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Supplementary Document

Disclinations, e-cones, and their interactions in extensible sheets

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1. Mechanical Characterization

The Young's modulus for the Mylar (biaxially-oriented polyethylene terephthalate) and cellulose acetate ribbons used in the experiments were obtained using a commercial Instron Universal Testing Instrument. In this test, the stress-strain relation is measured while the ribbon is clamped at its ends and stretched longitudinally with a prescribed velocity using a servo motor, while the tensile force is recorded using strain gauges.

To obtain the mechanical response of Mylar, we performed tensile tests using ribbons with width W = 7.0 ± 0.3mm, length L_0 = 50.0 ± 0.3mm, and thickness t = 254±3μm. The tension was increased with a speed 15μm/s. For cellulose acetate, we performed tensile tests using ribbons with width W = 20.0 ± 0.3mm, length L_0 = 70.0 ± 0.3mm, and thickness t = 254±3μm. The tension was increased with a speed 1mm/s in this case.

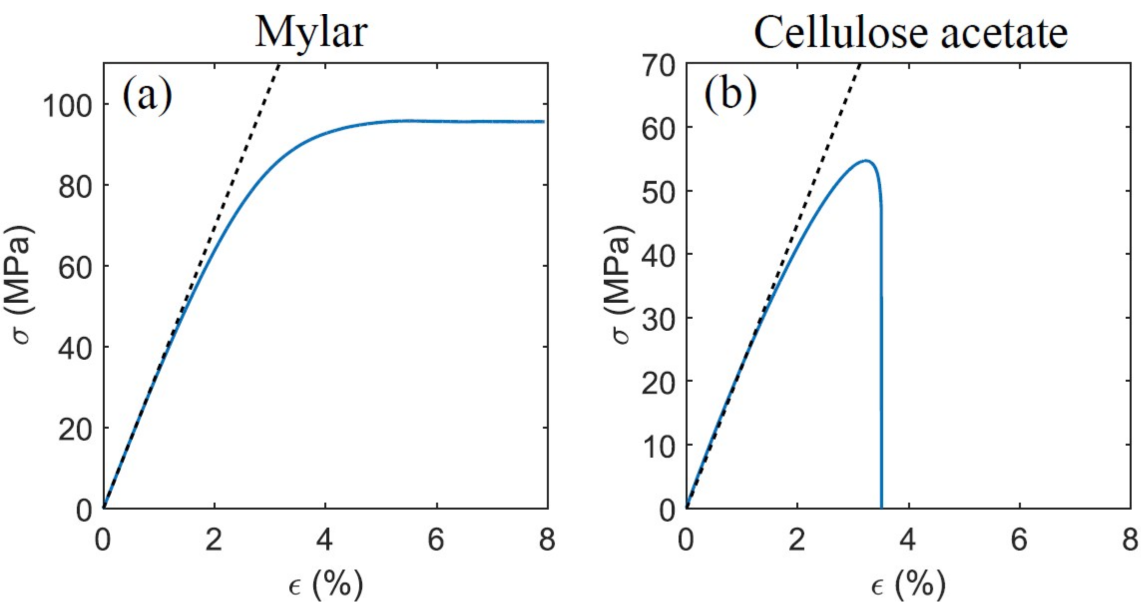

Figure S1: Stress-strain curves for (a) Mylar and (b) Cellulose acetate. (a) Loading curves of ribbon composed of Mylar showing a linear response for strains smaller than 2% and characterized by a Young's modulus E = 3.4GPa obtained by a linear fit (black dashed line). At larger strain, nonlinear response and plastic behavior are observed. (b) Loading curves of ribbon made of cellulose acetate showing a linear response for strain smaller than 2% and characterized by a Young's modulus E = 2.2GPa obtained by a linear fit (black dashed line). At larger strain, nonlinear response is observed rapidly followed by a complete rupture of the sample.

In Figure S1, we plot the stress-strain response of the ribbon of both materials. Here the stress is calculated as the tensile force F divided by the cross section area Wt and the strain is calculated using

$(L-L_0)/L_0$ where L_0 and L are the initial and stretched length of the ribbon, respectively. A linear behavior is observed for a strain greater than 2%. In this regime, a linear fit yields the Young's modulus E = 3.4GPa for Mylar, and E = 2.2GPa for cellulose acetate.

For strains greater than 2%, Mylar has a nonlinear response followed by a plateau at σ_{max} = 96MPa where plastic deformation occurs. This plateau is absent for cellulose acetate. Instead, a complete rupture of the sample is observed shortly after reaching a maximum stress σ_{max} = 55MPa corresponding to a strain ε = 3.2%.

