Electronic Supplementary Material (ESI) for RSC Advances.

Accurate Measurement of Specific Tensile Strength of Carbon Nanotube Fibers with Hierarchical Structures by Vibroscopic Method

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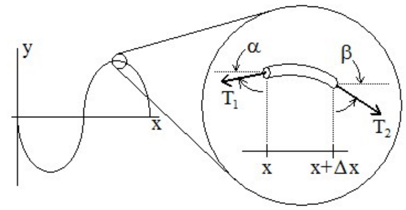
Electronic supplementary information contents

- Mathematical derivation of Merssene's law from vibrating string theory
- Table S1 Detailed synthesis conditions of CNT fibers by direct spinning method.
- Fig. S1 The TEM images and I_G/I_D ratios of synthesized CNTs.
- Fig. S2 The plots showing variations in the linear densities of the sample #1 to #4 measured by the microbalance.
- Fig. S3 Representative scanning electron microscopy (SEM) images of (a-b) surface and (c-d) cross-section of CNT fiber.
- Fig. S4 Measured intensity-frequency plots of the sample #1 to #4 at 0.5 % strain.
- Fig. S5 Measured intensity-frequency plots of the samples #1 to #4 at 0.05 % strain.
- Fig. S6 SEM image and linear density-strain graph of graphene oxide (GO) fiber.
- Fig. S7 SEM images of CNT fiber at various magnifications.
- Fig. S8 New proposed procedure for accurately measuring the linear density of fibers with hierarchical structures by vibroscopic method.
- Movie S1 Video of intensity-frequency graphs of CNT fiber with increasing tension measured by vibroscope

Mathematical derivation of Merssene's law from vibrating string theory

Equation 1. Force in x-axis



$$T_{1x} = T_1\cos(\alpha) \approx T \approx T_2\cos(\beta) = T_{2x}\ (where\ \alpha, \beta \approx 0)$$

Equation 2. Force in y-axis

$$\sum F_y = -T_{2y} - T_{1y} = -T_2\sin(\beta) - T_1\sin(\alpha)$$

Equation 3. Newton's second law for y-axis

$$\sum F_y = \Delta ma = \Delta\frac{mx}{x}\Delta a \approx \mu\Delta x\frac{\partial^2 y}{\partial t^2}(where\ \mu = linear\ density\ of\ fiber)$$

Equation 4. Dividing the equation 3 by T and applying the equation 1 and 2

$$\frac{\mu\Delta x\partial^2 y}{T\ \ \partial t^2} = -\frac{T_2\sin(\beta)}{T_2\cos(\beta)} - \frac{T_1\sin(\alpha)}{T_1\cos(\alpha)} = -\tan(\beta) - \tan(\alpha) = \frac{\partial y}{\partial x}\bigg|^{x+\Delta x} - \frac{\partial y}{\partial x}\bigg|^{x}$$

Equation 5. Dividing the equation 4 by Δx

$$\frac{\mu\partial^2 y}{T\partial t^2} = \frac{1}{\Delta x}\left(\frac{\partial y}{\partial x}\bigg|^{x+\Delta x} - \frac{\partial y}{\partial x}\bigg|^{x}\right) = \frac{\partial^2 y}{\partial x^2}(where\ \Delta x \approx 0)$$

From equation 5, function of x and t should be wave function.

Equation 6. The coefficient of the second time derivative term is equal to v^{-2} and the speed of propagation can be written as below equation.

$$\frac{\mu}{T} = \left(\frac{\partial^2 t}{\partial x^2}\right) = v^{-2} = \left(\frac{1}{\lambda f}\right)^2 = \left(\frac{1}{2Lf}\right)^2$$

