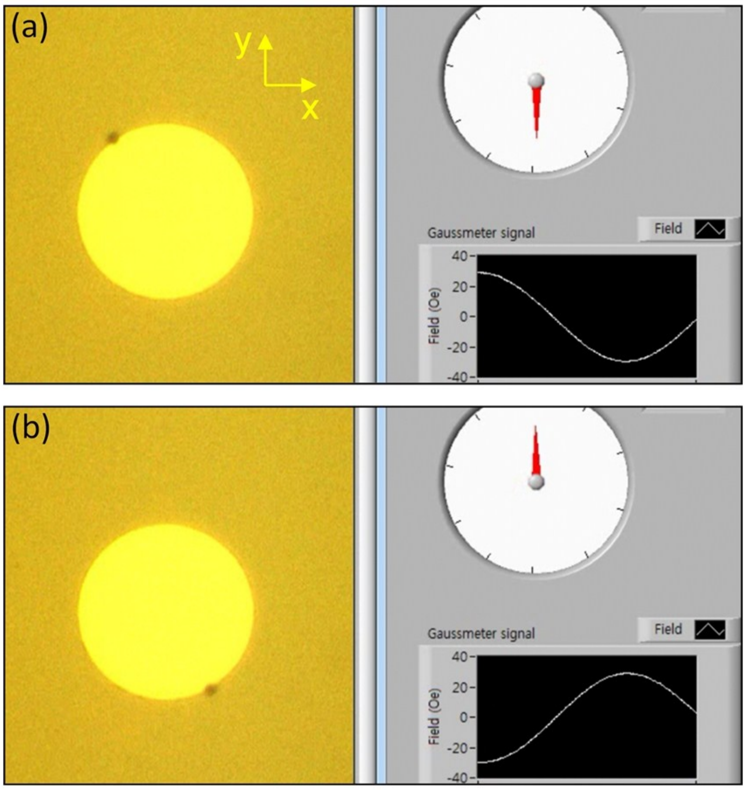

Figure S1. Measurements of the colloid phase-locked angles at the field directions of 0° (a) and 180° (b) from the y-axis (as the initial points to measure phase angles around whole periphery) under an H_{app} of 3 mT with a rotating frequency of 0.5 Hz. The phase lags of the colloids from the field directions were observed. The data was averaged for 20 cycles to obtain reliable statistics. Here, the wave peaks of the gaussmeter signal monitored in the y-axis represent that the fields are at 0° and 180° from the y-axis.

