SUPPORTING INFORMATION for:

Nanoparticle dispersion in disordered porous media with and without polymer additives

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1. Bed Characterization

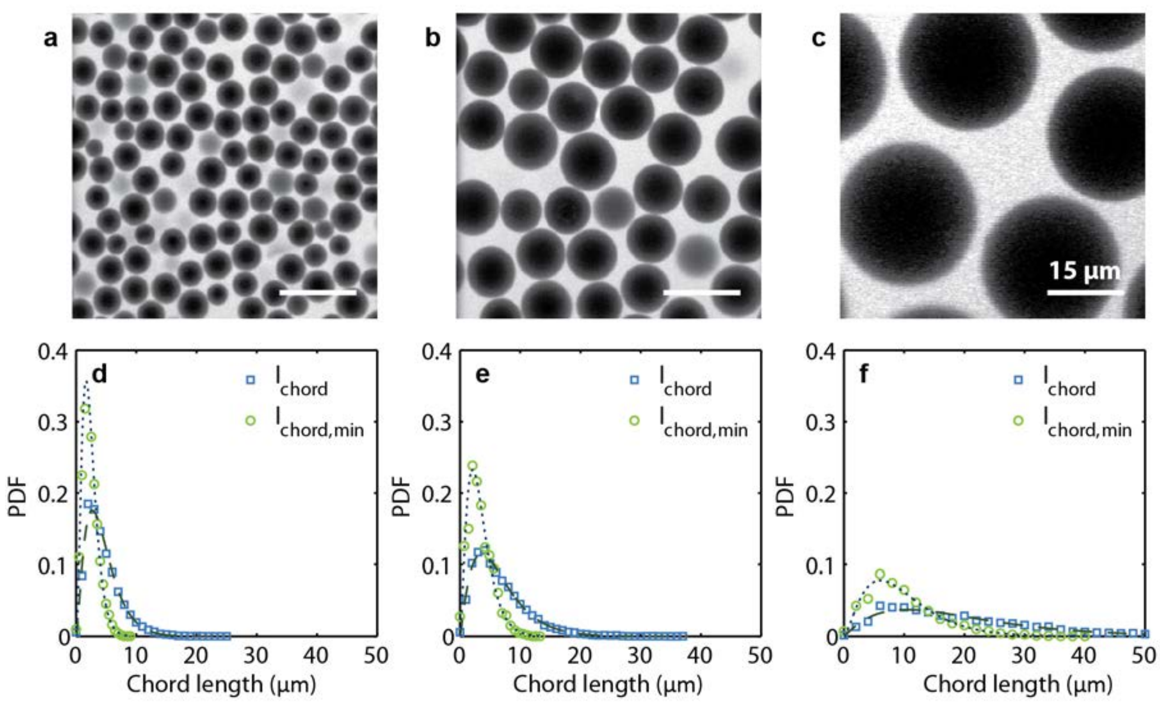

Figure S1. Characterization of porous media. (a–c) Confocal micrographs of porous media with bead diameters of (a) 5.4 μm, (b) 10 μm, and (c) 30 μm. (d–f) Probability distributions of the chord length and the minimum chord length for porous media with bead diameters of (d) 5.4 μm, (e) 10 μm, and (f) 30 μm. The dashed lines represent fits to a gamma distribution function.

2. Dimensionless Numbers

In this section we calculate dimensionless numbers that are important in flow through porous media. The Deborah number De, the ratio of polymer relaxation time to the residence time of the nanoparticles, and the Weissenberg number Wi, the ratio of the viscous forces to the elastic forces, are specifically defined for flow of elastic fluids through porous media. The Deborah and Weissenberg numbers are calculated as

$$De = \frac{\lambda}{t_{NP}} = \frac{\lambda v_{\text{avg}}}{r_b} \qquad \text{S-2}$$

$$Wi = \lambda\dot{\gamma} = \frac{\lambda v_{\text{avg}}}{l_c} \qquad \text{S-3}$$

where λ is the polymer relaxation time, t_{NP} is the residence time of the nanoparticles, v_{avg} is the average velocity of the nanoparticles, r_b is the characteristic radius of curvature of the streamlines $(d_b/2)$, $\dot{\gamma}$ is the characteristic shear rate, and l_c is the confinement length of the porous media.[1-3]

Table S1. De numbers of nanoparticles in HPAM flowed through porous media with bead diameters of 5.4, 10, and 30 µm at flow rates of 5 – 20 µl/hr.

Bead size d_b (µm)	**Flow Rate Q (µm/sec)**			
	5	10	15	20
5.4	89	120	160	230
10	42	80	120	70
30	11	16	26	35

Table S2. Wi numbers of nanoparticles in HPAM flowed through porous media with bead diameters of 5.4, 10, and 30 µm at flow rates of 5 – 20 µl/hr.

Bead size d_b (µm)	**Flow Rate Q (µm/sec)**			
	5	10	15	20
5.4	90	120	160	230
10	57	110	160	220
30	22	30	50	65

The Reynolds number Re is the ratio of inertial forces to viscous forces. For Newtonian solutions flowed through porous media the Reynolds number is calculated as

$$Re = \frac{\rho V_0 d_b}{\eta(1-\varphi)} \qquad \text{S-4}$$

where φ is the bed porosity, d_b is the diameter of glass beads, $V_0 \approx \varphi v_{\text{avg}}$ is the superficial velocity, ρ is the fluid density of, and η is the fluid viscosity. For non-Newtonian power-law fluids flowed through porous media the Reynolds number is calculated as[4]

$$Re = \frac{\rho V_0 d_b}{\eta_{app}(1-\varphi)} \qquad \text{S-5}$$

where $\eta_{app} = k\dot{\gamma}^{n-1} = k\left(\frac{v_{avg}}{l_c}\right)^{n-1}$ is the apparent viscosity of a shear-thinning fluid. Therefore, the Reynolds number can be simplified as:

$$Re = \frac{\rho\varphi v_{\text{avg}} d_b}{k\left(\frac{v_{avg}}{l_c}\right)^{n-1}(1-\varphi)} \qquad \text{S-6}$$

Table S3. Re numbers $\times 10^6$ of nanoparticles in HPAM flowed through porous media with bead diameters of 5.4, 10, and 30 μm at flow rates of 5 – 20 μl/hr.

Bead size d_b (μm)	**Flow Rate Q (μm/sec)**			
	5	10	15	20
5.4	0.23	0.37	0.62	1.10
10	0.27	0.77	1.48	2.55
30	0.36	0.63	1.38	2.21

Table S4. Re numbers $\times 10^6$ of nanoparticles in glycerol/water flowing through porous media with bead diameters of 5.4, 10, and 30 μm at flow rates of 5 – 20 μl/hr.

Bead size d_b (μm)	**Flow Rate Q (μm/sec)**			
	5	10	15	20
5.4	0.36	0.46	0.73	0.98
10	0.65	1.35	1.80	2.52
30	2.48	4.00	5.15	7.03

Table S5. Re numbers ×10^6 of nanoparticles in water flowing through porous media with bead diameters of 5.4, 10, and 30 μm at flow rates of 5 – 20 μl/hr.

Bead size d_b (μm)	Flow Rate Q (μm/sec)			
	5	10	15	20
5.4	50.90	74.30	99.72	125.11
10	93.07	163.50	196.05	268.71
30	321.42	542.37	716.59	848.63

The Péclet number Pe, the ratio of convective transport to diffusive transport, is calculated as

$$Pe = \frac{v_{\mathrm{avg}} l_c}{D_q} \quad \text{S-6}$$

where D_q is the quiescent diffusion coefficient of the nanoparticles in different porous media.

Table S6. Pe numbers ×10^{-2} of nanoparticles in HPAM flowed through porous media with bead diameters of 5.4, 10, and 30 μm at varying flow rates of 5 – 20 μl/hr.

