Supplementary Information

Self-enhancement of droplet jumping velocity: The interaction of liquid bridge and surface texture

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S.1 Formula derivation

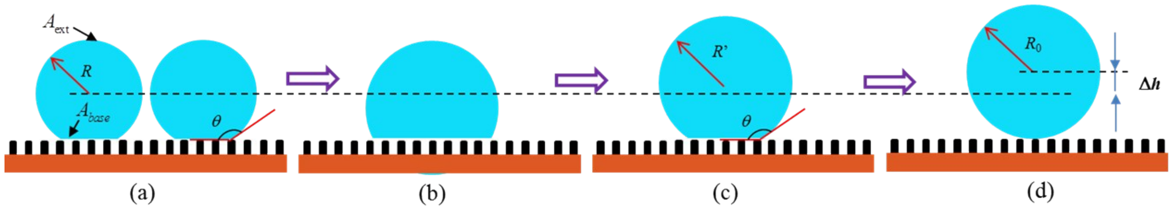

Fig. S1. Schematic of coalescence-induced droplet jumping on superhydrophobic surfaces.

(1) The released surface energy ΔE_s

The surface energy of a droplet which is in the Cassie state on superhydrophobic surfaces can be given by[1, 2]

$$E_s = \sigma_{lv} A_{lv} + \sigma_{sl} A_{sl} + \sigma_{sv} A_{sv}, \tag{S1}$$

where σ is the interfacial energy, A is the interfacial area and the subscripts s, l, and v denote the solid, liquid, and vapor, respectively. The solid-vapor contact area is given by

$$A_{sv} = A_{total} - A_{sl}, \tag{S2}$$

where A_{total} is the surface area of the superhydrophobic surface.

The solid-liquid interfacial energy can be derived from Young's equation[3]

$$\sigma_{sv} = \sigma_{sl} + \sigma_{lv} \cos\theta_Y, \tag{S3}$$

where θ_Y is Young contact angle. Substituting Eq. (S2) and Eq. (S3) into Eq. (S1), we obtain

$$E_s = \sigma_{lv} A_{lv} + \sigma_{sl} A_{sl} + \sigma_{sv}(A_{total} - A_{sl}) = \sigma_{lv}(A_{lv} - A_{sl}\cos\theta_Y) + \sigma_{sv} A_{total}. \tag{S4}$$

The total interfacial area A_{lv} and A_{sl} in Fig. S1a can be computed as follows:

$$A_{lv} = 2\times[A_{ext} + (1-\varphi)A_{base}] = 2\times[2\pi R^2(1-\cos\theta) + \pi R^2 \sin^2\theta(1-\varphi)], \tag{S5}$$

$$A_{sl} = 2\times\varphi\times A_{base} = 2\times(\pi R^2 \sin^2\theta\varphi), \tag{S6}$$

where A_{ext} is surface area of external droplet surface, A_{base} is surface area of droplet base area, R is the droplet radius before coalescence, θ is the apparent contact angle, φ is the solid-liquid fraction.

Substituting Eq. (S5)-(S6) into Eq. (S4), we obtain the surface energy of droplets before coalescence as shown in Fig. S1a:

$$E_a = 2\sigma_{lv}\pi[2(1-\cos\theta) + (1-\varphi-\varphi\cos\theta_Y)\sin^2\theta]R^2 + \sigma_{sv}A_{total} . \quad \text{(S7)}$$

In the same way, the surface energy of droplet after coalescence as shown in Fig. S1d is given by

$$E_d = 4\sigma_{lv}\pi R_0{}^2 + \sigma_{sv}A_{total} , \quad \text{(S8)}$$

where R_0 is the droplet radius after coalescence.