2. X-ray Instrument and Measurement of Curvatures

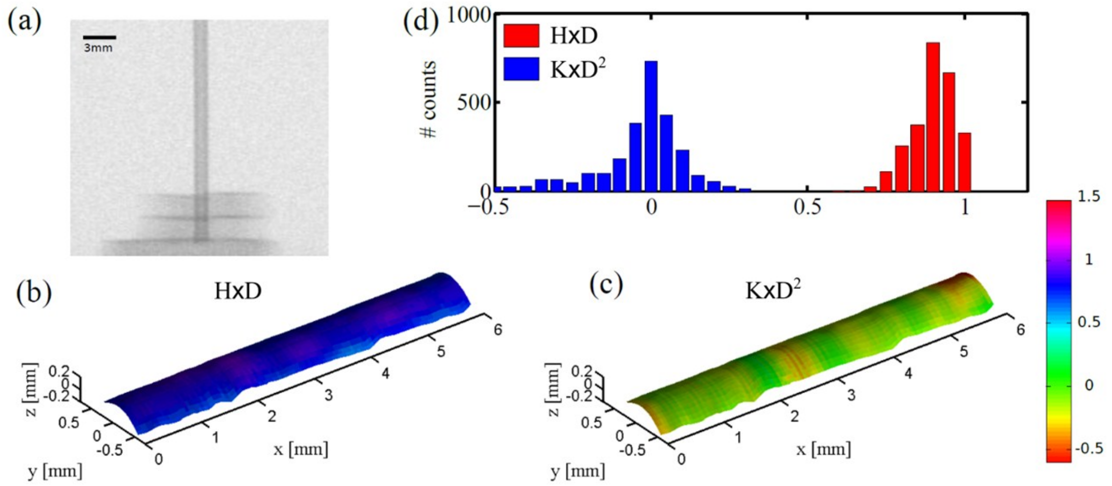

Figure S2: (a) A radiogram of a plastic tube used to demonstrate the accuracy of the curvature measurements. (b,c) Three dimensional rendering of the tube with the normalized map of the mean curvature *HxD* (b) and Gaussian curvature *KxD²* (c) where D = 1.27mm is the tube mean diameter. Histogram of the *HxD* (red) and *KxD²* (blue).

The x-ray scans were performed using a Varian Medical Systems BIR 200/130 micro x-ray Computed Tomography (CT) instrument. The instruments consist of a Kevex 130 kV micro-focus x-ray source and a Varian PaxScan 2520V amorphous silicon flat panel detector with a 1516 × 1900 pixels resolution. The sample is placed in between the source and the detector, and rotated through 360 degrees while radiograms of the attenuated rays moving through the sample are taken by the instrument. A set of integrated software provided by the manufacturer performs the three dimensional reconstruction of the sample from the radiograms. The resulting volume data consisting of a 16-bit resolution density map is then analyzed using further algorithms written to extract features of interest.

To demonstrate the efficacy of our surface and curvature measurement techniques, we performed an x-ray scan of a plastic tube composed of Polytetrafluoroethylene (PTFE) with mean diameter D = 1.27 mm and mean curvature H = 0.79mm^{-1} and no measurable Gaussian curvature K. A radiogram of the tube is shown in Figure S2(a) along with a rendering of the tube surface with the map of the mean curvature (b) and the Gaussian curvature (c) superimposed.

To measure the curvatures, we used a small region of the plastic tube with length 5.9mm, width 1.1mm, and height 0.4mm corresponding to a volume of interest of 120 × 23 × 9 pixel^3. Voxel size is 0.049mm^3. A quadratic polynomial of the form Ax^2+By^2+Cxy where A, B and C are free parameters is used to fit locally the surface over patches of lateral size 15 × 15pixel^2. A local rotation is applied to align the normal with the z axis before fitting. As H and K are intrinsic geometrical quantities, they are not affected by this transformation. Then, they are given by H = (A+B) and K = 4AB-C^2. The experimental mean curvature is $H_{exp} = 0.74\text{mm}^{-1}$ which is within 6% of the value based on direct measurement. The Gaussian curvature $K = 0 \pm 0.007\text{mm}^{-2}$.

Thus, our method allows us to precisely characterize the local geometry of strongly curved surfaces with spatial accuracy below 0.5mm which corresponds to only few times the thickness of the elastic sheets used in our experiment.