Bead size d_b (μm)	Flow Rate Q (μm/sec)			
	5	10	15	20
5.4	270	350	490	690
10	280	520	780	1080
30	340	480	780	1030

Table S7. Pe numbers ×10^{-2} of nanoparticles in glycerol/water flowed through porous media with bead diameters of 5.4, 10, and 30 μm at flow rates of 5 – 20 μl/hr.

Bead size d_b (μm)	Flow Rate Q (μm/sec)			
	5	10	15	20
5.4	47	68	92	110
10	71	120	150	200
30	170	280	370	440

Table S8. Pe numbers ×10^{-2} of nanoparticles in water flowed through porous media with bead diameters of 5.4, 10, and 30 μm at flow rates of 5 – 20 μl/hr.

Bead size d_b (μm)	Flow Rate Q (μm/sec)			
	5	10	15	20
5.4	0.80	1.2	1.6	2.0
10	1.2	2.1	2.5	3.4
30	1.8	3.1	4.1	4.8

3. Supplementary Results

Normalized distribution of longitudinal and transverse velocities in HPAM and glycerol/water at 10 µl/hr through porous media with bead diameters of 5.4, 10, and 30 µm

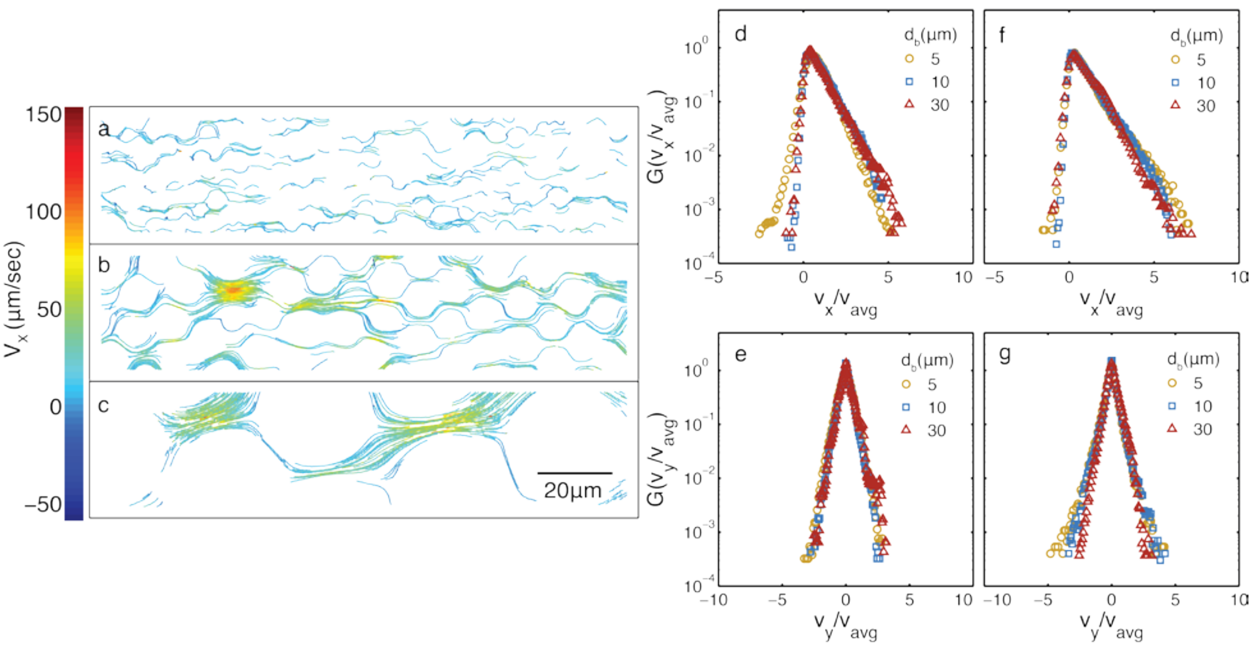

Figure S2. Dependence of normalized velocity distributions on pore size in glycerol/water and in polymer solutions. (a-c) Trajectories of nanoparticles flowed at an inlet flow rate of 10 µl/hr through porous beds with bead diameters (d_b) of (a) 5 µm, (b) 10 µm, and (c) 30 µm. Colors represent velocity variation along the flow direction (v_x). (d-g) Normalized velocity distributions for nanoparticles in (d, e) glycerol/water mixture (90 w/w%) and in (f, g) HPAM solution (0.1 w/w%; $c/c^* \sim 6.25$) flowed at an inlet flow rate of 10 µl/hr through porous beds with bead diameter of 5 µm (gold circles), 10 µm (blue squares), and 30 µm (red triangles). The top row shows the velocity distributions along the flow direction and the bottom row shows the velocity distributions transverse to the flow direction.

Normalized distribution of longitudinal and transverse velocities in HPAM and glycerol/water at 15 μl/hr through porous media with bead diameters of 5.4, 10, and 30 μm

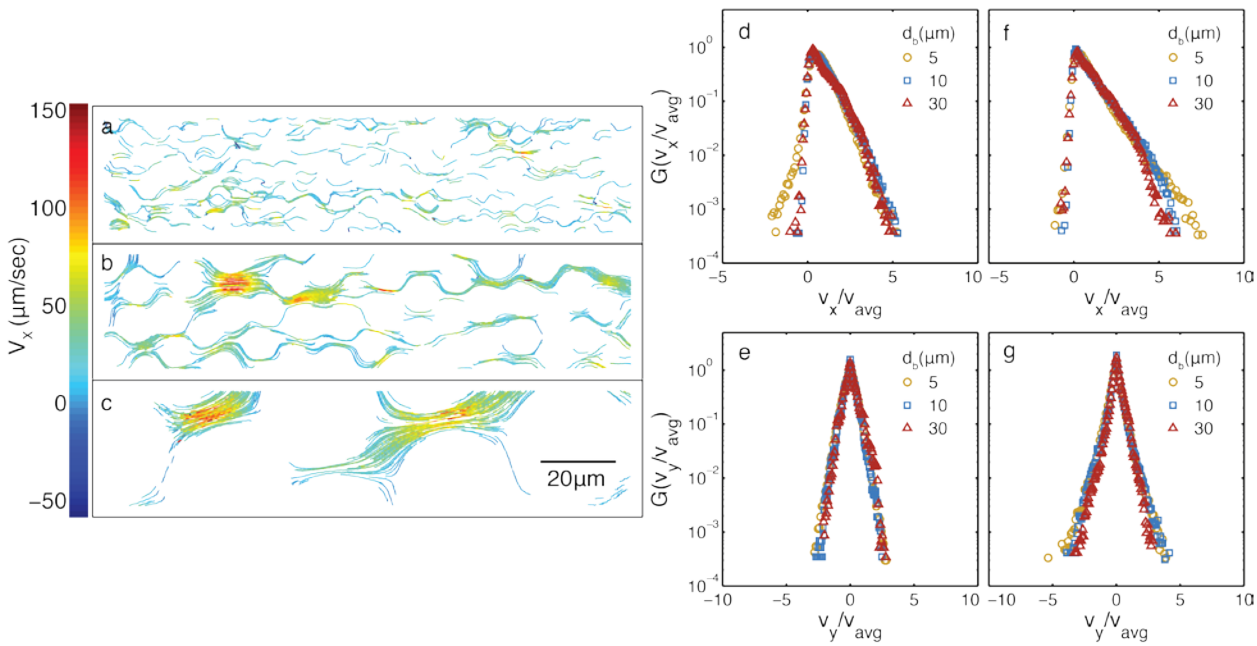

Figure S3. Dependence of normalized velocity distributions on pore size in glycerol/water and in polymer solutions. (a-c) Trajectories of nanoparticles flowed at an inlet flow rate of 15 μl/hr through porous beds with bead diameter (d_b) of (a) 5 μm, (b) 10 μm, and (c) 30 μm. Colors represent velocity variation along the flow direction (v_x). (d-g) Normalized velocity distributions for nanoparticles in (d, e) glycerol/water mixture (90 w/w%) and in (f, g) HPAM solution (0.1 w/w%; $c/c^* \sim 6.25$) flowed at an inlet flow rate of 15 μl/hr through porous beds with bead diameter of 5 μm (gold circles), 10 μm (blue squares), and 30 μm (red triangles). The top row shows the velocity distributions along the flow direction and the bottom row shows the velocity distributions transverse to the flow direction.