$(where\ \lambda = wavelength, f = frequency\ and\ L = length\ of\ string)$

Equation 7. Linear density calculation

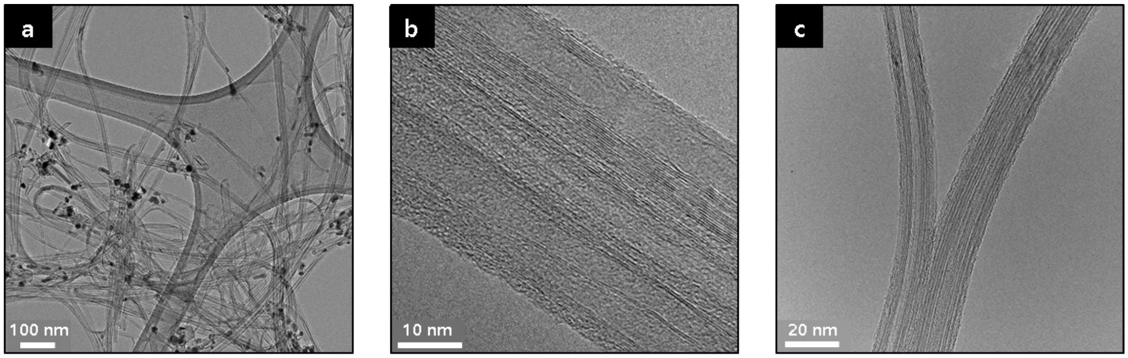
$$\mu = \frac{T}{4L^2f^2}$$

Reference 1. https://en.wikipedia.org/wiki/String_vibration

Fixed conditions	Ferrocene	Hydrogen	Reaction temperature	Spinning rate
	8.385×10^{-6} mol/min.	8.917×10^{-2} mol/min.	1170 ℃	5 m/min.

Experiment	Sample #1	Sample #2	Sample #3	Sample #4
Acetone	4.274×10^{-3} mol/min.	2.849×10^{-3} mol/min.	4.289×10^{-3} mol/min.	2.859×10^{-3} mol/min.
Thiophene	2.133×10^{-5} mol/min.	1.422×10^{-5} mol/min.	1.071×10^{-5} mol/min.	7.137×10^{-6} mol/min.

Table S1 Detailed synthesis conditions of CNT fibers by direct spinning method.



d

Sample	#1	#2	#3	#4
I_G/I_D ratio [a.u.]	2.75 ± 0.74	3.03 ± 0.58	2.66 ± 0.44	2.26 ± 0.41

Fig. S1 (a-c) The representative TEM images of the CNT fibers synthesized by the direct spinning method. The diameters of CNTs range 10 ~ 20 nm and CNTs are multi-walled for all CNT fiber samples. (d) The table of I_G/I_D ratio and standard deviations of each sample.

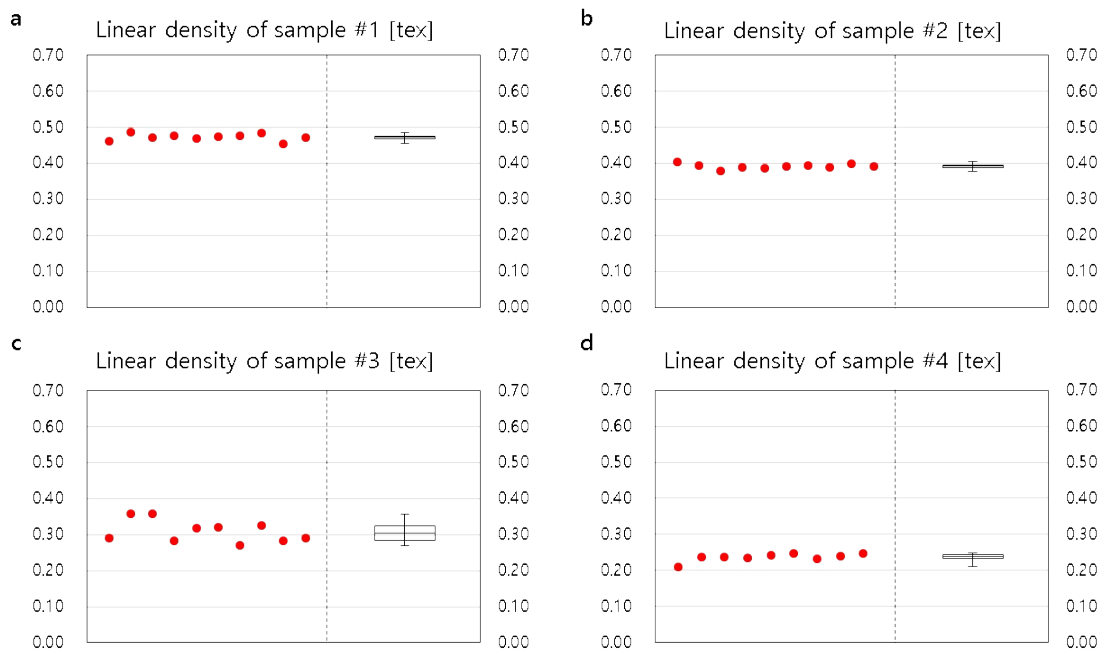


Fig. S2 (a-d) The plots showing variations in the linear densities of the sample #1 to #4 measured by the microbalance. For each samples, ten of CNT fibers having 50 cm length were measured by microbalance in order to confirm the uniformity of the linear density.

