Electronic Supplementary Information Novel Quantum Spin Hall Insulators in Two-dimensional Hydrogenated Molybdenum and Tungsten Dinitride MN₂H₂ (M = Mo, W) with High Stability

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Figure S1. The phonon spectra and projected phonon density of states (PhDOS) for (a) g-MoN₂H₂ and (b) g-WN₂H₂ monolayers.



Figure S2. The phonon spectra for (a) g-MoN₂ and (b) g-WN₂ monolayers.



Figure S3. The top views of monolayers after the *ab-initio* molecular dynamics simulation: (a) 500 K, (b) 1000 K and (c) 1500 K for g-MoN₂H₂ monolayer; (d) 500 K, (e) 1000 K and (f) 1500 K for g-WN₂H₂ monolayer.



Figure S4. Energy fluctuations with respect to time in *AIMD* simulations at 500 K, 1000 K and 1500K for the (a) g-MoN₂H₂ and (b) g-WN₂H₂ monolayers, respectively.



Figure S5. The zoom-in band structures for (a) g-MoN₂H₂ and (b) g-WN₂H₂ monolayers with SOC.



Figure S6. The band structures for (a) g-MoN₂H₂ and (b) g-WN₂H₂ monolayers based on PBE and HSE06.



Figure S7. The parities of fifty-four occupied bands at Γ (red color) and M (black color) points for g-MoN₂H₂ and g-WN₂H₂ sheets. The total number of +1 (-1) at Γ point is twenty-eight (twenty-six), while twenty-seven (twenty-seven) for M point. So it yields that the product of Γ and M points are + 1 and – 1 for g-MoN₂H₂ and g-WN₂H₂ sheets.



Figure S8. (a) Selected orbital-resolved band structures for g-WN₂H₂ monolayers under strain -8% and -6%. The red hexagonal dots represent the contributions from the W d_{z2} atomic orbitals; the blue hexagonal dots represent the contribution from W $d_{xy,x2-y2}$, atomic orbitals. (b) Schematic illustration of the band evolution from the atomic orbitals d_{z2} and $d_{xy,x2-y2}$, of the W atom around the Fermi level at the Γ point for g-WN₂H₂ monolayer. (c) Zoom-in orbital-resolved band structures for g-WN₂H₂ monolayers with and without SOC. Parity values are presented near the various orbitals.

Table S1. The contributions of the five *d* orbitals to the VBM and CBM based on PBE and HSE06 for g-MN₂H₂ sheets.

Comp.	$d_{\rm xy}$ (HSE06/PBE)	$d_{\rm yz}$	d_{z2} (HSE06/PBE)	$d_{\rm xz}$	d_{x2-y2} (HSE06/PBE)	Total (HSE06/PBE)	
g-MoN ₂ H ₂	0.055/0.062	0	0.645/0.618	0	0.056/0.063	0.855/0.854	CBM
	0.056/0.063	0	0.645/0.618	0	0.055/0.062	0.855/0.854	VBM
g-WN ₂ H ₂	0.055/0.058	0	0.613/0.612	0	0.055/0.060	0.835/0.829	CBM
	0.055/0.060	0	0.613/0.612	0	0.055/0.058	0.835/0.829	VBM