Electronic Supplementary Material (ESI) for Soft Matter.

Supplementary Information

Thickness measurement. After the sample was cut out and peeled from the glass slide, white light interferometer was used to locate the newly exposed glass surface and the free surface of remaining film on the glass slide. Thickness measurements were taken at various points on one slide and mean thickness of each sample was calculated.

Young's Modulus measurement. Young's modulus of the PDMS was measured by an indentation test. A spherical glass indenter was brought into contact with thicker slabs (>5mm) of PDMS made from the same liquid PDMS mixture and undergoing the same curing cycle as the films. During one measurement, the indenter slowly indented into the sample with uniform speed and was then slowly lifted up until detaching from the sample. The contact area was captured by a microscope and the indenting force P was measured using a load cell. The Young's modulus E and work of adhesion W_{ad} can be obtained by fitting the experimental data using JKR theory[28],

$$\left(P-\frac{4E^{*}a^{3}}{3R}\right)^{2}=8\pi W_{ad}E^{*}a^{3},\ \text{where}\ \frac{1}{E^{*}}=\frac{1-\nu^{2}}{E}=\frac{3}{4E} \tag{SI-1}$$

Solving von Karman plate equations. Normalize equations (5a, b) in the paper using the following scheme,

$$w=w_{c}\overline{w},\ r=c\eta, u=\frac{w_{c}^{2}}{c}\overline{u},\ \ T_{0}=\frac{Ehw_{c}^{2}}{c^{2}}\overline{T}_{0}, P=\frac{Ehw_{c}^{3}}{c^{2}}\overline{P} \tag{SI-2}$$

Hence equations (5a, b) in the paper become

$$\begin{gathered}\varepsilon\frac{d}{d\eta}\left[\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\frac{d\overline{w}}{d\eta}\right)\right]=\left[\overline{T}_{0}+\frac{1}{1-\nu^{2}}\left(\frac{d\overline{u}}{d\eta}+\frac{1}{2}\left(\frac{d\overline{w}}{d\eta}\right)^{2}+\nu\frac{\overline{u}}{\eta}\right)\right]\frac{d\overline{w}}{d\eta}+\frac{\overline{P}}{2\pi\eta}\\ \frac{d^{2}\overline{u}}{d\eta^{2}}+\frac{1}{\eta}\frac{d\overline{u}}{d\eta}-\frac{\overline{u}}{\eta^{2}}=-\frac{1-\nu}{2\eta}\left(\frac{d\overline{w}}{d\eta}\right)^{2}-\frac{d\overline{w}}{d\eta}\frac{d^{2}\overline{w}}{d\eta^{2}}\end{gathered} \tag{SI-3a, b}$$

$$\text{where}\ \varepsilon=\frac{1}{12\left(1-\nu^{2}\right)}\left(\frac{h}{w_{c}}\right)^{2} \tag{SI-4}$$

From equation (SI-4), the importance of bending depends on the square of the ratio of film thickness to imposed displacement at the inner edge.

Normalized boundary conditions are,

$$
\begin{aligned}
&\bar{w}(\eta=1)=1, \bar{u}(\eta=1)=\frac{u_c c}{w_{c^2}} \\
&\frac{d\bar{w}}{d\eta}(\eta=1)=\frac{c}{w_c}\sin^{-1}(c/R)\approx\frac{c^2}{Rw_c}(\text{small contact}) \\
&\bar{w}(\eta=a/c)=\frac{d\bar{w}}{d\eta}(\eta=a/c)=\bar{u}(\eta=a/c)=0
\end{aligned}
\tag{SI-5}
$$

Equations (SI-3a, b) need to be solved using an iterative numerical scheme for boundary value problems with MatLab®. The key idea is to increase w_c in small steps over a number of iterations. An analytical solution can be obtained, when a sufficiently small w_c is used, so that bending is dominant and nonlinear terms in equation (SI-3a) can be neglected,

$$
\varepsilon\frac{d}{d\eta}\left[\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\frac{d\bar{w}}{d\eta}\right)\right]=\bar{T}_0\frac{d\bar{w}}{d\eta}+\frac{\bar{P}}{2\pi\eta}
\tag{SI-6}
$$

Subject to boundary conditions,

$$
\begin{aligned}
&\bar{w}^0(\eta=1)=1 \\
&\frac{d\bar{w}^0}{d\eta}(\eta=1)=\frac{c^2}{Rw_c^0} \\
&\bar{w}^0(\eta=a/c)=\frac{d\bar{w}^0}{d\eta}(\eta=a/c)=0
\end{aligned}
\tag{SI-7}
$$

where a superscript ^{0} denotes the value used in the first iteration.