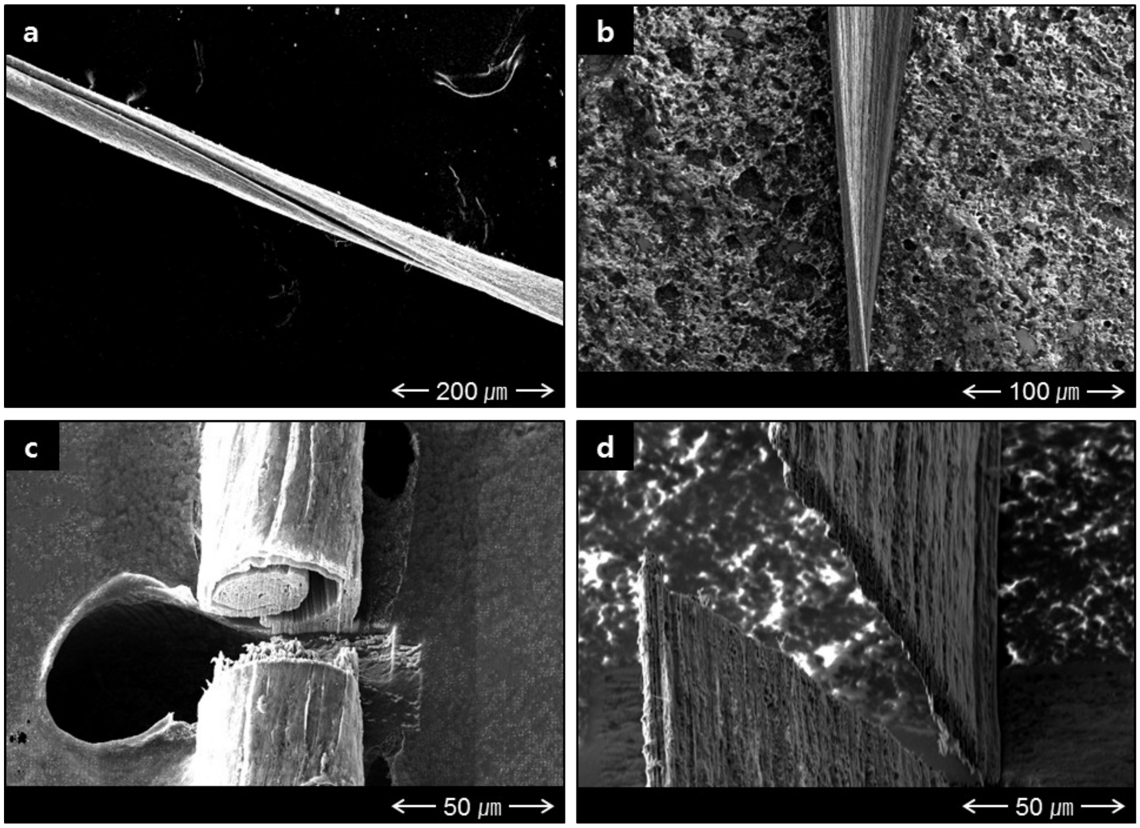


Fig. S3 Representative scanning electron microscopy (SEM, FEI Helios NanoLab650, operated at 10 kV) images of (a-b) surface and (c-d) cross-section of CNT fiber. The cross-section SEM images of CNT fibers were acquired after cutting and polishing the fibers by focused ion beam (FIB) milling (operated at 30 kV Ga^{+} ion beam) using the same SEM equipment.

