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Life and death of not so "bare" bubbles

Supplementary Information

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In this supplementary information, we compare the drainage data presented in the article to two different functional forms found in the literature for the film thickness h as a function of time t. The first one, proposed by Lhuissier & Villermaux[3], accounts for drainage driven by marginal regeneration. The second one was established by Hermans *et al.*[2] for the gravitational drainage of the hemispherical film coated onto a solid lens, in the presence of Marangoni and surface viscous stresses at the liquid/air interface.

1 Comparison of drainage data to the model by Lhuissier & Villermaux[3]

Studying the drainage and subsequent bursting of tap water bubbles, Lhuissier & Villermaux[3] observed convective motion at the bubble foot, due to the rising of thinner film portions created by marginal regeneration at the junction with the meniscus. They proposed a model to describe the overall bubble cap thinning due to this phenomenon, where the decrease in film thickness stems from (i) capillary succion through the pinching zone at the bottom of the bubble and (ii) replacement of thick film portions by thinner ones that rise because of boyancy. Assuming that the thickness of the pinching zone remains of the order of the overall bubble cap thickness h, they obtained the following scaling law

$$h(t) \sim \ell_c \left(\frac{\eta \ell_c}{\gamma t}\right)^{2/3} \left(\frac{R}{\ell_c}\right)^{7/3}, \tag{1}$$

which was found in reasonable agreement with experimental data on the drainage of tap water bubble of radius $R < 5\ell_c$.

To look into whether marginal regeneration contributes significantly to the drainage of our surfactant-stabilized bubbles, we plot in Figure 1 the time variation of the film thickness at the bubble apex in log-log scale. Figure 1 demonstrates that our experimental data – which correspond to $R \approx 4.5\ \ell_c$ – do not follow the $-2/3$ power law predicted by equation (1). This seems to indicate that marginal regeneration is not the prevailing thinning mechanism for concentrated bubbles, *i.e.* at least for concentrations $c > 0.8$ cmc.

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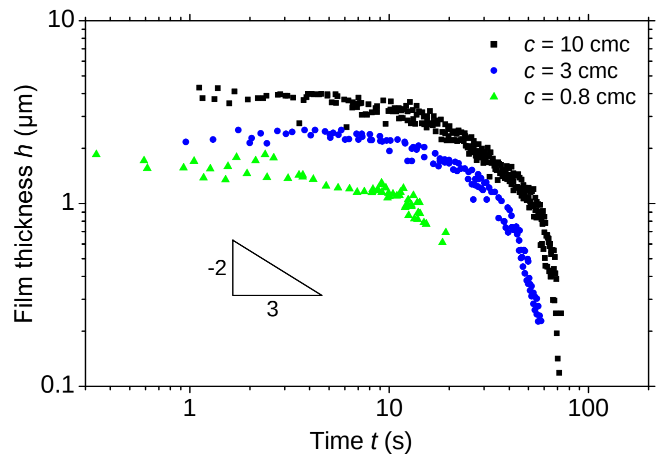

Fig. 1 Time evolution of the film thickness h at the bubble apex for various TTAB concentrations (colors). For each concentration, we superimpose the data corresponding to three different bubbles (symbols) and compare the drainage curve to the $-2/3$ power law predicted by Lhuissier & Villermaux (Equation (1)). Note that the potential error on the initial time $t = 0$ is of the order of 1 s, which is small compared to the overall drainage timescale.

2 Comparison of drainage data to the model by Hermans *et al.*[2]

Hermans *et al.*[2] studied the drainage of surfactant-stabilized thin films coated onto a solid lens. The geometry of their system ressembles the one of a hemispherical bubble, except that a liquid/air interface is replaced by a solid/liquid interface in their case. Relying on the previous work by Bhamla *et al.*[1], they show that, whatever the nature of the boundary condition at the liquid/air interface, the time variation of the film thickness h always has the form

$$h(t) = \frac{h_0}{\sqrt{1 + 4\alpha\, t/T_r}} \qquad \text{with} \qquad T_r = \frac{\eta R}{\rho g h_0^2}, \tag{2}$$

where α depends on the boundary condition at the liquid/air interface. Hermans *et al.* are able to relate α to the surface rheological properties of the surfactant solution – *e.g.* shear and dilational surface viscosities – and to Marangoni stresses. Note that for $\alpha = 1/12$, corresponding to a rigid liquid/air interface, we recover Equation (1) of our paper, which describes the drainage of a bubble with two rigid liquid/air interfaces.

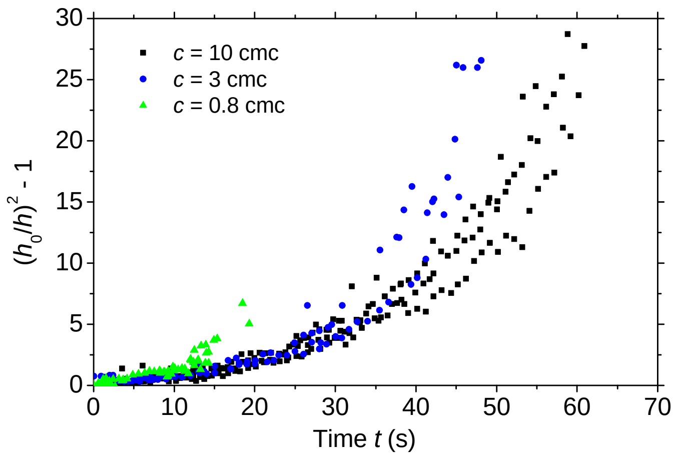

Fig. 2 Time evolution of the film thickness h at the bubble apex for various TTAB concentrations (colors). For each concentration, we superimpose the data corresponding to three different bubbles (symbols). The data are plotted under the form $(h_0/h)^2 - 1$, where h_0 is the initial film thickness. In these coordinates, the prediction of Hermans *et al.* (Equation (2)) would yield a straight line.

Figure 2 shows our drainage data (the same as in Figure 1) in a way such that Equation (2) would be represented by a straight line. The variation of the data in Figure 2 is clearly non linear, so they cannot be described by a function of the form of Equation (2), whatever the value of α. This is due to the fact that the functional form of Equation (2) essentially stems from the no-slip boundary condition at the solid/liquid interface of the coated film in Hermans *et al*. In the case of a hemispherical bubble, however, both interfaces have the same boundary condition, which is not rigid in the general case. Thus, there is an extensional contribution to the flow in the bubble cap, leading to a different functional form $h(t)$.

References

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