Normalized distribution of longitudinal and transverse velocities in HPAM and glycerol/water at 20 µl/hr through porous media with bead diameters of 5.4, 10, and 30 µm

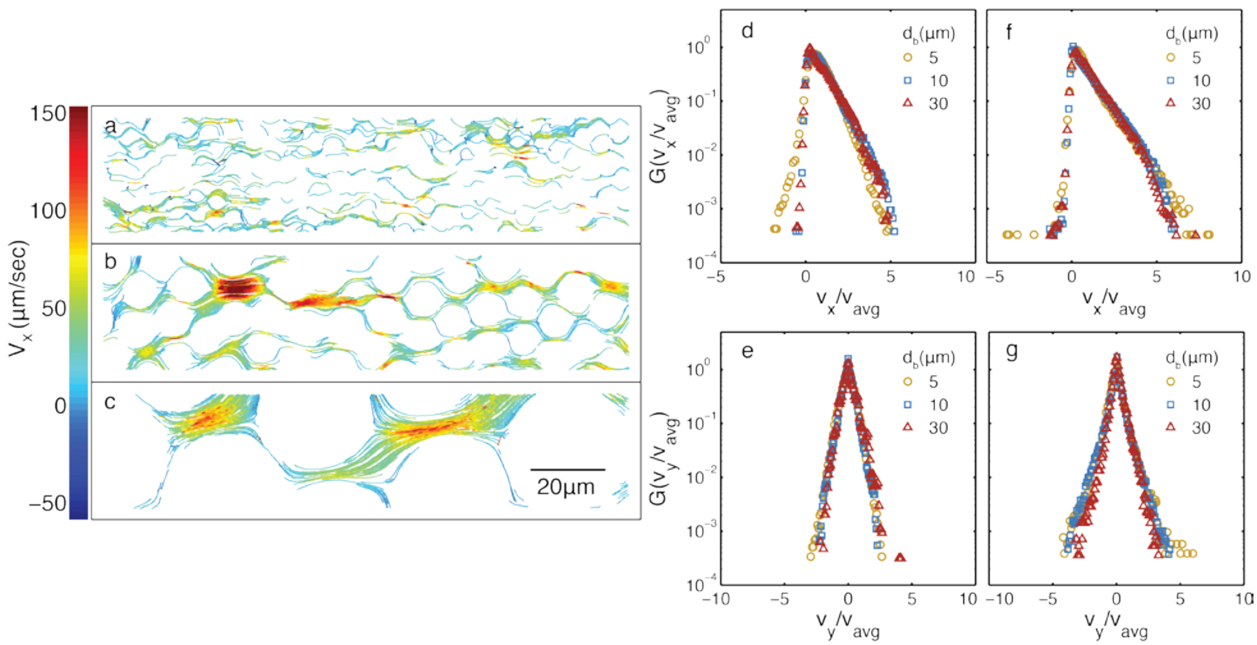

Figure S4. Dependence of normalized velocity distributions on pore size in glycerol/water and in polymer solutions. (a-c) Trajectories of nanoparticles flowed at an inlet flow rate of 20 µl/hr through porous beds with bead diameter (d_b) of (a) 5 µm, (b) 10 µm, and (c) 30 µm. Colors represent velocity variation along the flow direction (v_x). (d-g) Normalized velocity distributions for nanoparticles in (d, e) glycerol/water mixture (90 w/w%) and in (f, g) HPAM solution (0.1 w/w%; $c/c^* \sim 6.25$) flowed at an inlet flow rate of 20 µl/hr through porous bed with bead diameter of 5 µm (gold circles), 10 µm (blue squares), and 30 µm (red triangles). The top row shows the velocity distributions along the flow direction and the bottom row shows the velocity distributions transverse to the flow direction.

Normalized distribution of longitudinal and transverse velocities in water at different flow rates of 5 to 20 µl/hr through porous media with bead diameters of 5.4, 10, and 30 µm

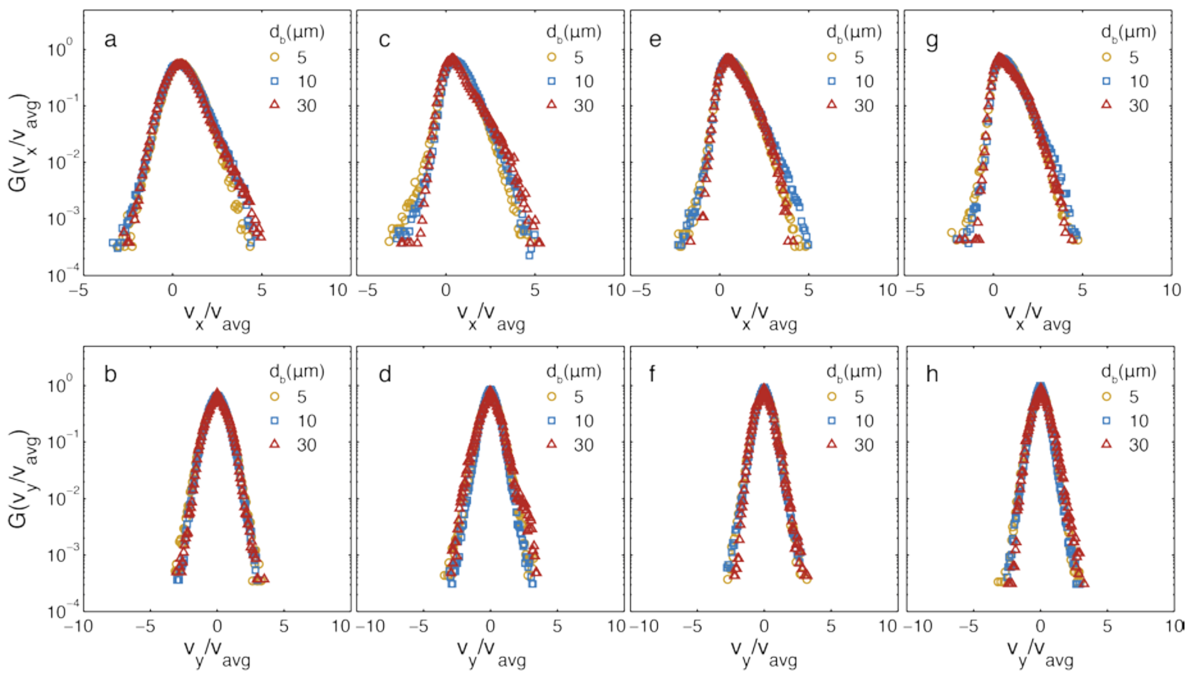

Figure S5. Dependence of normalized velocity distributions on pore size in water. Normalized velocity distributions of nanoparticles in water at inlet flow rates of 5 µl/hr (a, b), 10 µl/hr (c, d), 15 µl/hr (e, f), and 20 µl/hr (g, h) through porous media with bead diameter of 5 µm (gold circles), 10 µm (blue squares), and 30 µm (red triangles). Top row shows the velocity distributions along the flow direction and bottom row shows the velocity distributions transverse the flow direction.