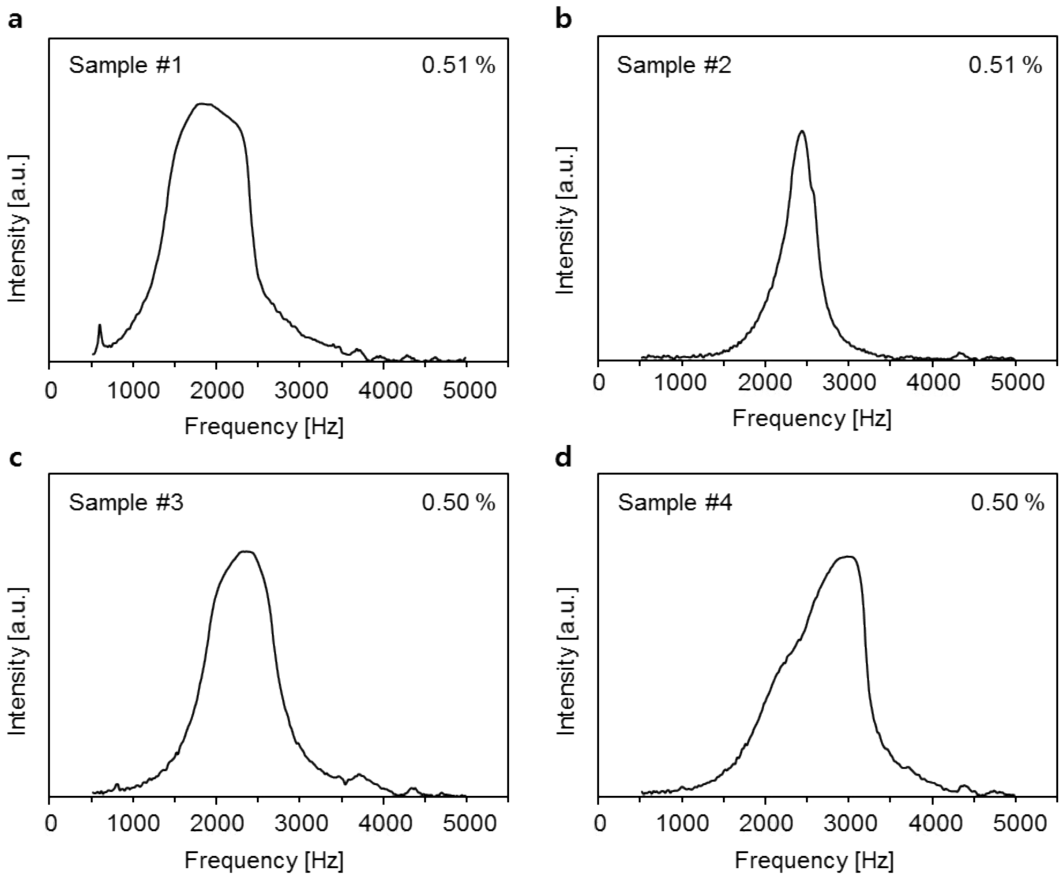


Fig. S4 (a-d) Measured intensity-frequency plots of the sample #1 to #4 at 0.5 % strain. Each graph shows a single characteristic peak, meaning that CNT fibers behave elastically and resonance frequency can be determined accurately.

