Supplementary Information

Wrinkling of structured thin films via contrasted materials

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1. Condition of inextensibility

The condition of inextensibility[1-3] implies the contraction caused by wrinkles depends on the compressive strain ε_c, written as

$$\left(\frac{A}{\lambda}\right)^2 \sim -\varepsilon_c \,. \tag{S1}$$

In our model, ε_c is the compressive strain caused by the interaction between two contrasted materials. To estimate the compressive strain simply, the periodic cell is assumed to be stretched with a strain ε. The elements with low elastic modulus (E_1) and high elastic modulus (E_2) will therefore deform with tensile strains $\frac{2E_r}{E_r+1}\varepsilon$ and $\frac{2}{E_r+1}\varepsilon$ ($E_r = E_2/E_1$) in stage 1 (Fig. 3), respectively. In stage 2, according to continuity and symmetry of deformation, the interaction between two vertical elements will cause the elements with low elastic modulus to be compressed with a strain $-\frac{E_r-1}{E_r+1}\varepsilon$ in the length direction. But this compressive strain also causes a tensile strain $k\frac{E_r-1}{E_r+1}\varepsilon$ in the width direction, where k is a constant. Oppositely, elements with high elastic modulus will be tensioned with a strain $\frac{E_r-1}{E_r+1}\varepsilon$ in the length direction and undergo a compressive strain $-k\frac{E_r-1}{E_r+1}\varepsilon$ in the width direction. As a result, in stage 3, the mismatched strains of the two elements in the width direction causes a compressive stress in the elements with low elastic modulus. The corresponding compressive strain caused by the compressive stress is $-2k\frac{E_r(E_r-1)}{(E_r+1)^2}\varepsilon$. Considering the influence of the ratio of length to width of the element, L_e/W_e, the condition of inextensibility can be expressed as

$$\left(\frac{A}{\lambda}\right)^2 \sim \frac{E_r(E_r-1)}{(E_r+1)^2}\left(\frac{L_e}{W_e}\right)\varepsilon \,. \tag{S2}$$

2. Numerical results for the wrinkling of the whole structured films

The wrinkling of the whole structured films under stretching is modeled by using the commercial FEM software ABAQUS (SIMULIA, Providence, RI, USA). The ratio of elastic moduli of the two materials (E_r) is designed from 1.25 to 2.50 with a fixed length of element (L_e) of 20.0 mm and L_e changes from 12.5 mm to 30.0 mm with a fixed E_r of 1.50. The shell element S4R and the linear elastic constitutive model are adopted. The size of each element is approximately 0.625 × 0.250 mm (length × width) in all regions. Geometric imperfections consisting of the superposition of several buckling modes with a maximum value of $0.01t$ are imposed at the nodes of the original planar mesh. We measure wavelengths of wrinkles for various structured films with increasing strain up to 20%. As shown in Fig. S1, the dimensionless wavelength $\lambda_0=\lambda(1-\nu^2)^{1/4}E_r^{1/4}L_e^{-1/2}t^{-1/2}$ is linear with $\varepsilon^{-1/4}$ with a prefactor of 2.75, which indicates that our theoretical scaling law with the prefactor 2.82 can be used to describe the wavelength of wrinkles.