Normalized distribution of longitudinal and transverse velocities in HPAM and glycerol/water at different flow rates of 5 to 20 μl/hr through porous media with bead diameters of 10 μm

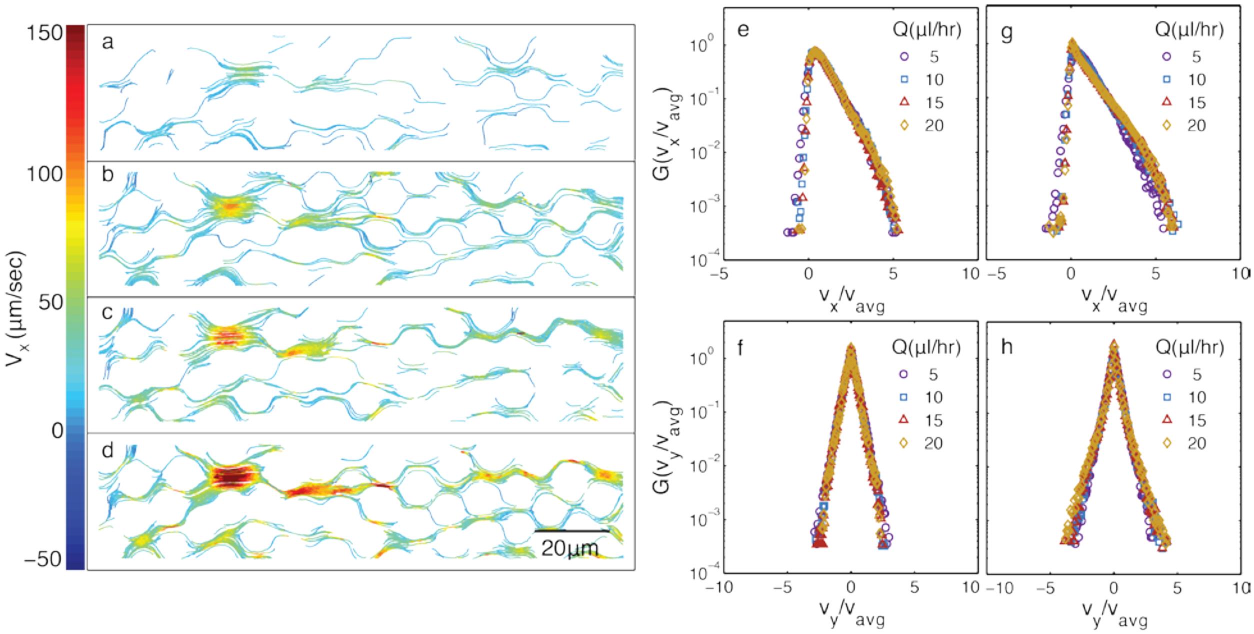

Figure S6. Dependence of normalized velocity distributions on flow rates in glycerol/water and in polymer solutions. (a-d) Trajectories of nanoparticles transported through porous beds with bead diameter of 10 μm at inlet flow rates of (a) 5 μl/hr, (b) 10 μl/hr, (c) 15 μl/hr, and (d) 20 μl/hr. Colors represent velocity variation along the flow direction (v_x). (e-h) Normalized velocity distributions for nanoparticles in (e, f) glycerol/water mixture (90 w/w%) and in (f, g) HPAM solution (0.1 w/w%; $c/c^* \sim 6.25$) flowed at inlet flow rates of 5 μl/hr (purple circles), 10 μl/hr (blue squares), 15 μl/hr (red triangles), and 20 μl/hr (gold diamonds). The top row shows the velocity distributions along the flow direction and the bottom row shows the velocity distributions transverse to the flow direction.

Normalized distribution of longitudinal and transverse velocities in HPAM and glycerol/water at different flow rates of 5 to 20 µl/hr through porous media with bead diameters of 30 µm

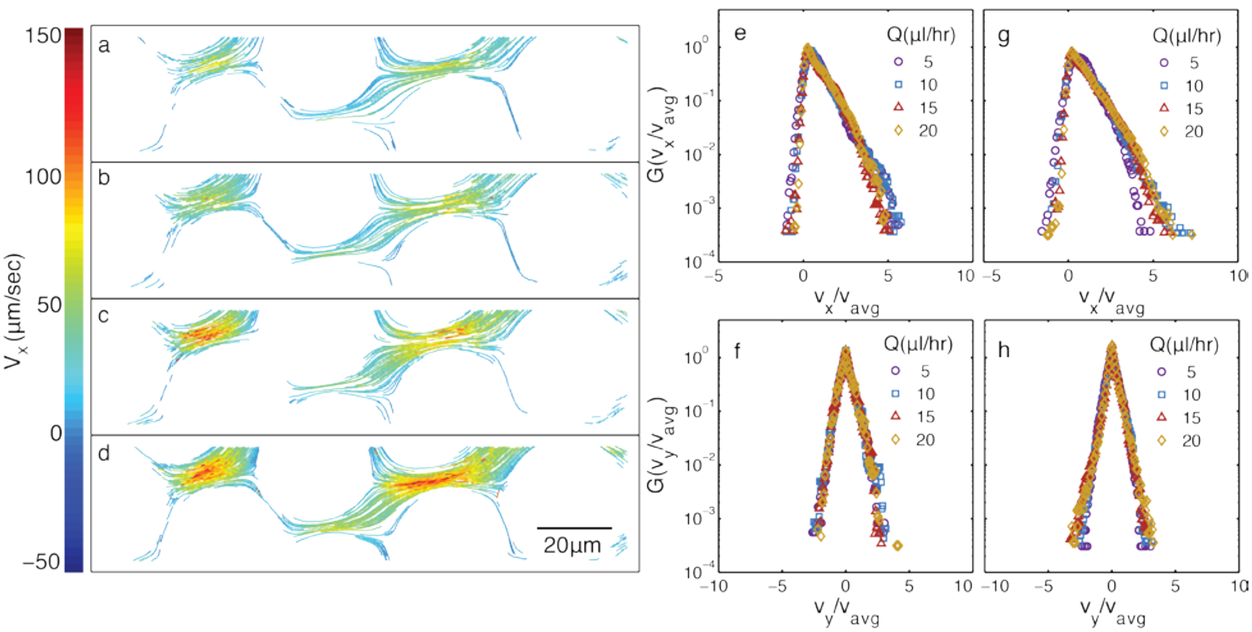

Figure S7. Dependence of normalized velocity distributions on flow rates in glycerol/water and in polymer solutions. (a-d) Trajectories of nanoparticles transported through porous beds with bead diameter of 30 µm at an inlet flow rate of (a) 5 µl/hr, (b) 10 µl/hr, (c) 15 µl/hr, and (d) 20 µl/hr. Colors represent velocity variation along the flow direction (v_x). (e-h) Normalized velocity distributions for nanoparticles in (e, f) glycerol/water mixture (90 w/w%) and in (f, g) HPAM solution (0.1 w/w%; $c/c^* \sim 6.25$) flowed at inlet flow rates of 5 µl/hr (purple circles), 10 µl/hr (blue squares), 15 µl/hr (red triangles), and 20 µl/hr (gold diamonds). The top row shows the velocity distributions along the flow direction and the bottom row shows the velocity distributions transverse to the flow direction.