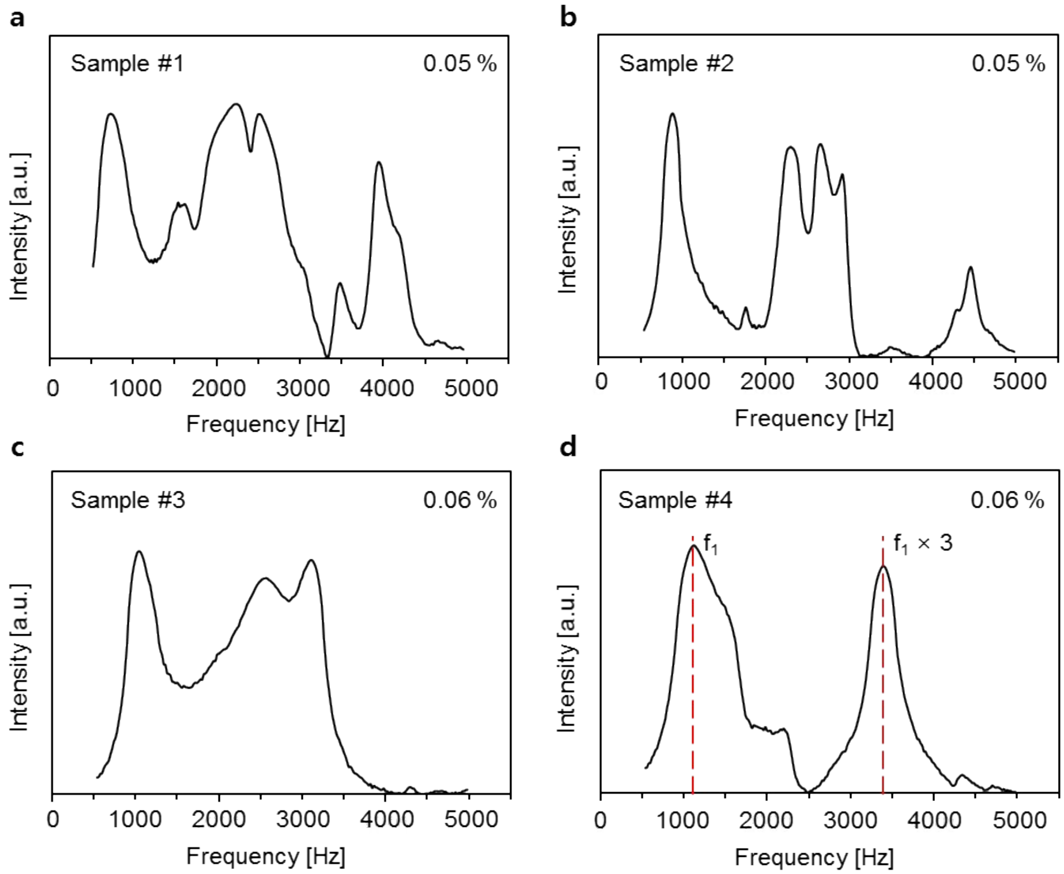


Fig. S5 (a-d) Measured intensity-frequency plots of the samples #1 to #4 at 0.05 % strain. All graphs except for the sample #4 show no distinct peaks with harmonics, meaning that CNT fibers cannot behave elastically and resonance frequency cannot be determined accurately.

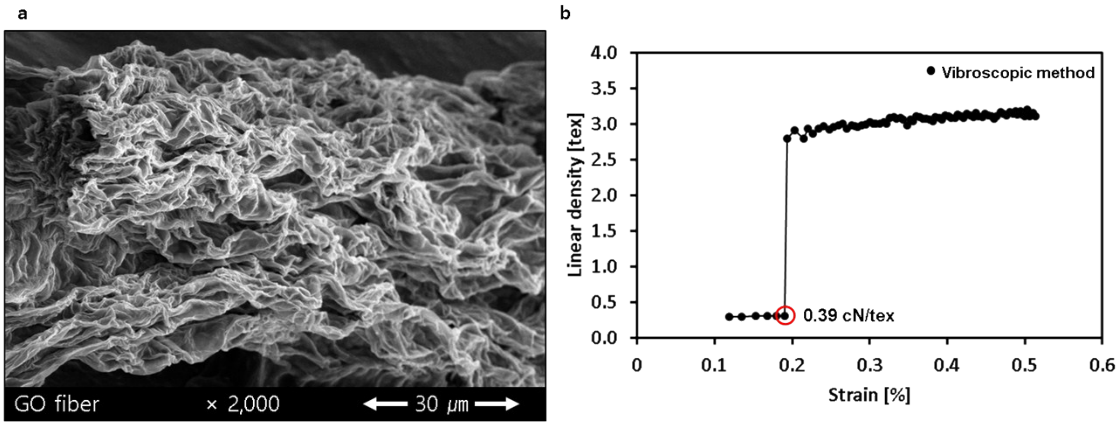


Fig. S6 (a) SEM image and (b) linear density versus strain graph of graphene oxide (GO) fiber spun by the solution spinning method from the liquid crystalline phase.

