Supporting Information

A Multiple Length Scale Description of the Mechanism of Elastomer Stretching

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S1. Details of Characterization



Figure S1. The applied crosslinkers for the silvlation based crosslinking reaction. Left: The three-functional crosslinker (Tris(dimethylsiloxy)phenylsilane). Right: The four-functional crosslinker (Tetrakis(dimethylsilyl) orthosilicate).



Figure S2. Mechanical properties. Reversing large amplitude oscillatory elongation on the strongest (Elastosil) and softest (V31) networks. The strain rate is $v = 0.02 \text{ s}^{-1}$.

Table S1: Properties of the applied polymers. The molecular weight M_n was determined by Size Exclusion Chromatography. The polydispersity index, PDI = M_W/M_n , was determined by ¹H NMR measurements. The methods are described in (13).

Polymer	M _n [g/mol]	PDI
V31	29,000	1.3
V35	49,500	1.5
V41	62,400	1.6

Table S2: Average bond lengths (r) and associated widths (σ) in the distance distributions as well as derived average bond angles for sample V31 in the loaded and un-loaded state. The parameters result from a fit of the isotropic experimental data in Fig 2a to the scattering arising from a model of the structural unit. ¹: Fit of σ is restricted to values larger than 0.08 Å, $|\Delta\sigma|$ to values greater than 10% of σ .

			Relative
		Δr,Δσ	change
	r ₀ , σ (λ=1)	(λ=2 - λ=1)	Δr/r ₀ , Δσ/σ
r _{siO} / Å	1.642	-6.0 10-4	-3.7 10 ⁻⁴
σ _{siO/} Å	0.111	4.5 10 ⁻³	4.0 10-2
r _{sic} / Å	1.869	-2.8 10 ⁻⁴	-1.5 10 ⁻⁴
σ _{siC/} Å	0.143	4.4 10 ⁻³	3.1 10-2
r _{sisi} / Å	3.160	-4.210 ⁻⁴	-1.3 10 ⁻⁴
σ _{siSi/} Å	0.139	3.5 10 ⁻³	2.5 10 ⁻²
r _{oo} /Å	2.736	1.2 10 ⁻³	4.6 10-4
σ _{00/} Å	¹ 0.080	-5.4 10 ⁻³	-6.8 10 ⁻²
r _{cc} / Å	3.055	-10.4 10 ⁻³	-3.4 10 ⁻³
σ _{cc/} Å	¹ 0.080	7.8 10 ⁻³	9.7 10 ⁻²
r _{oc} / Å	2.858	1.010-4	3.6 10 ⁻⁵
σ _{oc/} Å	0.200	1.210-2	6.0 10 ⁻²
<osio deg<="" td=""><td>112.8</td><td>7.9 10⁻²</td><td>7.0 10-4</td></osio>	112.8	7.9 10 ⁻²	7.0 10-4
<csic deg<="" td=""><td>109.6</td><td>-5.5 10⁻¹</td><td>-5.0 10⁻³</td></csic>	109.6	-5.5 10 ⁻¹	-5.0 10 ⁻³
<siosi deg<="" td=""><td>148.4</td><td>-5.3 10⁻²</td><td>-3.6 10⁻⁴</td></siosi>	148.4	-5.3 10 ⁻²	-3.6 10 ⁻⁴

Table S3: The results for the preferred orientation parameter is obtained by scaling the model described in the main text to the amplitude of g_2 in Fig S4b, and reflects the increased likelihood of a SiO bond to point in the tensile direction compared to an isotropic distribution at $\lambda=2$. ¹The values for V41 and Elastosil are extrapolated to $\lambda=2$.

Polymer	V31	V35	V41	Elastosil
Preferred orientation parameter / %	1.9	0.2	0.61	1.41

S2. Details of Entropic and Enthalpic Contributions

The entropy of stretching of polymer chains is generally written in the simplified form under the assumption that the polymer chains behave according to Gaussian random walks¹:

$$\Delta S = -\frac{3}{2}k\frac{R^2}{R_o^2} = -\frac{3}{2}k\frac{R^2}{Nb^2}$$
(S1)

Where k is the Boltzmann's constant, R and R_o are the perturbed and unperturbed end-to-end distances of the polymer, respectively, and N is the number of Kuhn steps of length b.