Normalized distribution of longitudinal and transverse velocities in water at different flow rates of 5 to 20 µl/hr through porous media with bead diameters of 5.4, 10, and 30 µm

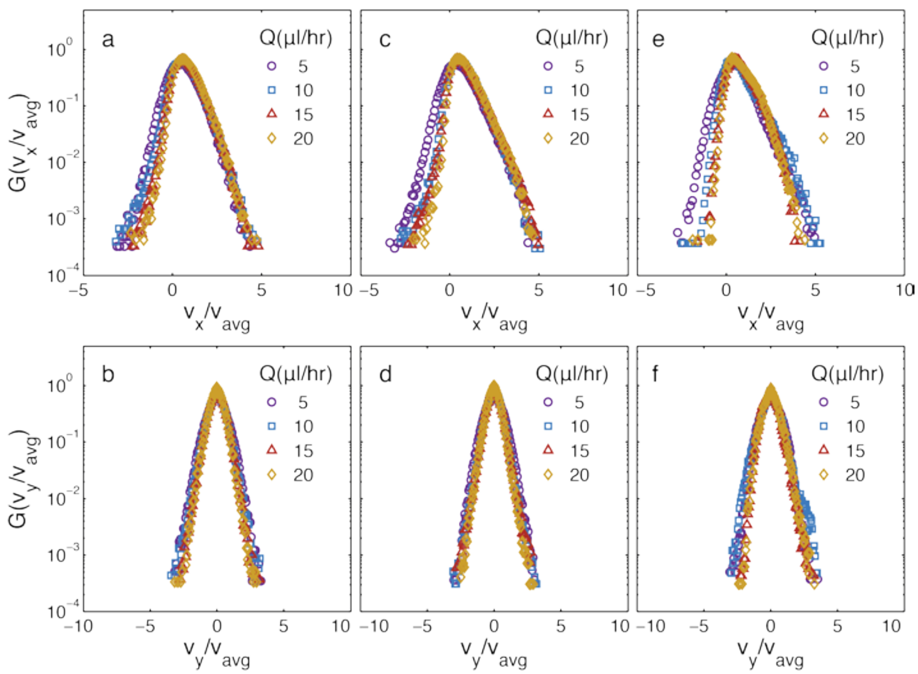

Figure S8. Dependence of normalized velocity distributions on flow rates in water. Normalized velocity distributions of nanoparticles flowed in water through porous media with bead diameters of 5 µm (a, b) 10 µm (c, d), and 30 µm (e, f) and at inlet flow rates of 5 µl/hr (purple circle), 10 µl/hr (blue square), 15 µl/hr (red triangle), and 20 µl/hr (gold diamond). The top row shows the velocity distributions along the flow direction and the bottom row shows the velocity distributions transverse to the flow direction.

Normalized distribution of longitudinal and transverse velocities in glycerol/water mixture and HPAM solutions at different flow rates of 5 to 20 µl/hr through porous media with bead diameters of 5.4, 10, and 30 µm

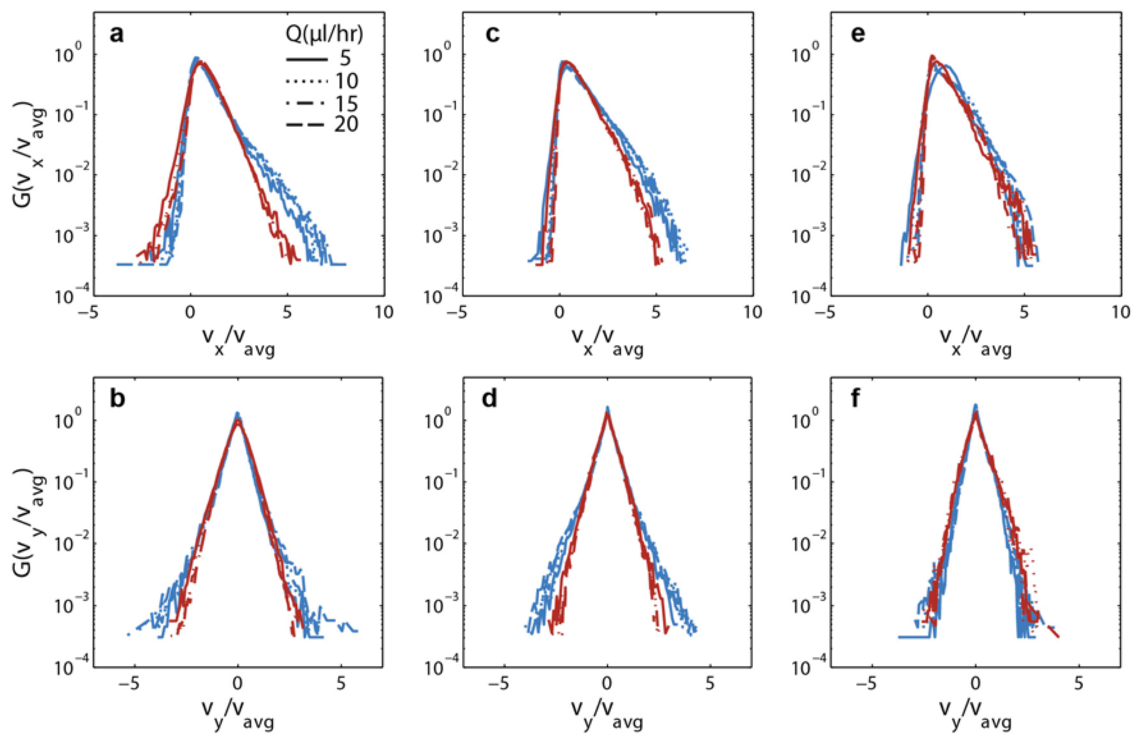

Figure S9. Dependence of normalized velocity distributions on solution characteristics. Normalized velocity distributions for nanoparticles flowed in glycerol/water mixture (red) and in HPAM polymer solution (blue) at inlet flow rates of 5 µl/hr (line), 10 µl/hr (dotted line), 15 µl/hr (dashed-dotted line), and 20 µl/hr (dashed line) through porous media with bead diameters of (a,b) 5.4 µm, (c,d) 10 µm, and (e,f) 30 µm. The top row shows the velocity distributions along the flow direction and the bottom row shows the velocity distributions transverse to the flow direction.

Time dependent longitudinal and transverse dispersion of nanoparticles in HPAM solutions and glycerol/water through porous media with bead diameter of 10 μm

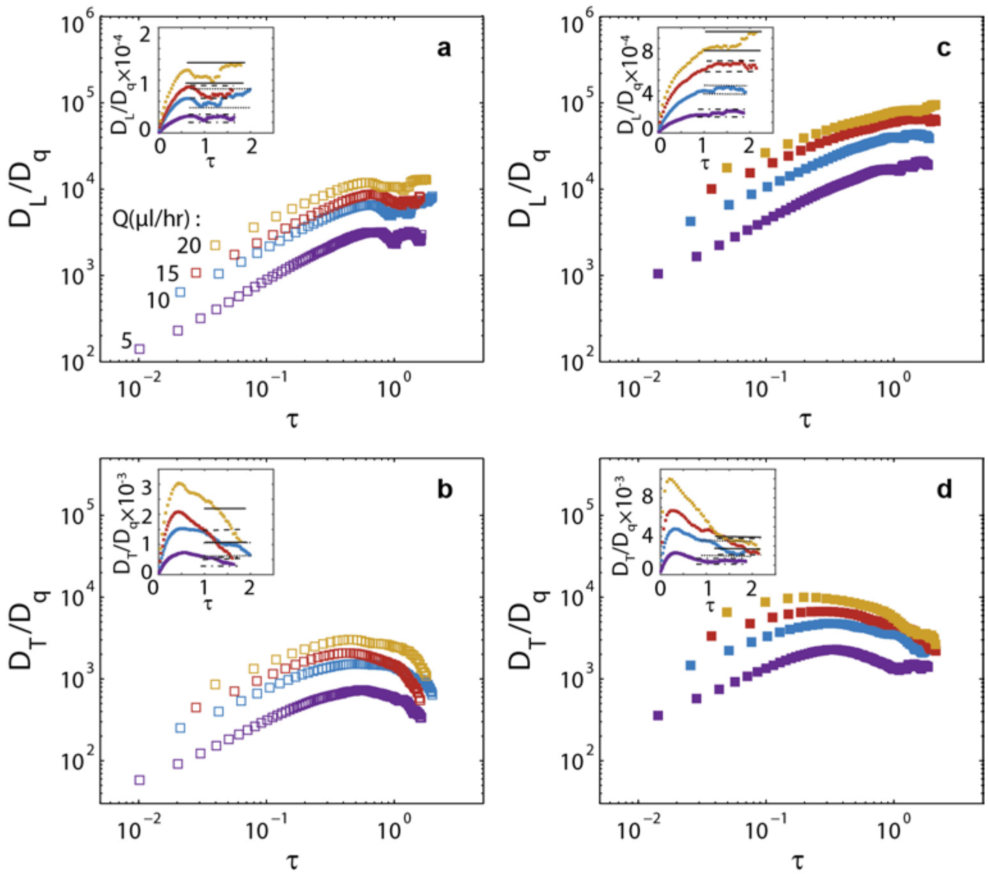

Figure S10. Time dependent dispersion of nanoparticles in glycerol/water and in polymer solutions. Normalized longitudinal (D_L/D_q, top row) and transverse (D_T/D_q, bottom row) dispersion coefficients of nanoparticles in (a, b) glycerol/water mixture (90 w/w%) and in (c, d) HPAM solution (0.1 w/w%; c/c* ~ 6.25) as a function of normalized lag time ($\tau = tv_{\text{avg}}/d_b$) for flow through porous bed with bead diameter of 10 μm. Colors represent inlet flow rates: 5 μl/hr (purple), 10 μl/hr (blue), 15 μl/hr (red), and 20 μl/hr (gold).