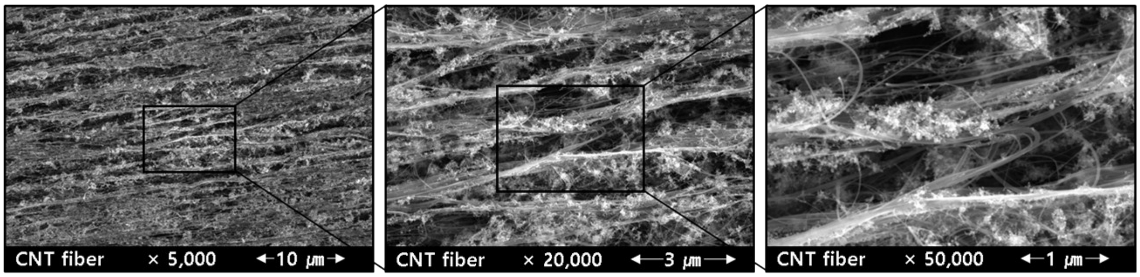


Fig. S7 SEM images of CNT fiber with the hierarchical structure at various magnifications.

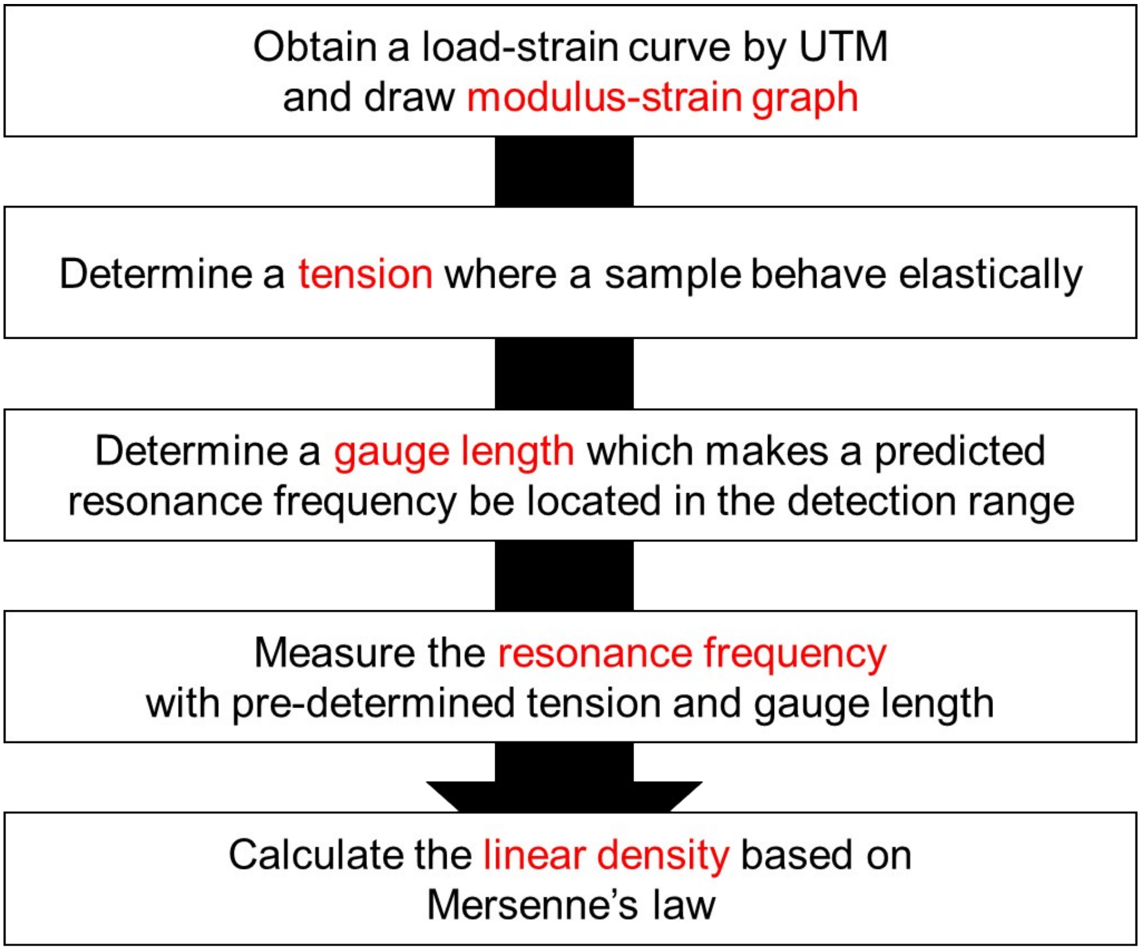


Fig. S8 New proposed procedure for accurately measuring the linear density of fibers with hierarchical structures by vibroscopic method.