Electronic Supplementary Material (ESI) for Soft Matter.

Electronic Supplementary Information

Model of the finite sliding and stress-stretch relation

Let us first consider Stage II of the deformation. The mesh layer is taken to be linear elastic:

$$f = k(\lambda - 1). \tag{S-1}$$

When the mesh is in a critical state and the island is about to fracture, the maximum membrane force in the non-sliding zone C reaches a critical value, $f = f^*$. The deformed length of zone C, r, is related to the undeformed length r_0 by $r = r_0(1 + f^*/k)$. In the sliding zone B, the lateral force balance of the mesh layer dictates a linear distribution of the membrane force:

$$f = \tau x. \tag{S-2}$$

Utilizing Eqs. (S-1) and (S-2) and considering (3), we obtain the relation between the deformed length L and the corresponding length before deformation L_0:

$$\frac{L}{L_0} = 1 + \frac{f^*}{2k}. \tag{S-3}$$

Now let us turn to the deformation in the tape. Due to the finite sliding, the part of the tape that was beneath the mesh island is significantly stretched. Denote the total deformed length of the sliding segment by l. The horizontal equilibrium of the tape gives the stress distribution

$$\sigma_{\text{tape}} = \begin{cases} \sigma^0 + \dfrac{\tau L}{H} & (L - l < x < 0) \\ \sigma^0 + \dfrac{\tau}{H}(L - x) & (0 < x < L) \end{cases} \tag{S-4}$$

where σ^0 is the stress in the non-sliding segment C. In the absence of relative sliding, the strain in the tape in zone C matches with that in the mesh, and σ^0 is the corresponding axial stress. When a certain constitutive model for the VHB tape selected, and the stress-stretch relation $\sigma(\lambda)$ is prescribed, Eq. (S-4) can be used to determine the deformation field in the tape layer. Here by referring to the test result of the VHB tape, we select the incompressible neo-Hookean model, of which the uniaxial nominal stress is related to stretch as

$$\sigma_{\text{tape}} = \mu\left(\lambda - \frac{1}{\lambda^2}\right). \tag{S-5}$$

When the mesh in zone C reaches the critical point, $f = f^*$, the stress in the underlying tape layer is

$$\sigma^0 = \mu\left(\lambda^* - \frac{1}{\lambda^{*2}}\right), \tag{S-6}$$

where $\lambda^* = 1 + f^*/k$ is the fracture stretch of the mesh.

By integrating the inverse of stretch over the deformed length of the sliding zone on the tape, a relation between its original length L_0 and the deformed length l can be obtained:

$$L_0 = \int_{L-l}^{L} \frac{dx}{\lambda}. \tag{S-7}$$

The integral can be evaluated numerically with the stretch values obtained from Eqs. (S-4-S-6). The system has two dimensionless parameters: the stretch at fracture of the mesh λ^*, and the ratio between the strength of the mesh and the stiffness of the tape $f^*/\mu H$. Just as the first network in the DN gel, the mesh provides the overall stiffness of the composite and acts as the sacrificial component. With limited stretchability, the range of the critical stretch λ^* is relatively small and is thus not a major contributor to the toughness of the composite. On the other hand, the strength-stiffness ratio $f^*/\mu H$ plays an important role in the stretchability and toughness of the composite. Here by taking a representative value $\lambda^* = 1.2$ and integrating (10) numerically, we plot the average stretch l/L_0 in the sliding zone of the VHB tape as a function of the dimensionless parameter $f^*/\mu H$ in Fig. S1. As shown by Fig. S1, for relatively large $f^*/\mu H$, l/L_0 approximately scales linearly with $f^*/\mu H$.

Subdividing of larger islands continues until each island becomes too small to be subdivided, i.e. the length of the island becomes smaller than $2f^*/\tau$. For an island even smaller, the axial force accumulated from the shear lag can no longer reach f^*. The cessation of the island-fragmentation mechanism corresponds to the end point of the plateau stage on the stress-strain curve, and the deformation goes into Stage III. In a small island where the non-sliding zone r shrinks to 0, and thus the stretches in the two layers do not need to match. The minimum stress in the tape σ^0 is no longer determined by Eq. (S-6). Instead, the stress distribution in the tape layer can be rewritten as

$$\sigma_{\text{tape}} = \begin{cases} \sigma & (L - l < x < 0) \\ \sigma - \dfrac{\tau}{H} & (0 < x < L) \end{cases}. \tag{S-8}$$

The stress in the tape layer maximizes in the gap between neighboring mesh islands, and the maximum stress σ is just

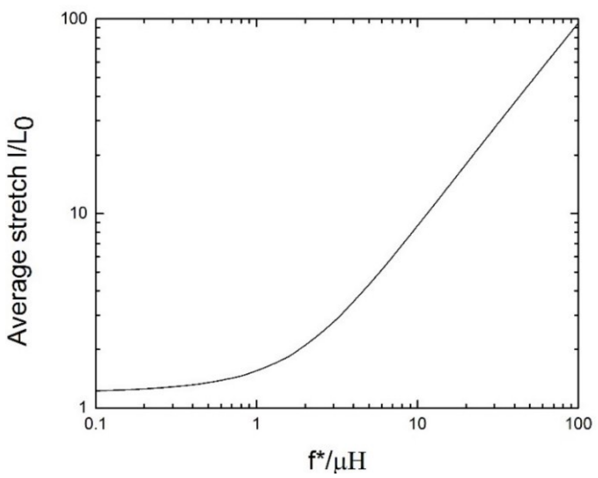

Figure S1. Average stretch in the sliding segment of the tape, plotted as a function of the ratio between the strength of the mesh and the stiffness of the tape, $f^*/\mu H$.

the overall axial stress of the composite. As the maximum force in the mesh is bounded by level limited by the island size, no further fragmentation is possible, the stress in the tape layer and thus the overall stress of the composite will keep increasing in Stage III. The stress-stretch relation could be evaluated numerically by combining Eqs. (6) and (8) and substituting into Eq. (7). Ultimately, the composite fails by rupturing the tape layer, when the maximum stress in the tape σ reaches its strength s_{tape}. Due to the stress concentration in the 2D non-homogeneous deformation, it is expected that the actual fracture stress of the composite is lower than the strength of the VHB tape.

By using the model presented above, the effective stress-stretch curve of the composite can be reconstructed theoretically, as shown by Fig. S2 and Fig. 11 in the paper.

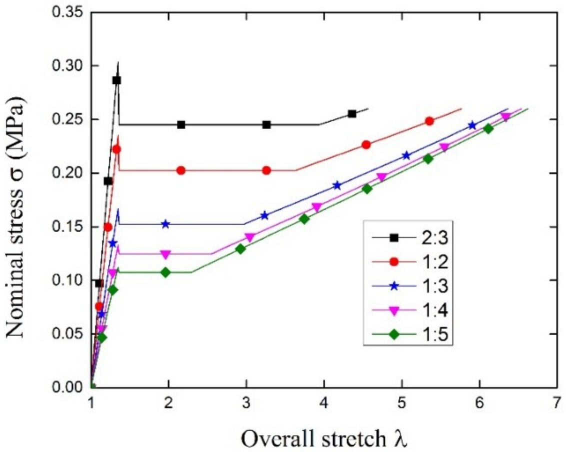

Figure S2. Theoretical prediction by considering the interaction in the sliding zone with a shear-lag model of constant sliding stress. Material parameters are extracted from independent experiments on base materials.