Electronic Supplementary Information for

"Estimation of Delays in Generalized Asynchronous Boolean Networks"

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1 Coherent type 1 feed forward loop (C1 FFL)

The GABN model for coherent type 1 FFL with AND input logic is given by

$$\begin{cases} X(t) &= S_x(t) \\ Y(t) &= X(t-0.92) \wedge X(t-0.51) \\ Z(t) &= X(t-0.69) \wedge Y(t-0.69) \end{cases} \quad (1)$$

and the GABN model for coherent type 1 FFL with OR input logic is given by

$$\begin{cases} X(t) &= S_x(t) \\ Y(t) &= X(t-0.69) \\ Z(t) &= X(t-0.69) \vee Y(t-0.69) \end{cases} \quad (2)$$

where S_x, represented by a unit rectangular pulse of duration six time units, is given by $S_x(t) = A * rect(\frac{t-5}{T})$ where A=1 and T=6.

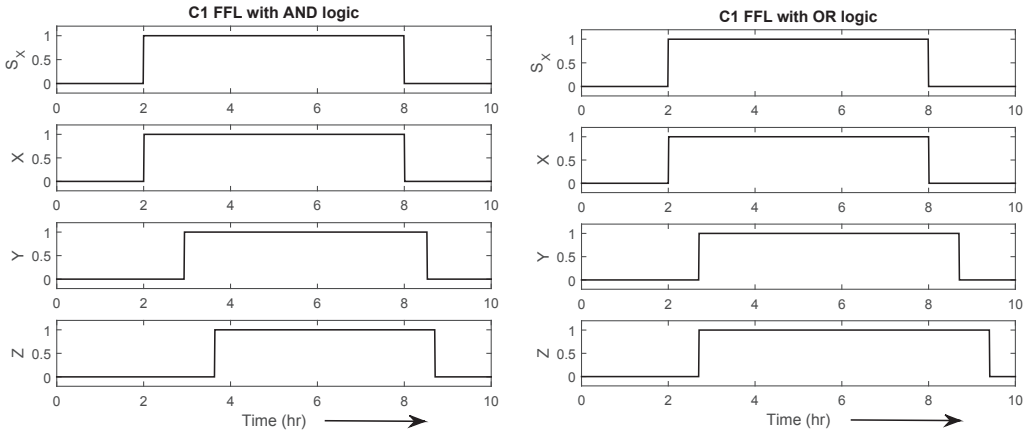


Figure 1: The simulation result of GABN model for coherent type 1 FFL with (a) AND input logic where $K_{yz} = 1.8$ and (b) OR input logic where $K_{yz} = 1.5$. $\alpha_x = \alpha_y = \alpha_z = 1$, $\beta_x = 1$, $\beta_y = \beta_z = 3$, $K_{xy} = K_{xz} = 0.5$.

2 Incoherent type 1 feed forward loop (I1 FFL)

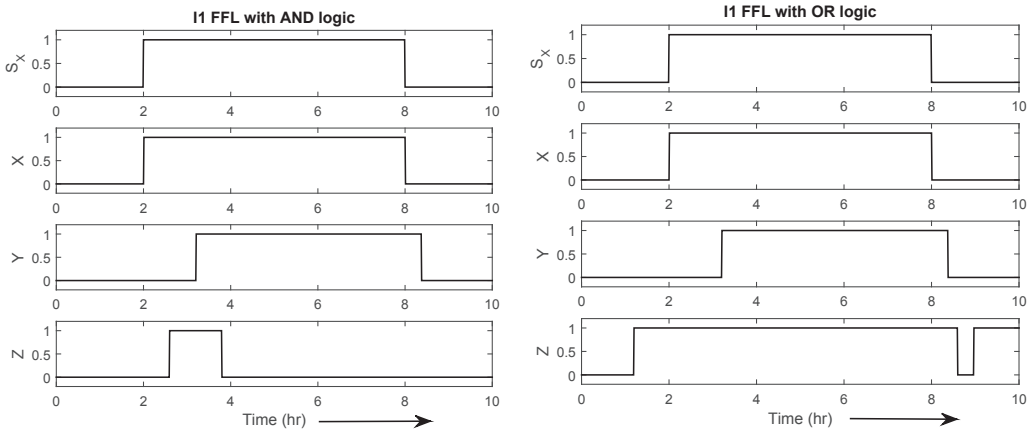
The GABN model for incoherent type 1 FFL with AND input logic is given by