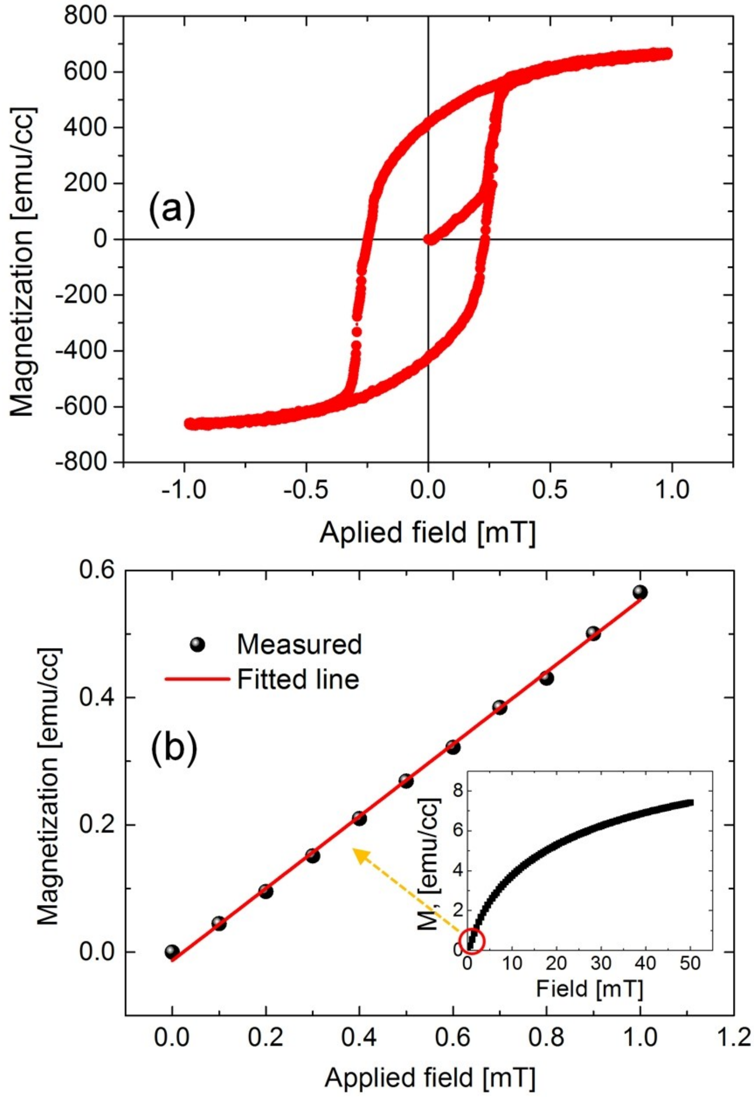

Figure S2. (a) Magnetic hysteresis loop for a 100-nm-thick NiFe film. The hysteresis loop shows that the film has a coercivity of 0.23 mT and a saturation magnetization of 668 emu/cc. (b) Magnetization curve for 2.8 μm superparamagnetic colloids in the low-field region. The inset shows the initial magnetization curve up to saturation.

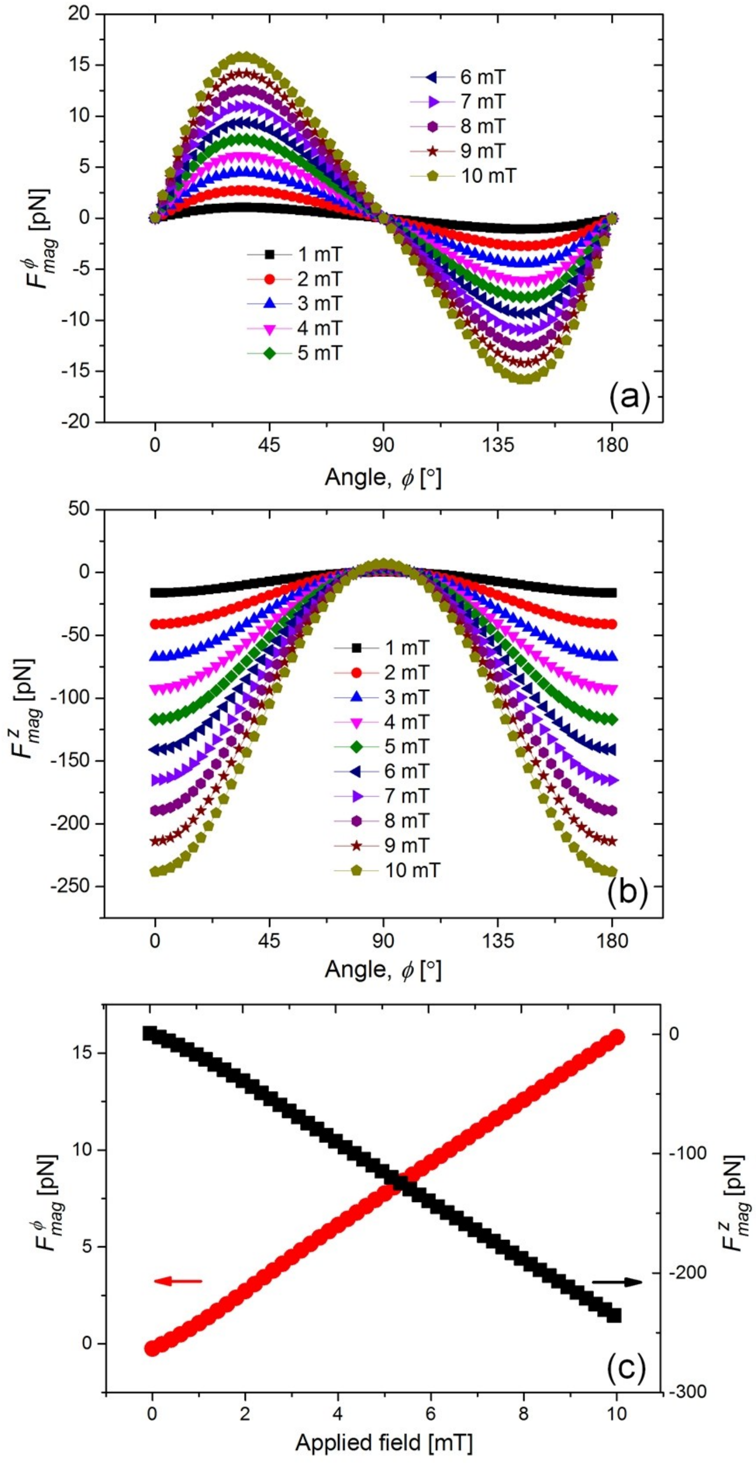

Figure S3. (a, b) Angular dependence of the magnetic forces, $F^{\emptyset}_{mag}$ and F^{z}_{mag}, on a superparamagnetic colloid with a diameter of 2.8 µm and a magnetic susceptibility of χ_v = 0.7 around a micro-magnet under in-plane fields from 1 to 10 mT. (c) Maximum magnitudes of $F^{\emptyset}_{mag}$ and F^{z}_{mag} as a function of the field strength, H_{app}.