Time dependent longitudinal and transverse dispersion of nanoparticles in HPAM solutions and glycerol/water through porous media with bead diameter of 30 μm

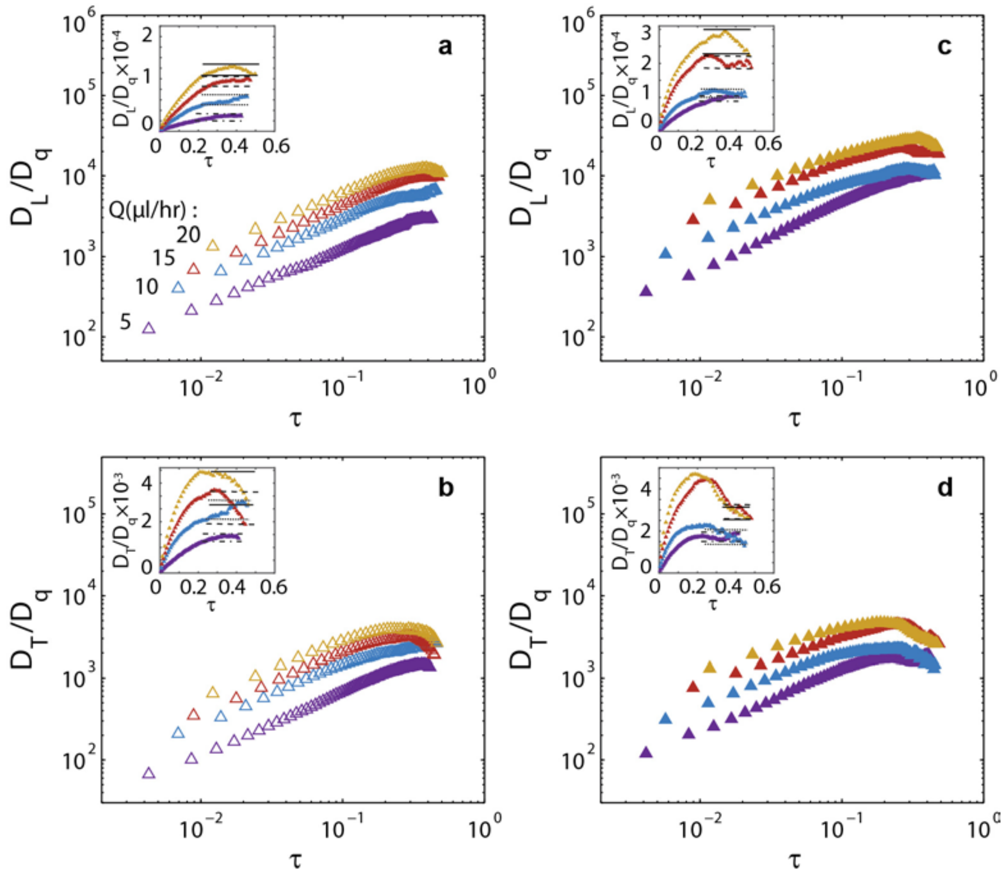

Figure S11. Time dependent dispersion of nanoparticles in glycerol/water and in polymer solutions. Normalized longitudinal (D_L/D_q, top row) and transverse (D_T/D_q, bottom row) dispersion coefficients of nanoparticles in (a, b) glycerol/water mixture (90 w/w%) and in (c, d) HPAM solution (0.1 w/w%) as a function of normalized lag time ($\tau = tv_{\text{avg}}/d_b$) for flow through porous bed with bead diameter of 30 μm. Colors represent inlet flow rates: 5 μl/hr (purple), 10 μl/hr (blue), 15 μl/hr (red), and 20 μl/hr (gold).

Time dependent longitudinal and transverse dispersion of nanoparticles in water through porous media with bead diameter of 5.4, 10, and 30 μm

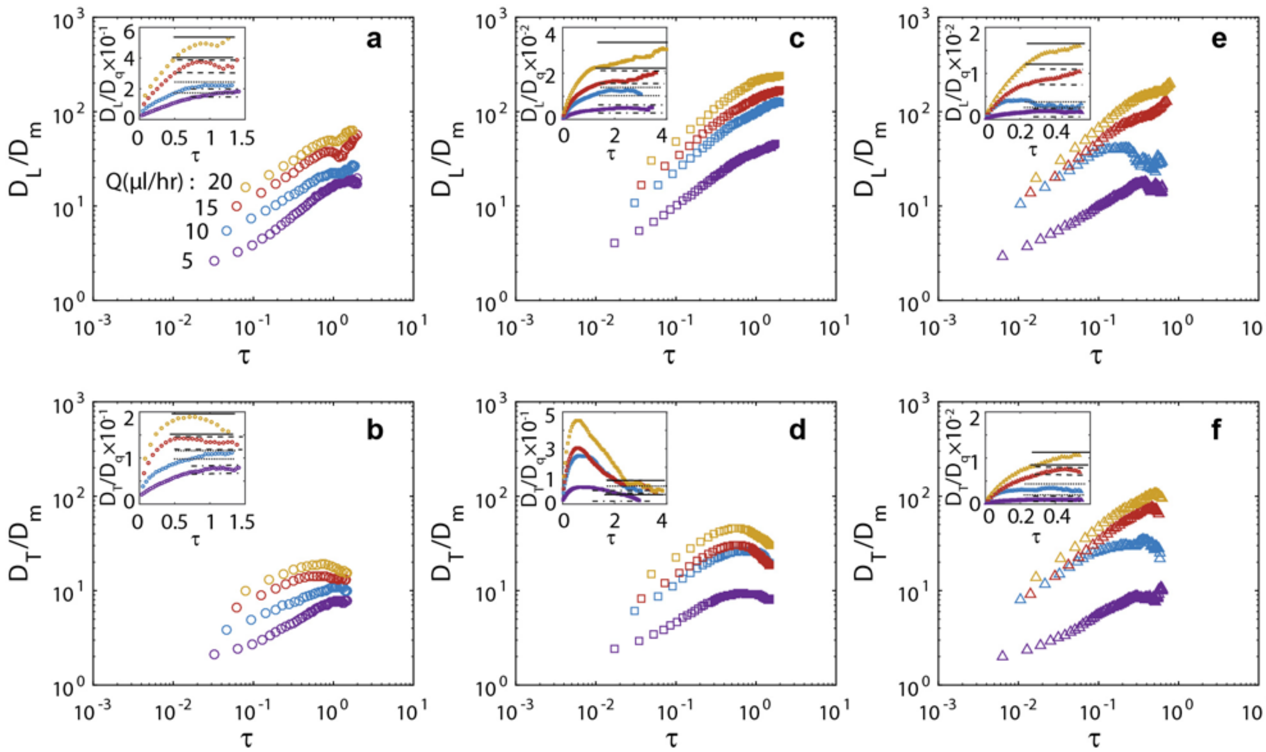

Figure S12. Time dependent dispersion of nanoparticles in water. Normalized longitudinal (D_L/D_q, top row) and transverse (D_T/D_q, bottom row) dispersion coefficients of nanoparticles in water as a function of normalized lag time ($\tau = t v_{\mathrm{avg}}/d_b$) for flow through porous beds with bead diameters of 5 μm (a, b), 10 μm (c, d), and 30 μm (e, f). Colors represent inlet flow rates: 5 μl/hr (purple), 10 μl/hr (blue), 15 μl/hr (red), and 20 μl/hr (gold).

