

Figure S1. Optimized transition state structures of interconversion reactions of keto or enol forms themselves obtained by B3LYP/6-311++G(d,p) calculations.



Figure S2. Optimized atomic distances [Å] of enol form structures II-VII.



Figure S3. Optimized atomic distances [Å] of keto form structures A-C.

Keto-enol tautomerism reactions



Figure S4. Optimized transition state structures of keto-enol tautomerism reactions obtained by B3LYP/6-311++G(d,p) calculations.

Isomerization reactions



Figure S5. Optimized structures of reactant, transition state, and product systems of mono hydrated isomerization reactions obtained by B3LYP/6-311++G(d,p) calculations.

Keto-enol tautomerism reactions



Figure S6. Optimized structures of reactant, transition state, and product systems of mono hydrated keto-enol tautomerism reactions obtained by B3LYP/6-311++G(d,p) calculations.

Table S1. Optimized exponent α (α_{opt}) values of quantum protons and deuterons in reactant molecule of isomerization and keto-enol tautomerism reactions catalyzed by a single water molecule optimized by MC_B3LYP/6-311++G(d,p) method.

| | α_{opt} values for H species | | | | | | | α_{opt} values for D species | | | | | | | |
|------|-------------------------------------|------|------|------|------|------|------------|-------------------------------------|------|------|------|------|------|--|--|
| | H1 | H2 | Н3 | H4 | Н5 | H6 | . <u>-</u> | D1 | D2 | D3 | D4 | D5 | D6 | | |
| I-II | 24.1 | 24.1 | 23.1 | 22.7 | 23.5 | 21.7 | | 35.8 | 35.7 | 34.3 | 33.8 | 34.9 | 32.5 | | |
| IV-V | 24.1 | 24.2 | 23.6 | 22.5 | 23.5 | 21.8 | | 35.7 | 35.8 | 35.0 | 33.7 | 34.9 | 32.7 | | |
| B-C | 24.0 | 24.0 | 24.2 | 21.7 | 23.5 | 22.7 | | 35.6 | 35.6 | 35.9 | 32.5 | 34.9 | 33.9 | | |
| A-VI | 23.9 | 23.9 | 24.0 | 23.7 | 23.1 | 22.9 | | 35.4 | 35.4 | 35.5 | 35.2 | 34.4 | 34.0 | | |
| B-V | 23.9 | 24.0 | 23.9 | 23.8 | 23.1 | 23.2 | | 35.5 | 35.6 | 35.5 | 35.2 | 34.3 | 34.5 | | |
| C-IV | 23.9 | 24.0 | 24.0 | 23.8 | 23.1 | 23.2 | | 35.5 | 35.5 | 35.6 | 35.2 | 34.3 | 34.4 | | |
| | | | | | | | | | | | | | | | |

Table S2. Optimized exponent α (α_{opt}) values of quantum protons and deuterons in product molecule of isomerization and keto-enol tautomerism reactions catalyzed by a single water molecule optimized by MC_B3LYP/6-311++G(d,p) method.

| | α_{opt} values for H species | | | | | | | α_{opt} values for D species | | | | | | | |
|------|-------------------------------------|------|------|------|------|------|--|-------------------------------------|------|------|------|------|------|--|--|
| | H1 | H2 | Н3 | H4 | Н5 | H6 | | D1 | D2 | D3 | D4 | D5 | D6 | | |
| I-II | 24.1 | 24.1 | 23.2 | 21.5 | 23.5 | 22.6 | | 35.7 | 35.8 | 34.5 | 32.3 | 34.9 | 33.7 | | |
| IV-V | 24.1 | 24.2 | 23.6 | 21.8 | 23.5 | 22.6 | | 35.6 | 35.8 | 35.0 | 32.7 | 34.9 | 33.7 | | |
| B-C | 24.0 | 24.0 | 24.2 | 22.7 | 23.5 | 21.7 | | 35.6 | 35.6 | 35.9 | 33.8 | 34.9 | 32.5 | | |
| A-VI | 24.1 | 24.2 | 23.4 | 23.4 | 22.3 | 23.0 | | 35.6 | 35.8 | 34.8 | 34.8 | 33.3 | 34.3 | | |
| B-V | 24.1 | 24.2 | 23.4 | 23.4 | 22.6 | 23.3 | | 35.7 | 35.8 | 34.8 | 34.8 | 33.7 | 34.6 | | |
| C-IV | 24.2 | 24.2 | 23.4 | 23.4 | 22.7 | 23.3 | | 35.8 | 35.8 | 34.8 | 34.8 | 33.7 | 34.6 | | |