Let $\eta=\bar{r}\sqrt{\frac{\varepsilon}{\bar{T}_0}}$, the solution to equation (SI-6) is,

$$
\bar{w}^0(\bar{r})=\frac{-a_1K_0(\bar{r})+a_2(I_0(\bar{r})-1)}{\bar{T}_0}-\frac{\bar{P}}{\bar{T}_0}f(\bar{r})+\frac{A}{\bar{T}_0}
\tag{SI-8}
$$

where

$$
f(\bar{r})=\frac{G_{3,5}^{2,3}\left(\bar{r},\frac{1}{2}\middle|\begin{matrix}1/2,1,1\\1,1,0,0,0\end{matrix}\right)}{8\pi^{3/2}}+\frac{\sqrt{\pi}G_{5,7}^{2,3}\left(\bar{r},\frac{1}{2}\middle|\begin{matrix}\frac{1}{2},1,1,\frac{3}{4},\frac{5}{4}\\1,1,0,0,0,\frac{3}{4},\frac{5}{4}\end{matrix}\right)}{4}+\frac{\log(\bar{r})}{2\pi}
\tag{SI-9}
$$

$$\begin{bmatrix} a_1 \\ a_2 \\ \overline{P} \\ A \end{bmatrix} = \begin{bmatrix} K_1(\overline{r}_0) & I_1(\overline{r}_0) & -J(\overline{r}_0) & 0 \\ K_1(\overline{r}_\infty) & I_1(\overline{r}_\infty) & -J(\overline{r}_\infty) & 0 \\ -K_0(\overline{r}_0) & I_0(\overline{r}_0)-1 & -f(\overline{r}_0) & 1 \\ -K_0(\overline{r}_\infty) & I_0(\overline{r}_\infty)-1 & -f(\overline{r}_\infty) & 1 \end{bmatrix}^{-1} \begin{bmatrix} \dfrac{c^2\sqrt{\varepsilon \overline{T}_0}}{Rw_c^0} \\ 0 \\ \overline{T}_0 \\ 0 \end{bmatrix}, \quad \overline{r}_0 = \sqrt{\frac{\overline{T}_0}{\varepsilon}}, \overline{r}_\infty = \frac{a}{c}\sqrt{\frac{\overline{T}_0}{\varepsilon}} \qquad \text{(SI-10)}$$

Solution to equation (SI-3b) is

$$\overline{u}^0(\overline{r}) = \frac{1}{\overline{T}_0}\sqrt{\frac{1}{\varepsilon \overline{T}_0}}\left(b_1\overline{r} + b_2/\overline{r} - 2\overline{r}\int_{\overline{r}_0}^{\overline{r}} F(\overline{r}')d\overline{r}' + 2\overline{r}^{-1}\int_{\overline{r}_0}^{\overline{r}} (\overline{r}')^2 F(\overline{r}')d\overline{r}' \right) \qquad \text{(SI-11)}$$

where

$$\begin{aligned} F(\overline{r}) &\equiv -\frac{1-\nu}{2\overline{r}}\left(\Phi(\overline{r})\right)^2 - \Phi(\overline{r})\frac{d\Phi(\overline{r})}{d\overline{r}} \\ \Phi(\overline{r}) &= a_1 K_1(\overline{r}) + a_2 I_1(\overline{r}) - \overline{P}\frac{K_1(\overline{r})I_0(\overline{r}) + I_1(\overline{r})K_0(\overline{r})}{2\pi} \\ b_1 &= \left[2\int_{\overline{r}_0}^{\overline{r}_\infty} F(\overline{r}')d\overline{r}' - \frac{2}{\overline{r}_\infty^2}\int_{\overline{r}_0}^{\overline{r}_\infty} (\overline{r}')^2 F(\overline{r}')d\overline{r}' - \frac{\overline{r}_0 c\overline{T}_0\sqrt{\varepsilon \overline{T}}u_c}{w_c^{\,2}\overline{r}_\infty^2} \right] \Big/ \left(1 - \frac{\overline{r}_0^2}{\overline{r}_\infty^2} \right) \\ b_2 &= -b_1\overline{r}_0^2 + \overline{r}_0\frac{c\overline{T}_0\sqrt{\varepsilon \overline{T}}}{w_c^{\,2}}u_c \end{aligned} \qquad \text{(SI-12)}$$

In our iteration scheme, equations (SI-8) and (SI-11) were used as the initial guess to solve equations (SI-3a, b) with the 'bvp4c' function in MatLab® in the first iteration. At the following iterations of increasing w_c, equations (SI-3a, b) were solved numerically using the solution obtained from the previous iteration as the initial guess.

Fitting experimental profile. Fitting the numerical solution to experimental film profile was performed through a least square fitting scheme using $\{T_0, u_c, c\}$ as fitting parameters. While T_0 and u_c were free to vary, the contact radius c was constrained based on the approximated measurement in a range of ±5 μm. Fitting was performed on each film sample at 5-11 different indentation depths where deflections showed good axisymmetry.

Approximate measurement of residual strain in cured PDMS. A thin layer of PDMS (~200µm) was moulded into a silicon master patterned with parallel microchannel structures of width 10µm and centre-to-centre spacing 20µm. After the PDMS layer was cured in the same manner as the film samples, it was peeled off from the silicon master. We then aligned and re-attached the PDMS layer to its silicon master. Misalignment induced by shrinkage was monitored using a microscope, and the residual strain was computed to be approximately (0.2±0.001) %.