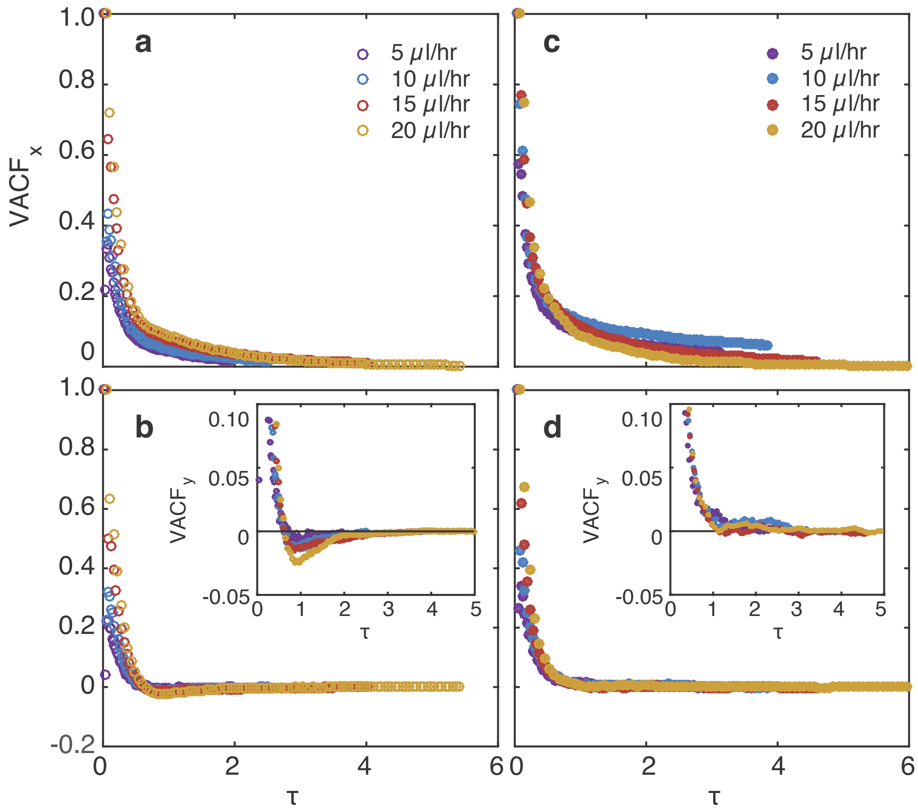

Figure S13. Velocity autocorrelation function in longitudinal ($VACF_x$, top row) and transverse ($VACF_y$, bottom row) direction for nanoparticles in (a, b) glycerol/water mixture (90 w/w%) and in (c, d) HPAM solution (0.1 w/w%) as a function of normalized lag time ($\tau = tv_{\mathrm{avg}}/d_b$) for flow through porous bed with bead diameter of 5 μm. Insets in (b) and (d) show the same data on restricted *x* and *y* axes. Colors represent inlet flow rates: 5 μl/hr (purple), 10 μl/hr (blue), 15 μl/hr (red), and 20 μl/hr (gold).

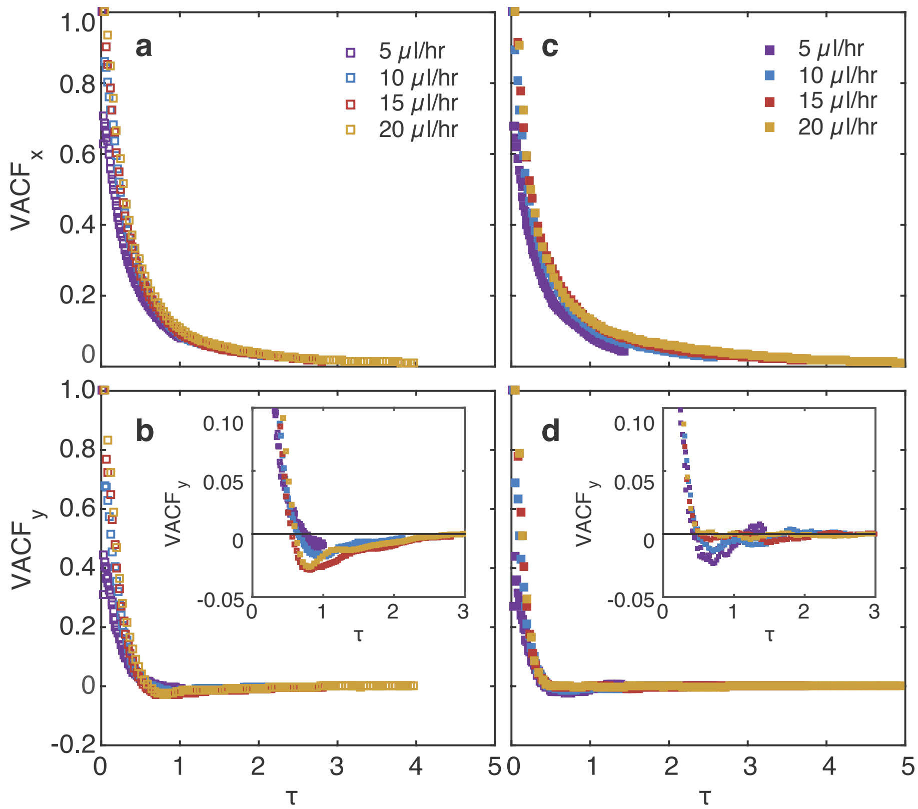

Figure S14. Velocity autocorrelation function in longitudinal ($VACF_x$, top row) and transverse ($VACF_y$, bottom row) direction for nanoparticles in (a, b) glycerol/water mixture (90 w/w%) and in (c, d) HPAM solution (0.1 w/w%) as a function of normalized lag time ($\tau = tv_{\mathrm{avg}}/d_b$) for flow through porous bed with bead diameter of 10 μm. Insets in (b) and (d) show the same data on restricted *x* and *y* axes. Colors represent inlet flow rates: 5 μl/hr (purple), 10 μl/hr (blue), 15 μl/hr (red), and 20 μl/hr (gold).

