Electronic Supplementary Information for

Magnetic microkayaks: propulsion of microrods precessing near a surface by kilohertz frequency, rotating magnetic fields

L. O. Mair[a,†], E. E. Evans[b], A. Nacev[a], P. Y. Stepanov[a], R. Hilaman[a],
S. Chowdhury[a], S. Jafari[a], W. Wang[c,d], B Shapiro[c], I. N. Weinberg[a]

[a]Weinberg Medical Physics, Inc., North Bethesda, Maryland 20817 USA
[b]Department of Physics, Elon University, CB#2625, Elon, North Carolina 27244 USA
[c]School of Material Sciences and Engineering, Harbin Institute of Technology, Shenzhen Graduate School, Shenzhen, China 518055
[d]Center for Soft and Living Matter, Institute of Basic Sciences, Ulsan, Korea
[e]Fischell Department of Bioengineering, University of Maryland, College Park, Maryland USA
Corresponding Author email: Lamar.Mair@gmail.com

Materials and Methods:

Particle Synthesis: Multisegmented gold-nickel-gold rods were grown via the well documented technique of electrochemical deposition of metals into the pores of anodic aluminum oxide (AAO) membranes[1] following previously demonstrated methods[2,3]. Briefly, a silver working electrode was thermally evaporated (600 nm) onto one side of the membrane. Subsequent electroplating was performed using a two-electrode cell, with a platinum wire serving as the counterelectrode. Gold was deposited from a commercial plating solution (Technic Orotemp 24 RTU Rack) at a current density of 0.5 mA/cm^2 for tens of minutes. Nickel was deposited from a solution containing 1.1 M nickel sulfate, 0.2 M nickel chloride, and 0.75 M boric acid, at 0.6 mA/cm^2 for 4 minutes. Nickel segments are shorter than ~1/2 rod diameters, ensuring that the magnetic easy axis of the nickel layer is in the radial direction[2,3]. Following nickel plating, a second gold segment was grown. Microrods were released by etching the silver working electrode (in nitric acid) and AAO (in 1 M NaOH). Microrods were stored in DI water.

Rotating Magnetic Field: A pair of Helmholtz coils rotates the microkayaks. A Matlab generated signal was amplified (QSC RMX4050HD audio amplifier) and used to power the Helmholtz coils. In order to generate kayaking motion, one pair of coils was oriented in the y-direction, another pair oriented in the z-direction. Microkayak paddling motion remains in phase with the applied field up to 1 kHz.

Microkayaking Experiments: Microkayaking experiments are performed in DI water. Imaging is performed using transmitted light microscopy (40x air and 63x oil objectives). Videos of microrod rotation were acquired at 62 frames/s using a CCD camera (Allied Vision Technologies Prosilica GC, 480x752 pixels). In order to observe induced advective flows and microvortices, we seeded samples with 1 µm diameter polystyrene beads (Bangs Laboratories). Polystyrene particle tracking and microrod velocity measurements were performed using SpotTracker (freely available for download at http://cismm.cs.unc.edu/downloads/?dl_cat=3).

Hydrodynamic Model:

In modeling the rotational motion of an elongated rod near a surface, it is helpful to begin with two extreme geometries. The first is the case of a sphere near a rigid plane boundary. The sphere model has been solved previously, though even this simple geometry does not result in closed-form analytical solutions. Dean and O'Neill first published series expansions for components of the mobility matrix[4], and several authors have subsequently worked to improve accuracy or computational speed in the case of diminishing sphere-floor separation, *h*[5–7]. Using results by Chaoui and Feuillebois[7], we find that a 280 nm diameter sphere would need to be located within 50 nm of the surface to produce the translational velocities we observe. This distance is incompatible with our observations of 1 μm diameter tracer particles traversing the space below the rod, and with our previous measurements of microrod-floor separation distances[8].

Because our particle is an elongated rod with aspect ratio (AR) ≈ 20, an infinite cylinder may seem a better approximation. However, an infinite cylinder rotating on axis (θ = 0) near a floor experiences no net fluid force due to the rotation[9]. This counterintuitive result is due to a pressure gradient which is induced by the rotation and generates a Poiseuille flow in a direction opposite to the surface-driven Couette flow. Adding a second rigid boundary (a ceiling) to the infinite cylinder scenario does result in a translational velocity[10]. However, in our geometry, this model predicts velocities only ~1% of the translational velocity we observe experimentally. Thus, the infinite cylinder model rotating axially, parallel to the floor, does not accurately explain the observed velocities.