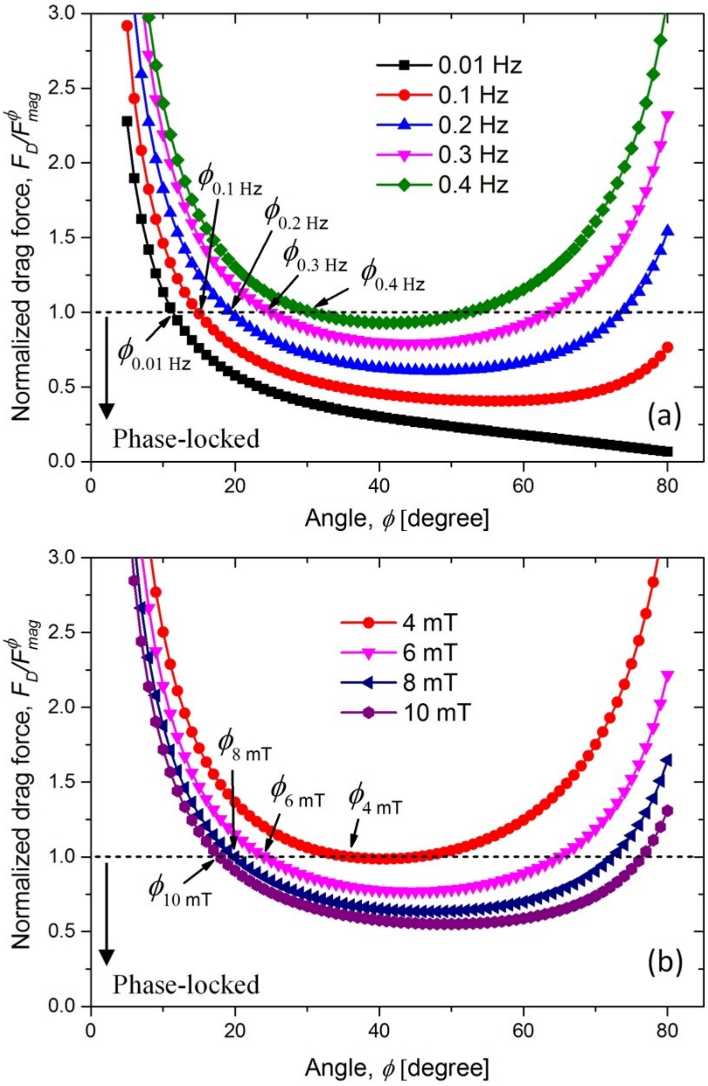

Figure S4. Angular dependence of the normalized drag by tangential force, $(-\mu_k F^{z}_{mag} + F_{vis})/F^{\phi}_{mag}$, under 3 mT field with different frequencies (a), and at 0.6 Hz frequency with different fields (b). Here, F^{z}_{mag} dominates the vertical force, $N \approx - F^{z}_{mag}$, causing the frictional force of $F_f = - \mu_k F^{z}_{mag}$. Thus, the angle to satisfy in Eq. (s3a) corresponds to the phase-locked angle. When the tangential force is smaller than the sum of frictional and viscous forces, the colloid can't rotate, as in Eq. (s4).

$$\frac{\mu_k \left| F^{z}_{mag}(\phi, H_{app}) \right|}{F^{\phi}_{mag}(\phi, H_{app})} + \frac{F_{vis}}{F^{\phi}_{mag}(\phi, H_{app})} > 1 \qquad \text{(s4)}$$