$$\begin{cases} X(t) &= S_x(t) \\ Y(t) &= X(t-1.19) \wedge X(t-0.36) \\ Z(t) &= X(t-0.58) \wedge Y'(t-0.58) \end{cases} \tag{3}$$

and the GABN model for incoherent type 1 FFL with OR input logic is given by

$$\begin{cases} X(t) &= S_x(t) \\ Y(t) &= X(t-1.19) \wedge X(t-0.36) \\ Z(t) &= X(t-0.58) \vee Y'(t-0.58) \end{cases} \tag{4}$$

where S_x, represented by a unit rectangular pulse of duration six time units, is given by $S_x(t) = A * rect(\frac{t-5}{T})$ where A=1 and T=6.



(a) (b)

Figure 2: The simulation result of GABN model for incoherent type 1 FFL with (a) AND logic and (b) OR logic. $\alpha_x = \alpha_y = 1$, $\alpha_z = 1.2$, $\beta_x = 1$, $\beta_y = 3$, $\beta_z = 3.5$, $K_{yz} = 2.0948$, $K_{xy} = K_{xz} = 0.5$.

3 Multi-output feed forward loop

The GABN model for multi-output FFL with OR input logic is given by

$$\begin{cases} Y(t) &= X(t-0.76) \vee X(t-1.54) \\ Z_1(t) &= \big\{X(t-0.69) \vee X(t-1.35)\big\} \vee \big\{Y(t-1.83) \wedge Y(t-0.58)\big\} \\ Z_2(t) &= \big\{X(t-0.98) \vee X(t-1)\big\} \vee \big\{Y(t-1.13) \wedge Y(t-0.91)\big\} \\ Z_3(t) &= \big\{X(t-1.19) \wedge X(t-0.69)\big\} \vee \big\{Y(t-0.69) \vee Y(t-1.18)\big\} \end{cases} \tag{5}$$

and the GABN model for multi-output FFL with AND input logic is given by

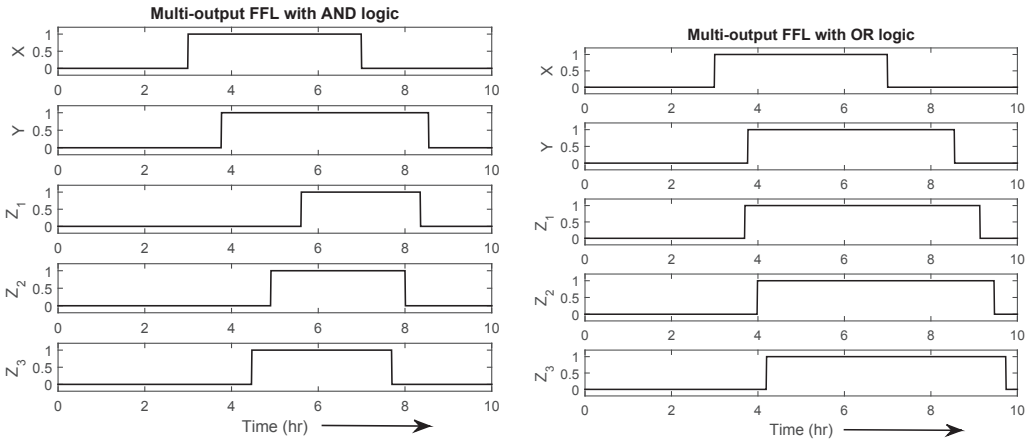
$$\begin{cases} Y(t) &= X(t-0.76) \vee X(t-1.54) \\ Z_1(t) &= \big\{X(t-0.69) \vee X(t-1.35)\big\} \wedge \big\{Y(t-1.83) \wedge Y(t-0.58)\big\} \\ Z_2(t) &= \big\{X(t-0.98) \vee X(t-1.0)\big\} \wedge \big\{Y(t-1.13) \wedge Y(t-0.91)\big\} \\ Z_3(t) &= \big\{X(t-1.19) \wedge X(t-0.69)\big\} \wedge \big\{Y(t-0.69) \vee Y(t-1.18)\big\} \end{cases} \tag{6}$$

where X, represented by a unit rectangular pulse of duration four time units, is given by $X(t) = A * rect(\frac{t-5}{T})$ where A=1 and T=4.