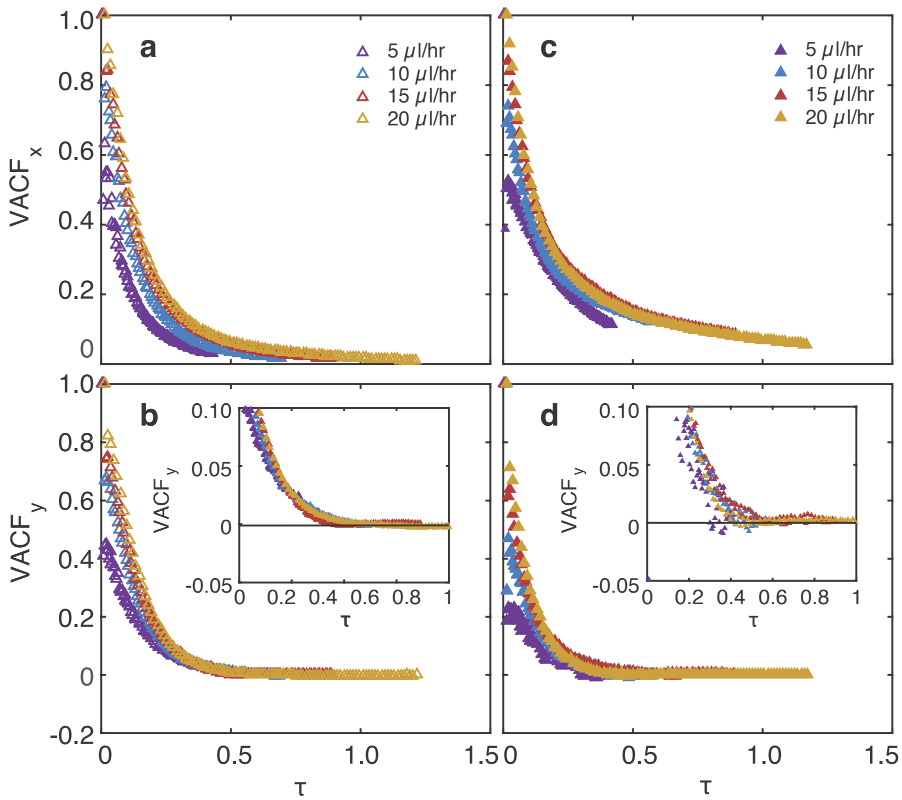

Figure S15. Velocity autocorrelation function in longitudinal ($VACF_x$, top row) and transverse ($VACF_y$, bottom row) direction for nanoparticles in (a, b) glycerol/water mixture (90 w/w%) and in (c, d) HPAM solution (0.1 w/w%) as a function of normalized lag time ($\tau = tv_{\text{avg}}/d_b$) for flow through porous bed with bead diameter of 30 μm. Insets in (b) and (d) show the same data on restricted *x* and *y* axes. Colors represent inlet flow rates: 5 μl/hr (purple), 10 μl/hr (blue), 15 μl/hr (red), and 20 μl/hr (gold).

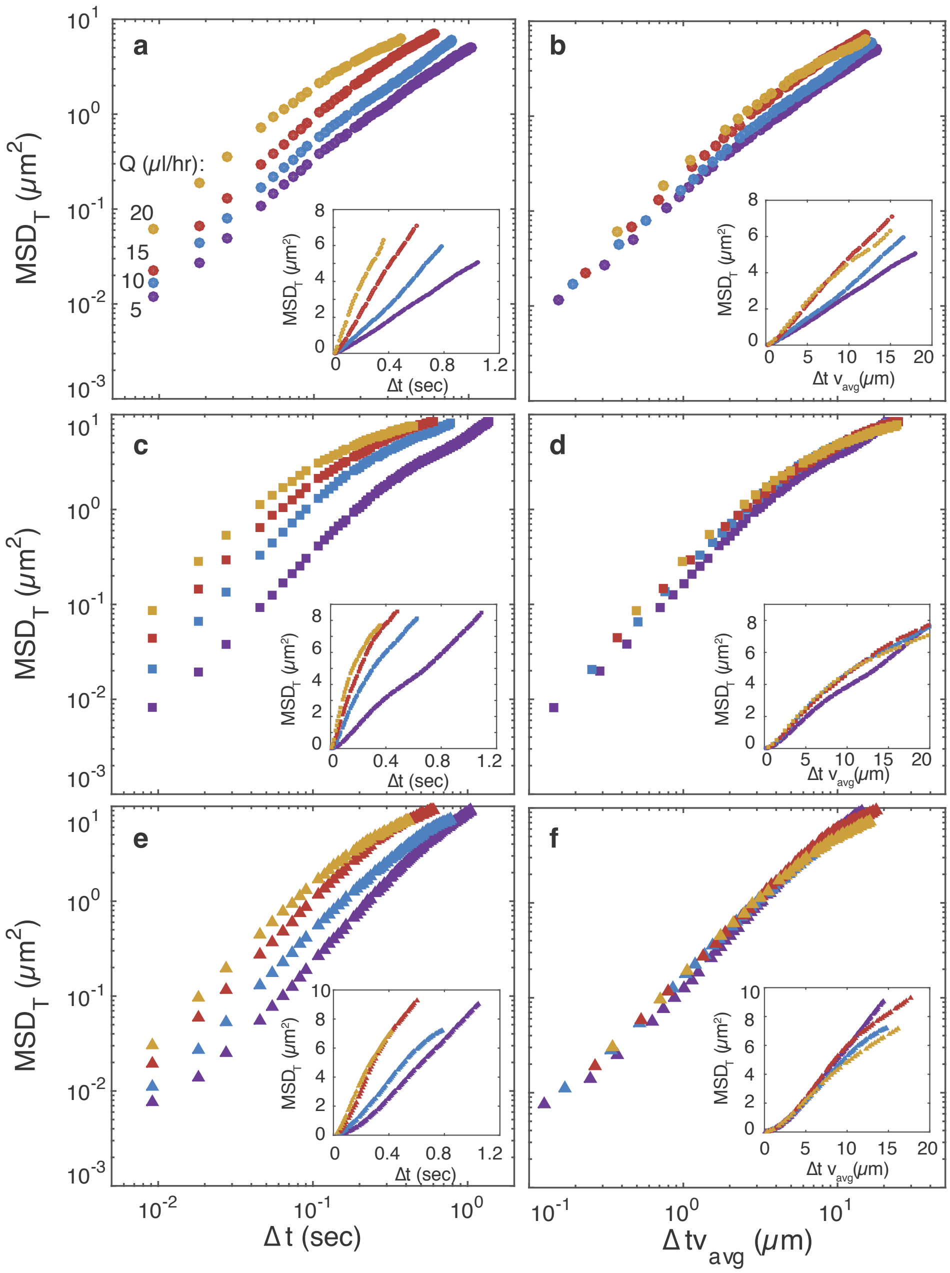

Figure S16. Transverse mean-squared displacement MSD_T as a function of lag time Δt (left, a, c, e) and normalized time $\Delta t v_{\text{avg}}$ (right, b, d, f) in HPAM for flow through porous beds with bead diameters of 5 μm (a, b), 10 μm (c, d), and 30 μm (e, f). Colors indicate inlet flow rates: 5 μl/hr (purple), 10 μl/hr (blue), 15 μl/hr (red), and 20 μl/hr (gold). The insets show the same data on linear axes.

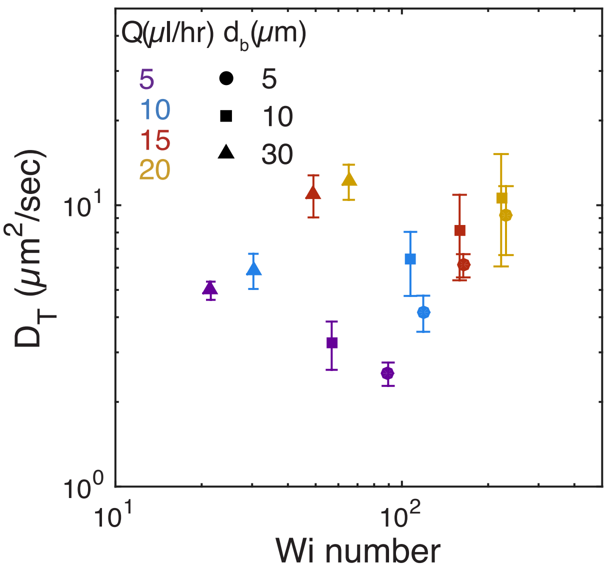

Figure S17. Transverse dispersion coefficient D_T as function of Weissenberg number Wi in HPAM for flow through porous beds with bead diameters of 5 μm (circle), 10 μm (square), and 30 μm (triangle). Colors indicate inlet flow rates: 5 μl/hr (purple), 10 μl/hr (blue), 15 μl/hr (red), and 20 μl/hr (gold).

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