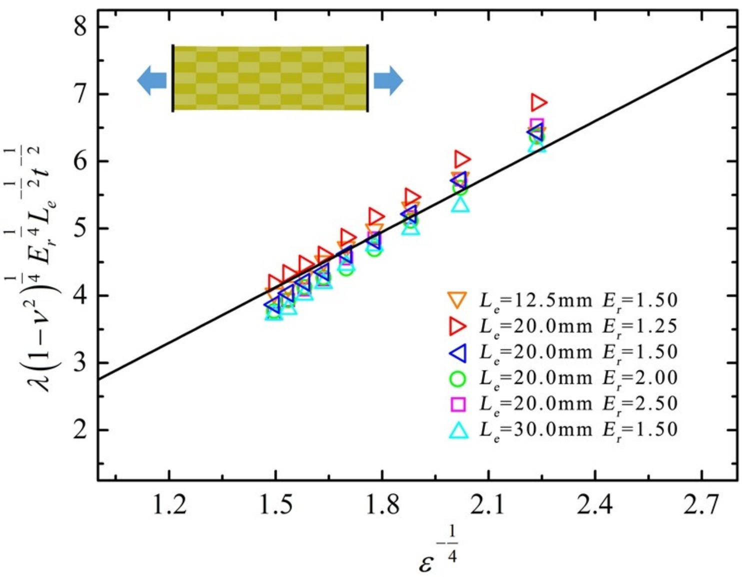

Fig. S1 Plot of the dimensionless wavelength $\lambda_0=\lambda(1-\nu^2)^{1/4}E_r^{1/4}L_e^{-1/2}t^{-1/2}$ against $\varepsilon^{-1/4}$ for various geometries and material properties. The data are calculated by FEM based on the whole structured films and the applied tensile strain is up to 20% (full line: scaling law $\lambda_0=2.75\varepsilon^{-1/4}$). W_e = 10.0 mm and t = 0.025 mm.

3. Numerical results for the wrinkling of a periodic cell

In order to simplify the numerical calculation, we also use a periodic cell to calculate the wrinkling of the structured film under stretching. The periodic boundary condition is applied to all of the boundaries of the cell. The mid-perpendicular lines of the elements with high elastic modulus are constrained by using zero out-of-plane displacement. We measure the wavelengths of the wrinkles in the periodic cell, as shown in Fig. S2a, and find that the value $\lambda_0=\lambda(1-\nu^2)^{1/4}E_r^{1/4}L_e^{-1/2}t^{-1/2}$ remains linear with $\varepsilon^{-1/4}$ for various geometries and material properties, as shown in Fig. S2b, but the prefactor is 2.63, which is smaller than the 2.75 obtained by the FEM based on the whole structured film. The wrinkling satisfies our scaling law well, although the FEM calculation based on the periodic cell results in a smaller wrinkle wavelength. The reason is that the constraint of out-of-plane displacement on the mid-perpendicular line of the elements with high elastic modulus enhances the stretching energy of R_2.

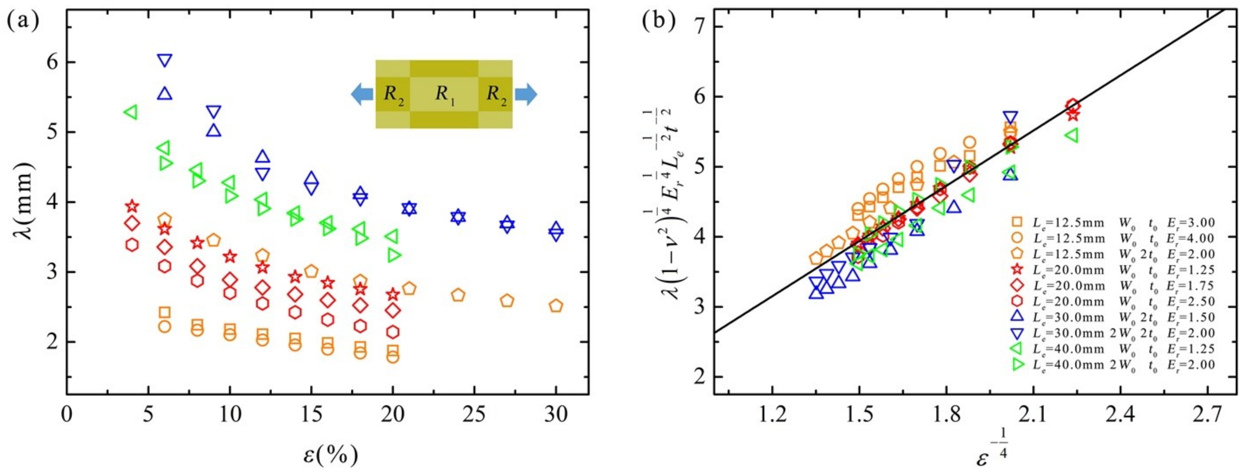

Fig. S2 Plots of (a) wavelength λ against ε and (b) dimensionless wavelength $\lambda_0=\lambda(1-\nu^2)^{1/4}(E_r)^{1/4}L_e^{-1/2}t^{-1/2}$ against $\varepsilon^{-1/4}$ for various geometries and material properties. The data are calculated by FEM based on a periodic cell and the applied tensile strain is up to 30% (full line: scaling law $\lambda_0=2.63\varepsilon^{-1/4}$). W_0 = 10.0 mm and t_0 = 0.025 mm.

