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Supplementary information

A novel micromixer based on the alternating current-flow field

effect transistor

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Theoretical background

Electric field

The electrical potential in the bulk electrolyte can be calculated by solving Laplace's equation, assuming the bulk concentration is homogeneous.

$$\nabla \cdot (\sigma \mathbf{E}) = -\sigma \nabla^2 \phi = 0 , \qquad (S1)$$

where **E** is the electric field, ${}^{{\it P}}$ is the bulk potential, and ${}^{{\it \sigma}}$ is the liquid conductivity.

The Laplace equation is subjected to the boundary conditions as follows: Although it is desirable to take into account the existence of a double layer on all electrode surfaces, for simplicity we ignored it on the driving electrodes, which is also a reasonable hypothesis considering that the double layer on the driving electrodes will be disrupted due to the faradaic reactions arising from high driving potentials. Therefore, the known potentials assigned to the driving electrodes are as shown in Fig 1b.

One: $\phi(t) = V_1 \cos(wt)$, and the other ground: $\phi(t) = 0$. (S2a) At the insulating surface, the boundary condition can be simplified to

$$\frac{\partial \phi}{\partial y} = 0 \tag{S2b}$$

The double layer on the gate electrode describes the charging of the induced double layer owing to the current in the bulk. In the case of low voltage across the diffuse layer, the surface conservation equation is:

$$C_{D}\frac{d\zeta}{dt} = C_{0}\frac{d\psi_{0}}{dt} = \frac{d(\phi_{g} - \phi)}{dt}\frac{C_{D}}{1 + \delta} = -\sigma\hat{n}\cdot\nabla\phi = \sigma \boldsymbol{E}_{n},$$
(S2c)

where $\zeta = V_g \cos(wt + \theta_g) - \phi(t)$ is the zeta potential, θ_g is the phase gap in the applied gate potential with reference to the driving signal. V_1 and V_g are the voltage amplitude applied to the driving electrode and gate electrode, respectively. $V_g = V_{g1}$ for the left gate electrode and $V_g = V_{g2}$ for the right gate electrode (Fig 1b). $\phi(t)$ is the potential in the bulk just outside the double layer. $C_0 = \frac{C_S C_D}{C_S + C_D} = \frac{C_D}{1 + \delta}$ is the capacitance per unit of area of the whole induced double layer, the parameter $\delta = C_D/C_S$ is the ratio of the diffuse layer capacitance ($C_D = \varepsilon/\lambda_D$) to Stern layer capacitance (C_S), ε is the permittivity, $\lambda_D = \sqrt{D\varepsilon/\sigma}$ is the Debye screening length, $D = 2 \times 10^{-9} \text{ m}^2/\text{s}$ is the bulk diffusivity and $\varepsilon = 7.08 \times 10^{-10} \text{ F/m}$ is the permittivity.

As for fixed-potential ICEO, an analytical solution for the induced zeta potential in the DC limit can be achieved as follows,

(1)When the gate electrode is floating, the induced zeta potential is

$$\zeta(t) = \frac{1}{1+\delta} \left(\frac{V_1}{2} \cos(wt) - \phi(t) \right) = \frac{1}{1+\delta} Ex \cos(wt)$$
(S3)

The ICEO slip velocity on the surface of the floating electrode is given by the Helmholtz-Smoluchowski equation:

$$\langle \mathbf{v}_s \rangle = \frac{-\varepsilon}{\eta} \left\langle \frac{1}{1+\delta} Ex \cos(wt) \cdot E_t \right\rangle = \frac{-\varepsilon E^2 x}{2\eta (1+\delta)}.$$
 (S4)

(2) Assuming the phase gap $\theta_g = 0$, when an electric signal $V_g \cos(wt)$ is applied to the gate electrode, the zeta potential becomes:

$$\zeta(t) = \frac{1}{1+\delta} \left(V_g \cos(wt) - \phi(t) \right) = \frac{1}{1+\delta} \left(Ex\cos(wt) + \left(V_g - \frac{V_1}{2} \right) \cos(wt) \right)$$
(S5)

The ICEO slip expression becomes:

$$\langle \mathbf{v}_{s} \rangle = \frac{-\varepsilon}{\eta} \left\langle \frac{1}{1+\delta} \left(Ex\cos(wt) + \left(V_{g} - \frac{V_{1}}{2} \right) \cos(wt) \right) \cdot E_{t} \right\rangle = \frac{-\varepsilon E}{2\eta \left(1+\delta \right)} \left(Ex + V_{g} - \frac{V_{1}}{2} \right)$$
(S6)

Flow field

For a Newtonian incompressible flow, the liquid motion can be obtained by solving the Navier-Stokes equations and the continuity equation in the limit of small Reynolds number.

$$\rho \left(\frac{\partial \overline{\boldsymbol{v}}}{\partial t}\right) = \eta \nabla^2 \overline{\boldsymbol{v}} - \nabla p , \qquad \nabla \cdot \overline{\boldsymbol{v}} = 0 , \qquad (S7)$$

P is the pressure, \overline{v} is the velocity.

For thin double layers and weak fields, the time averaged slip velocity at the interface between the bulk and the double layer can be derived from the Helmholtz-Smoluchowski formula³⁰.

$$\langle \boldsymbol{v}_{s} \rangle = \frac{-\varepsilon}{\eta} \frac{1}{2} \operatorname{Re} \left(\boldsymbol{\hat{\mathcal{C}}} \boldsymbol{E}_{t}^{*} \right) = \frac{-\varepsilon}{\eta} \frac{1}{1+\delta} \frac{1}{2} \operatorname{Re} \left(\left(\boldsymbol{\hat{\phi}}_{g}^{*} - \boldsymbol{\hat{\mathcal{C}}}^{*} \right) \left(\boldsymbol{\hat{E}} - \boldsymbol{\hat{E}} \cdot \boldsymbol{n} \cdot \boldsymbol{n} \right)^{*} \right)$$
(S8a)

where E_i is the tangential electric field, ζ is the zeta potential, η is the viscosity of the fluid, the asterisk indicates complex conjugate. Although some equilibrium potential may exist on the channel walls, it can be safety neglected here due to linear electroosmotic slip, whose time-averaged value is zero in an AC field. Thus, the above equation can be set as the boundary condition on the surfaces of the electrodes.

$$\overline{v} = \overline{v}_0$$
 at inlet and $p = 0$ at outlet, (S8b)

whereas, all other surfaces should be treated as no slip walls.

Concentration field

Assuming that electrophoretic effect is neglected, the mass transport equation can be written as

$$\frac{\partial C}{\partial t} + \overline{v}\nabla C = D\nabla^2 C \tag{59}$$

where, *C* is the concentration of the species. The boundary conditions: the concentration values are set as 0 and 1 at the two inlets respectively, the channel walls and outlet are specified as $\mathbf{n} \cdot (-D\nabla C) = 0$.



Fig. S1 The influence of the DC bias voltage on the mixing performance and mixing images at different conditions: (a) Case A, (b) Case B, (c) Case C, (d) Case D when the inlet velocity is 2000μ m/s. Scale bar, 200μ m.

Table SI. The parameters used in experiments for investigating the influence of DC bias volta

Case	<i>f</i> (Hz)	V_1 (V)	V_{g1} (V)	$V_{_{DC1}}$ (V)	V_{g^2} (V)	$V_{_{DC2}}$ (V)	Ме
A	0	0	0	0	0	0	45%
В	500	5	4	0	1	0	84%
С	500	5	3.5	1	1.5	-1	89%
D	500	5	4	1	1	-1	92%