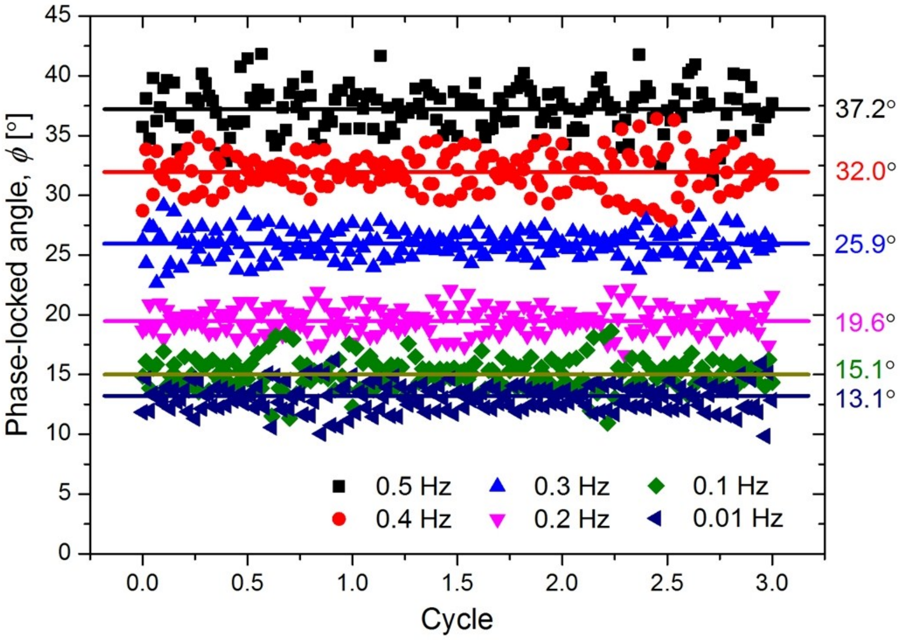

Figure S5. Experimentally determined phase-locked angles of the colloid as a function of the rotating cycle under a 3 mT field at rotating frequencies of 0.01, 0.1, 0.2, 0.3, 0.4 and 0.5 Hz. The phase-locked angles increase with the rotating frequency because the increased viscous force on the colloid reduces the net tangential force, thus causing higher angles.

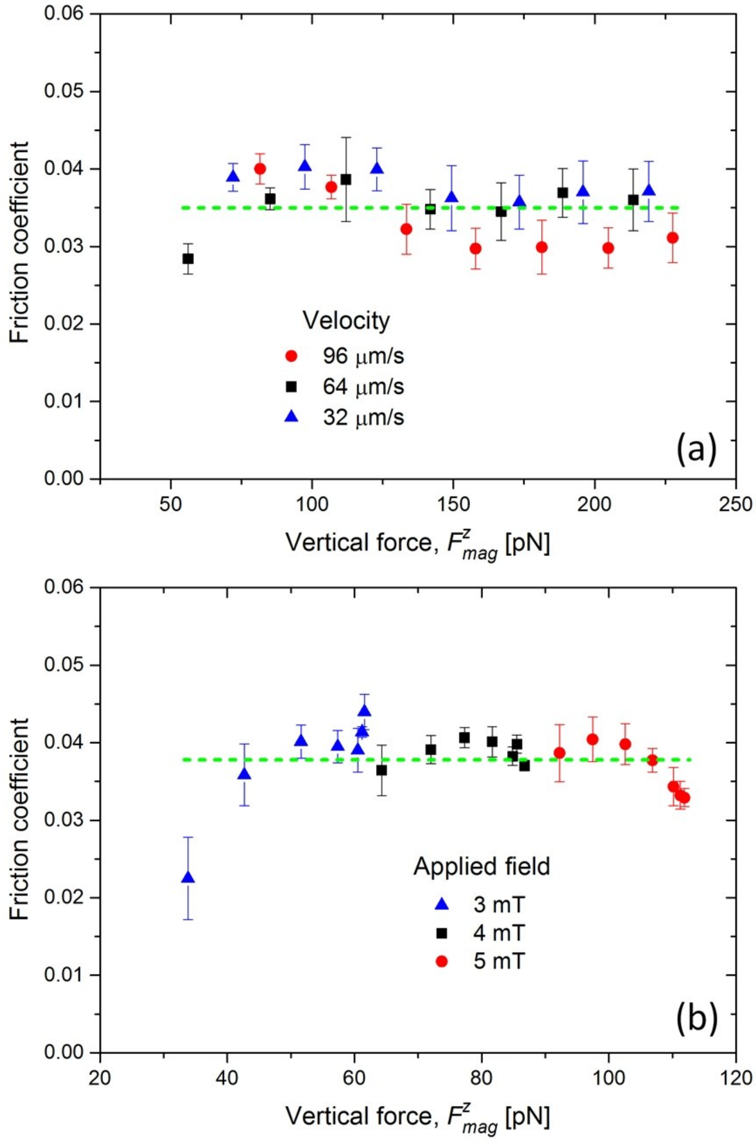

Figure S6. (a) Friction coefficient as a function of vertical force at velocities of 32, 64 and 96 µm/s. The dotted line shows the average coefficient of 0.035. (b) Friction coefficient as a function of vertical force under applied fields of 3, 4 and 5 mT. The dotted line shows the averaged coefficient of 0.038.