4. **Scaling law for the amplitude of wrinkles and numerical results**

The scaling law for the wavelength of the wrinkles is given as

$$\lambda \sim \left(1-\nu^2\right)^{-\frac{1}{4}} E_r^{-\frac{1}{4}} L_e^{\frac{1}{2}} t^{\frac{1}{2}} \varepsilon^{-\frac{1}{4}}. \tag{S3}$$

Substituting this expression of λ into eqn (S2), the amplitude of wrinkles is expressed as

$$A \sim \left[\frac{E_r\left(E_r-1\right)^2}{\left(1-\nu^2\right)\left(E_r+1\right)^4}\right]^{\frac{1}{4}} L_e W_e^{-\frac{1}{2}} t^{\frac{1}{2}} \varepsilon^{\frac{1}{4}}. \tag{S4}$$

From the scaling law, the amplitude of wrinkles on the structured film also depends on the dimensions of the elements, stretching strain and material properties. This scaling law is effective when $E_r \leqslant 5$, because a larger E_r than 5 will make the wrinkles deform seriously and deviate the sine-shape morphology. The amplitude is increased with the increasing E_r, L_e and ε. We also use a periodic cell to calculate the wrinkling and measure the amplitude of the wrinkles. The value $A_0=A(1-\nu^2)^{1/4}E_r^{-1/4}(E_r-1)^{-1/2}(E_r+1)L_e^{-1}W_e^{1/2}t^{-1/2}$ is linear with $\varepsilon^{1/4}$ for various geometries and material properties with a prefactor 2.04, as shown in Fig. S3. However, the accuracy of the scaling law for the amplitude is much less than that for the wavelength and needs to be studied further.

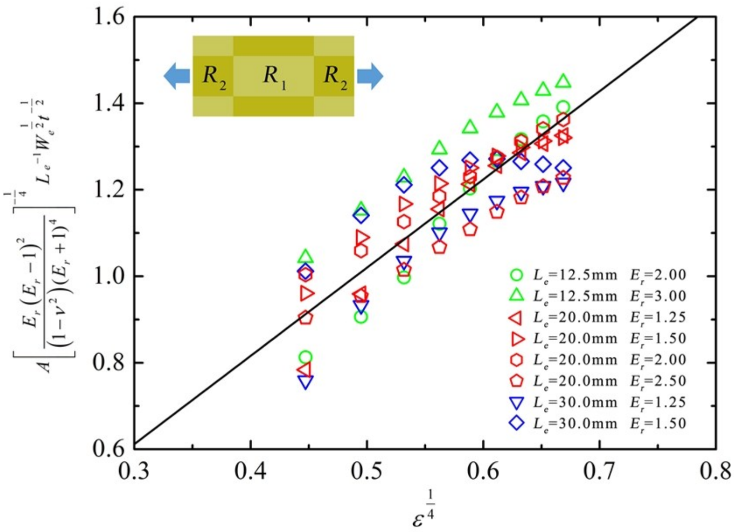

Fig. S3 Plot of the dimensionless amplitude $A_0=A(1-\nu^2)^{1/4}E_r^{-1/4}(E_r-1)^{-1/2}(E_r+1)L_e^{-1}W_e^{1/2}t^{-1/2}$ against $\varepsilon^{1/4}$ for various geometries and material properties. The data are calculated by FEM based on a periodic cell and the applied tensile strain is up to 20% (full line: scaling law $A_0=2.04\varepsilon^{1/4}$). W_e = 10.0 mm and t = 0.025 mm.

References

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