4 Bifan

The GABN model for bifan with AND input logic is given by

$$\begin{cases} Z_1(t) &= \big\{X_1(t-0.5) \vee X_1(t-1.07)\big\} \wedge \big\{X_2(t-1.19) \wedge X_2(t-0.5)\big\} \\ Z_2(t) &= \big\{X_1(t-1.16) \wedge X_1(t-0.69)\big\} \wedge \big\{X_2(t-0.69) \vee X_2(t-1.25)\big\} \end{cases} \tag{7}$$



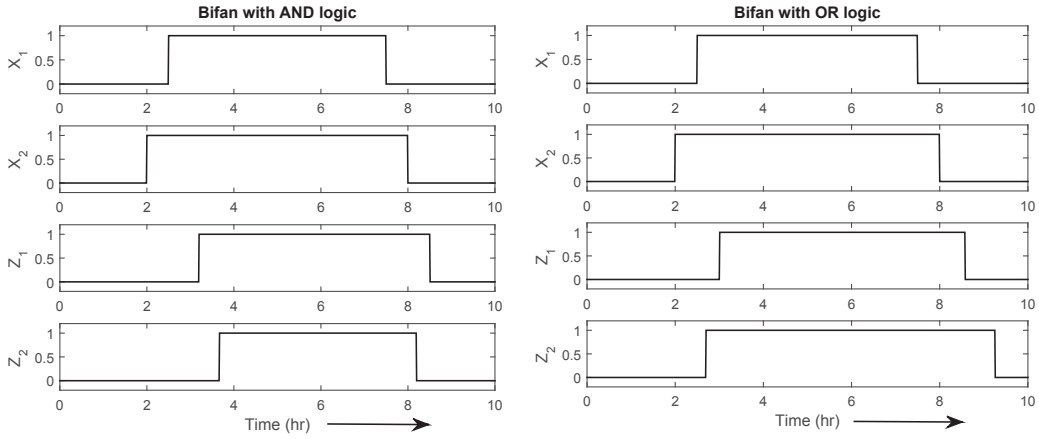
(a) (b)

Figure 3: The simulation result of GABN model for multi-output FFL with (a) AND logic and (b) OR logic. $K_{xy} = 0.3$, $K_1 = 0.6$, $K_2 = 1.0$, $K_3 = 1.4$, $K_1' = 2.6$, $K_2' = 2.1$, $K_3' = 1.6$, $\alpha_x = 1.1$, $\alpha_y = 1$, $\beta_x = 3.2$, $\beta_y = 3$, $\alpha_{z_1} = 1.2$, $\alpha_{z_2} = 1$, $\alpha_{z_3} = 1$, $\beta_{z_1} = 4$, $\beta_{z_2} = 3.1$, $\beta_{z_3} = 2.8$.

and the GABN model for bifan with OR input logic is given by

$$\begin{cases} Z_1(t) &= \big\{X_1(t-0.5) \vee X_1(t-1.07)\big\} \vee \big\{X_2(t-1.19) \wedge X_2(t-0.5)\big\} \\ Z_2(t) &= \big\{X_1(t-1.16) \wedge X_1(t-0.69)\big\} \vee \big\{X_2(t-0.69) \vee X_2(t-1.25)\big\} \end{cases} \tag{8}$$

Here X_1, represented by a unit rectangular pulse of duration five time units, is given by $X_1(t) = A * rect(\frac{t-5}{T})$ where A=1 and T=5. X_2, represented by a unit rectangular pulse of duration six time units, is given by $X_2(t) = A * rect(\frac{t-5}{T})$ where A=1 and T=6.



(a) (b)

Figure 4: The simulation result of GABN model for bifan with (a) AND logic and (b) OR logic. $K_{11} = 0.9487$, $K_{12} = 1.8974$, $K_{21} = 2.0968$, $K_{22} = 1.1982$, $\alpha_{x_1} = 1.2$, $\alpha_{x_2} = 1$, $\beta_{x_1} = 3.8$, $\beta_{x_2} = 3$, $\alpha_{z_1} = 1$, $\beta_{z_1} = 2.8$, $\alpha_{z_2} = 1.4$, $\beta_{z_2} = 4.4$.