If the chains are confined by entanglements moving affinely with the deformation, the change in entropy upon stretching of a polymer chain with N/N_e entangled chains can be expressed as:

$$\Delta S' = -\frac{3}{2} k \frac{N}{N_e} \frac{R_n^2}{N_e b^2} \tag{S2}$$

Where N_e is the number of Kuhn steps in an entanglement and R_n is the perturbed end-to-end length of the chain segment with the characteristic entanglement length.

If we assume simplistically that we allow one entanglement to slide upon the deformation, the two surrounding polymer segments, each of length $N_e b$, are replaced with two polymer segments of approximate lengths 0 and $2N_e b$. The entropy of stretching of such a configuration can be written:

$$\Delta S'' = -\frac{3}{2}k \left(\left(\frac{N}{N_e} - 2 \right) \frac{R_n^2}{N_e b^2} + \frac{2R_n^2}{2N_e b^2} \right) = -\frac{3}{2}k \left(\frac{N}{N_e} - 1 \right) \frac{R_n^2}{N_e b^2}$$
(S3)

Since entropy is a state function, the gained entropy from entanglement sliding in the stretched state can be calculated as:

$$\Delta S_{slide} = \Delta S'' - \Delta S' = \frac{3}{2}k \frac{R_n^2}{N_e b^2}$$
(S4)

In other words, when the entanglement is sliding, the decrease in entropy upon stretch is minimized and thus entropy is maximized and this can be facilitated by stretching the chains.

The magnitude of enthalpy is evaluated by the expression from Khazanovich²:

$$\frac{F_e}{F} = -\frac{\varepsilon \partial g}{f \partial \varepsilon} / \frac{\partial f}{\partial g}$$
(S5)

Where *F* is the total stretching force, F_e is the enthalpic contribution to the stretching force, ε is the reduced stiffness, *f* is dimensionless force, and *g* is the degree of chain stretching given by:

$$\varepsilon = \frac{b^2 K_s}{kT} \tag{S6}$$

$$f = \frac{bF}{kT}$$
(S7)

$$g = \frac{\langle h_f \rangle}{Nb} \tag{S8}$$

Where K_s is the stiffness coefficient of the Kuhn segment and h_f is the projection of the vector connecting the ends of the chains onto the stretching axis.

The solution to the above equation in terms of the degree of chain stretching can be written under the assumption that $\varepsilon >>1$:

$$g = \frac{f}{\epsilon} - \frac{1}{f} + \frac{(\epsilon + 1) + \coth(f)}{\epsilon + f \coth(f)}$$
(S9)

The assumption is not strictly correct as the Si-O bonds are rather flexible but the assumption does not change the order of magnitude of the solution.

If we assume that the allowed stretching force is of the order of several kT according to the results for the entropy gain from sliding, the coth(f) reduces to 1 and the expression can be simplified to:

$$g = \frac{f}{\epsilon} - \frac{1}{f} + \frac{(\epsilon+1)+f}{\epsilon+f} = 1 + \frac{f}{\epsilon} - \frac{1}{f} + \frac{1}{\epsilon+f}$$
(S10)

Further assuming that the reduced stretching force is greater than the reduced stiffness, i.e. $f \gg \epsilon$, following simplified equation is obtained:

$$g = 1 + \frac{f}{\epsilon} = 1 + \frac{\frac{bF}{kT}}{\frac{b^2K_s}{kT}} = 1 + \frac{F}{bK_s} \sim 1 + \frac{T\Delta S_{slide}/b}{bK_s} \sim 1 + \frac{T\Delta S_{slide}}{b^2K_s}$$
(S11)

Hereby it is shown by a very simplistic approach that the degree of chain stretching grows linearly with the entropy gain from allowing sliding of entanglements.

List of references:

- S1. Flory, P.J., *Principles of Polymer Chemistry*, Ithaca: Cornell Univ. Press, 1953.S2. Khazanovich, T.N., Doklady Physical Chemistry 2009, 425, 66–68.