Based on the law of mass conservation, we obtain

$$R_0 = R(\frac{2-3\cos\theta+\cos^3\theta}{2})^{1/3} . \quad \text{(S9)}$$

The released surface energy ΔE_s before and after droplets coalesce on a flat SHS shown in Fig. S1 can be given by

$$\Delta E_s = E_a - E_d = 2\sigma_{lv}\pi[2(1-\cos\theta) + (1-\varphi-\varphi\cos\theta_Y)\sin^2\theta - 2\times(\frac{2-3\cos\theta+\cos^3\theta}{2})^{2/3}]R^2 . \quad \text{(S10)}$$

(2) The viscous dissipation E_{vis}

The viscous dissipation energy for each droplet can be estimated as[4, 5]

$$E_{vis}^{s} = \int_0^{T_c}\int_\Omega \Phi \mathrm{d}\Omega \mathrm{d}t \approx \Phi\Omega T_c , \quad \text{(S11)}$$

in which Φ is the dissipation function:

$$\Phi = \frac{\mu}{2}(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})^2 \approx 12\mu(\frac{U}{R})^2 , \quad \text{(S12)}$$

Ω is the volume of each droplet, μ is the viscosity of the liquid, and T_c is the coalescence time. The coalescence time is defined as time takes from the beginning of coalescence to the merging of droplets into a spherical segment as shown in Fig. S1c. In Wang *et al.*'s study, the coalescence time T_c is approximated to the characteristic capillary time scale τ ,

$$T_c \approx \tau = \sqrt{\frac{\rho R^3}{\sigma_{lv}}}. \tag{S13}$$

As the coalescence starts, the capillary pressure inside the droplet ($\Delta P = \frac{2\sigma_{lv}}{R}$) will accelerate the droplet along the horizontal direction. Thus, the average velocity U of each droplet can be obtained as

$$U \approx \tau \cdot \Delta P \cdot \pi R^2 \frac{1}{4\rho\pi R^3 / 3} = \frac{3}{2}\sqrt{\frac{\sigma_{lv}}{\rho R}}. \tag{S14}$$

Substituting Eq.(S12)-(S14) in to Eq.(S11), we obtain

$$E_{vis}^{s} \approx 36\pi\mu\sqrt{\frac{\sigma_{\mathrm{lv}} R^3}{\rho}}. \tag{S15}$$

So, the total viscous dissipation energy during droplet coalescence can be estimated as[5]

$$E_{vis} = 2E_{vis}^{s} \approx 72\pi\mu\sqrt{\frac{\sigma_{\mathrm{lv}} R^3}{\rho}}. \tag{S16}$$

(3) The work of adhesion E_{ad}

The work of adhesion E_{ad} can be estimated by the Young-Dupré Equation,[6]

$$E_{ad} = A_{sl}(1+\cos\theta_Y)\sigma_{lv}. \tag{S17}$$

The solid-liquid contact area of the merged droplet shown in Fig. S1c is defined as $A_{sl} = \varphi A_{base} = \sqrt[3]{4}\pi R^2 \sin^2\theta\varphi$. So, we can get work of adhesion E_{ad}

$$E_{ad} = \sqrt[3]{4}\pi(1+\cos\theta_Y)\sigma_{lv}\varphi\sin^2\theta R^2. \tag{S18}$$

(4) The increased gravity potential energy ΔE_h

The height of the center of gravity of the droplet in Fig. S1a can be given by

$$h_1 = \frac{R(3+\cos\theta)(1-\cos\theta)}{4(2+\cos\theta)}. \tag{S19}$$

The height of the center of gravity of the droplet in Fig. S1d can be given by

$$h_2 = R_0 = R(\frac{2-3\cos\theta+\cos^3\theta}{2})^{1/3}. \quad \text{(S20)}$$

ΔE_h is the gravity potential energy increase which is given by

$$\Delta E_h = mg(h_2 - h_1) = \frac{4}{3}\pi R^4 \rho g[(\frac{2-3\cos\theta+\cos^3\theta}{2})^{1/3} - \frac{(3+\cos\theta)(1-\cos\theta)}{4(2+\cos\theta)}], \quad \text{(S21)}$$

S.2 Supplementary Video

Description:

Video S1 (S1.avi, 245KB, for Fig. 3b). Side-view imaging of the coalescence-induced droplet jumping on flat SHSs with only nanostructures. The video was captured at 15000 fps and played back at 3000fps.

Video S2 (S2.avi, 208KB, for Fig. 5a). Side-view imaging of the coalescence-induced droplet jumping on SHSs with a rectangular groove configuration. The video was captured at 12000 fps and played back at 3000fps.

Video S3 (S3.avi, 265KB, for Fig. 5b). Side-view imaging of the coalescence-induced droplet jumping on SHSs with a triangular prism configuration. The video was captured at 15000 fps and played back at 3000fps.

References

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