Several manuscripts describe the motion of prolate ellipsoids near a floor, and yet most of these feature end-over-end rotation without the generality required to include our system[11–13]. Those reports that do account for rotation about a long axis parallel to the floor often do not contain the cross terms $\boldsymbol{Z_C}$ and $\boldsymbol{Z_C^T}$ which are necessary to relate rotation and translation[14,15]. We are aware of only one manuscript which includes all the necessary components in the required geometry. Work by Corato et al. provides graphical solutions for all components of the mobility matrix for a prolate ellipsoid (AR = 8) rotating on- axis and parallel to a floor[16]. Using these solutions, we find that such an ellipsoid, rotating axially, would need to be within a couple tens of nanometres of the surface to produce the translational velocities that we see in our experiments. These distances are consistent with solutions for a rotating sphere discussed above, but they remain incompatible with our observations of 1 μm diameter tracer particles completing orbits about the long axes of our rods, as well as previous measurements on microrod height above the surface[8].

At separations (*d*) on the order of 1 μm, both the sphere and axially rotating ellipsoid models indicate that hydrodynamic interactions with the floor are far too weak to produce the velocities that we observe, suggesting that rotation about an easy axis parallel to the floor does not accurately represent microrod motion in our experiments. If the long axis of the cylinder is tilted even a small angle θ, such that the rod rotation is no longer parallel to the floor of the sample, then rotation about the y-axis will induce a precessing motion, and this motion will generate significantly larger translational velocities. In this case, work by Yang and Leal includes all components of the mobility matrix for a cylindrical slender body rotating near a floor[17]. While the slender-body approximation precludes translational velocity resulting from an on-axis rotation (θ = 0; $\vec{\omega} = \omega_y \hat{y}$) at the experimental separation distance, it is well-suited to describing the "kayaking" case in which θ is small and non-zero.

As described in the main text, the resistance matrix for motion is

$$\begin{bmatrix} \vec{F} \\ \vec{\tau} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z_a} & \boldsymbol{Z}_c \\ \boldsymbol{Z}_c^T & \boldsymbol{Z}_b \end{bmatrix} \begin{bmatrix} \vec{v} \\ \vec{\omega} \end{bmatrix} \qquad \text{(Eqn. 1a)}$$

Expanded, Eqn 1a is

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \left[\begin{array}{ccc|ccc} Z_a^{xx} & Z_a^{xy} & Z_a^{xz} & Z_c^{xx} & Z_c^{xy} & Z_c^{xz} \\ Z_a^{yx} & Z_a^{yy} & Z_a^{yz} & Z_c^{yx} & Z_c^{yy} & Z_c^{yz} \\ Z_a^{zx} & Z_a^{zy} & Z_a^{zz} & Z_c^{zx} & Z_c^{zy} & Z_c^{zz} \\ \hline Z_c^{xx} & Z_c^{yx} & Z_c^{zx} & Z_b^{xx} & Z_b^{xy} & Z_b^{xz} \\ Z_c^{xy} & Z_c^{yy} & Z_c^{zy} & Z_b^{yx} & Z_b^{yy} & Z_b^{yz} \\ Z_c^{xz} & Z_c^{yz} & Z_c^{zz} & Z_b^{zx} & Z_b^{zy} & Z_b^{zz} \end{array}\right] \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \text{(Eqn. 1b).}$$

In our experiments, we observe translation along the *x* axis (v_x) and rotation around the *y* axis (ω_y). We find the relationship between v_x and ω_y by solving the first row of Eqn 1b: We solve

$$v_x = -\left(\frac{Z_c^{xy}}{Z_a^{xx}}\right)\omega_y \qquad \text{(Eqn. 3),}$$

From Yang and Leal[17], the relevant coefficients of the resistance matrix are

$$Z_c^{xy} = -2\pi\eta \sin\theta\ \epsilon^2 \int_{-l}^{l} xB(x,l,\theta,d)\ dx + \mathcal{O}(\epsilon)$$

and

$$Z_a^{xx} = -8\pi\eta l\epsilon \left[1 - \epsilon\left(\ln 2 - \frac{1}{2} + \frac{1}{4l}\int_{-l}^{l} B(x,l,\theta,d)\ dx\right)\right] + \mathcal{O}(\varepsilon^3)$$

Here, η is the viscosity of the fluid, l is the half-length of the cylinder, and $\epsilon = 1/ln\left(\frac{2l}{R}\right)$ is the slenderness parameter, which depends on the cylinder length, $2l$, and radius, R. The function $B(x, l, \theta, d)$ is given as