5 Transcription cascade

The GABN model for a cascade of activators is given by

$$\begin{cases} Y(t) &= X(t-0.69) \vee X(t-0.7) \\ Z(t) &= Y(t-0.58) \end{cases} \tag{9}$$

and the GABN model for a cascade of inhibitors is given by

$$\begin{cases} Y(t) &= X'(t-0.69) \\ Z(t) &= Y'(t-0.58) \end{cases} \tag{10}$$

where X, represented by a unit rectangular pulse of duration six time units, is given by $X(t) = A * rect(\frac{t-5}{T})$ where A=1 and T=6.

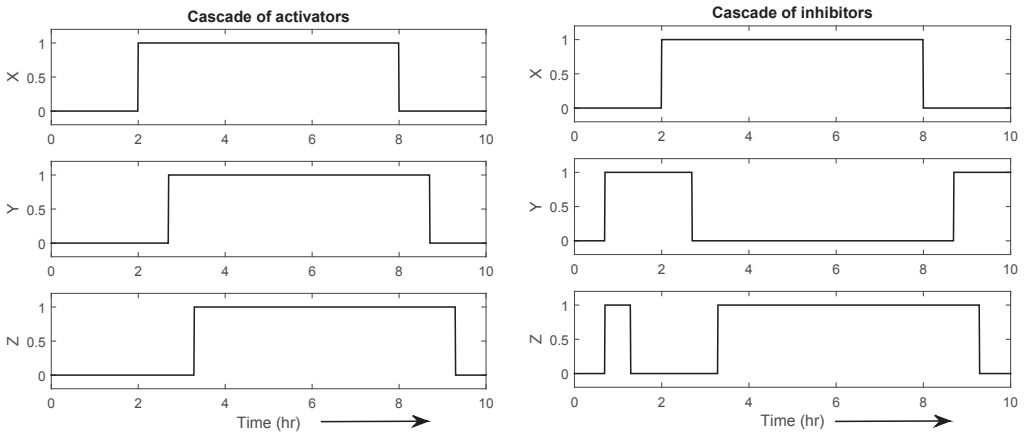


Figure 5: The simulation result of GABN model for a cascade of (a) activators with $K_{yz} = 1.495$ and (b) inhibitors with $K_{yz} = 1.5$. $K_{xy} = 1.16$, $\alpha_x = 1.1$, $\alpha_y = 1.0$, $\beta_x = 3.2$, $\beta_y = 3.0$, $\alpha_z = 1.2$, $\beta_z = 3.5$

6 Single input module

The GABN model for SIM network motif with three target genes is given by

$$\begin{cases} Z_1(t) &= X(t-0.58) \vee X(t-1.68) \\ Z_2(t) &= X(t-0.9) \vee X(t-1.1) \\ Z_3(t) &= X(t-0.96) \wedge X(t-0.5) \end{cases} \tag{11}$$

where X (same as S_x), represented by a unit rectangular pulse of duration six time units, is given by $X(t) = A * rect(\frac{t-5}{T})$ where A=1 and T=6.

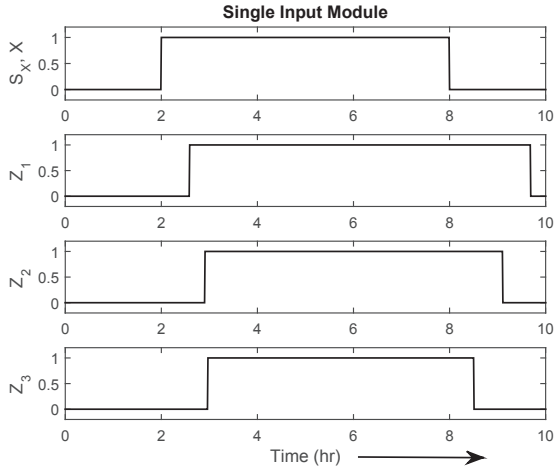


Figure 6: The simulation result of GABN model for SIM generating LIFO temporal order of gene expression. $K_1 = 0.5$, $K_2 = 1.0$, $K_3 = 1.5$, $\alpha_x = 1$, $\beta_x = 3.2$, $\alpha_{z_1} = 1.2$, $\beta_{z_1} = 3.8$, $\alpha_{z_2} = 1$, $\beta_{z_2} = 3$, $\alpha_{z_3} = 1.4$, $\beta_{z_3} = 3.6$.