$$B(x,l,\theta,d) =$$
$$sinh^{-1}\left|\frac{l - x\cos 2\theta - 2d\ sin\theta}{2cos\theta(d - x\ sin\theta)}\right| +$$
$$sinh^{-1}\left|\frac{l + x\ cos2\theta + 2d\ sin\theta}{2\ cos\theta(d - x\ sin\theta)}\right| -$$
$$2\ sin\theta\ (d - x\ sin\theta)\left[\frac{1}{\sqrt{(l - x\ cos\ 2\theta - 2d\sin\theta)^2 + \left(2\ cos\ \theta\ (d - x\ sin\ \theta)\right)^2}} - \frac{1}{\sqrt{(l + x\ cos\ 2\theta + 2d\sin\theta)^2 + \left(2\ cos\ \theta\ (d - x\ sin\ \theta)\right)^2}}\right] -$$
$$\frac{\cos 2\theta}{2\ cos^2\theta}\left[\frac{l - x\ cos\ 2\theta - 2d\sin\theta}{\sqrt{(l - x\ cos\ 2\theta - 2d\sin\theta)^2 + \left(2\ cos\ \theta\ (d - x\ sin\ \theta)\right)^2}} - \frac{l + x\ cos\ 2\theta + 2d\sin\theta}{\sqrt{(l + x\ cos\ 2\theta + 2d\sin\theta)^2 + \left(2\ cos\ \theta\ (d - x\ sin\ \theta)\right)^2}}\right]$$

These solutions are valid for $\epsilon \ll 1$ and for distances above the floor *d* larger than a few radii. In our experiments, $\epsilon \sim 0.03$ and we anticipate that $d \sim 1\ \mu$m, and so both conditions are satisfied.

Descriptions of Supplementary Information Video:

Supplementary Information video contains four separate video segments. Video segments 1 and 2 show microrods translating due to 300 Hz and 1000 Hz rotating magnetic fields, respectively. Video segments 3 and 4 show polystyrene microbeads advected in the rotational flow surrounding the microrods.

References

1. Hurst, S. J., Payne, E. K., Qin, L. & Mirkin, C. a. Multisegmented one-dimensional nanorods prepared by hard-template synthetic methods. *Angew. Chemie - Int. Ed.* **45,** 2672–2692 (2006).
2. Love, J. C., Urbach, A. R., Prentiss, M. G. & Whitesides, G. M. Three-Dimensional Self-Assembly of Metallic Rods with Submicron Diameters Using Magnetic Interactions. *J. Am. Chem. Soc.* **125,** 12696–12697 (2003).
3. Ahmed, S. *et al.* Steering Acoustically Propelled Nanowire Motors toward Cells in a Biologically Compatible Environment Using Magnetic Fields. *Langmuir* **29,** 16113–8 (2013).
4. Dean, W. R. & O'Neill, M. E. A slow motion of viscous liquid caused by the rotation of a solid sphere. *Mathematika* **10,** 13–24 (1963).
5. Goldman, A. J., Cox, R. G. & Brenner, H. Slow Viscous Motion of a Sphere Parallel to a Plane Wall I: Motion through a Quiscent Fluid. *Chem. Eng. Sci.* **22,** 637–651 (1967).
6. Lee, S. H. & Leal, L. G. Motion in the Presence of a plane interface 2: An exact solution in bipolar coordinates. *J. Fluid Mech.* **98,** (1980).
7. Chaoui, M. & Feuillebois, F. Creeping flow around a sphere in a shear flow close to a wall. *Q. J. Mech. Appl. Math.* **56,** 381–410 (2003).
8. Mair, L. O. *et al.* Highly controllable near-surface swimming of magnetic Janus nanorods: application to payload capture and manipulation. *J. Phys. D. Appl. Phys.* **44,** 125001 (2011).
9. Jeffrey, D. J. & Onishi, Y. The slow motion of a cylinder next to a plane wall. *Q. J. Mech. Appl. Math.* **34,** 129–137 (1981).
10. Yang, J., Huber, G. & Wolgemuth, C. W. Forces and Torques on Rotating Spirochete Flagella. *Phys. Rev. Lett.* **107,** (2011).
11. Hsu, R. & Ganatos, P. The motion of a rigid body in viscous-fluid bounded by a plane wall. *J. Fluid Mech.* **207,** 29–72 (1989).
12. Hsu, R. & Ganatos, P. Gravitational and zero-drag motion of a spheroid adjacent to an inclined plane at low-Reynolds Number. *J. Fluid Mech.* **268,** 267–292 (1994).
13. Gavze, E. & Shapiro, M. Particles in a shear flow near a solid wall: Effect of nonsphericity on forces and velocities. *Int. J. Multiph. Flow* **23,** (1997).
14. Padding, J. T. & Briels, W. J. Translational and rotational friction on a colloidal rod near a wall. *J. Chem. Phys.* **132,** (2010).
15. Mitchell, W. H. & Spagnolie, S. E. Sedimentation of spheroidal bodies near walls in viscous fluids: Glancing, reversing, tumbling, and sliding. *J. Fluid Mech.* **772,** 600–629 (2015).
16. De Corato, M., Greco, F., D'Avino, G. & Maffettone, P. L. Hydrodynamics and Brownian motions of a spheroid near a rigid wall. *J. Chem. Phys.* **142,** (2015).
17. Yang, Y. G. & Leal, L. G. Particle Motion in Stokes-Flow Near a Plane Fluid Fluid Interface. 1. Slender Body in a Quiescent Fluid. *J. Fluid Mech.* **136,** 393/421 (1983).