SI Text

Derivation to the distribution of flow velocity and turgor pressure

The Navier-Stokes (N-S) equation is given by

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\frac{1}{\rho}\nabla p + v\nabla^2 u + f \tag{S1}$$

where ρ is the fluid density, $v = \frac{\eta}{\rho}$ is the kinematic viscosity ($m^2 \cdot s^{-1}$). The five terms above represents the local inertia force, inertia force of convection, pressure force, viscosity force, and body force. In order to estimate the relative value of inertia force and viscosity force, the characteristic time t_o, characteristic length L and characteristic velocity U are introduced. And we obtain

$$\frac{local\ inertial\ force}{viscosity\ force} = \frac{\frac{\partial u}{\partial t}}{v\nabla^2 u} = \frac{\frac{U\partial u^*}{t_0 \partial t^*}}{\frac{vU}{L^2}\nabla^{*2}u^*} \propto \frac{L^2}{vt_0} = Ns$$

(Stokes number)

$$\frac{inertia\ force\ of\ convection}{viscosity\ force} = \frac{(u \cdot \nabla)u}{v\nabla^2 u} = \frac{\frac{U^2}{L^2}(u^* \cdot \nabla^*)u^*}{\frac{vU}{L^2}\nabla^{*2}u^*} \propto \frac{UL}{v} = Re$$

(Reynolds number)

$Ns = St \cdot Re$, where $St = \frac{L}{Ut_0}$, so

When $Re \to 0, Ns \to 0$, we neglect the terms of the local inertia force and inertia force of convection. We also discard the body force. **Eq.** (S1) is simplified as

$$\eta\nabla^2 u = \nabla p \tag{S2}$$

Where η is the dynamic viscosity ($Pa \cdot S$).

The incompressible condition of Newtonian fluid is given as

$$\nabla \cdot u = 0 \tag{S3}$$

Eqs. (S2) and (S3) describe the Stokes flow in pollen tube.

- **Boundary conditions**

Sliding wall boundary at the pollen tube wall

$$u_z = \bar{u}_z \tag{S4}$$

If V_{Pole} is the growth rate of pollen tube, the flux through the section of pollen tube

$$\int_0^R 2\pi r u_z dr = V_{Pole} \pi R^2 \tag{S5}$$

- **Solution**

In the idealized pollen tube, velocity components are $u_z = u_z(r), u_r = u_\theta(r) = 0$ The velocity components automatically meet continuity equation **Eqs.** (S3). According to momentum equation **Eqs.** (S2), pressure distribution is $p = p(z)$ **Eqs.** (S2) becomes

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{1}{\eta}\frac{dp}{dz} \tag{S6}$$

Integrating **Eqs.** (S6), we get

$$u_z = \frac{1}{\eta}\frac{dp}{dz}\frac{r^2}{4} + A\ln r + B \tag{S7}$$

Because of $u_z(r = 0) \neq \infty \Rightarrow A = 0$ and $u_z(r = R) = \bar{u}_z \Rightarrow B = \bar{u}_z - \frac{1}{\eta}\frac{dp}{dz}\frac{r^2}{4}$, we get

$$u_z = \bar{u}_z - \frac{1}{4\eta}\frac{dp}{dz}\left(R^2 - r^2\right) \tag{S8}$$

Substituting exp. (S8) into **Eqs.** (S5), we get

$$\frac{dp}{dz} = \frac{8\eta}{R^2}\left(\bar{u}_z - V_{Pole}\right) \tag{S9}$$

Substituting **Eqs.** (S9) into exp. (S8), we get

$$\frac{u_z}{\bar{u}_z} = 1 - 2\left(1 - \frac{V_{Pole}}{\bar{u}_z}\right)\left(1 - \frac{r^2}{R^2}\right) \tag{S10}$$

Integrating **Eqs.** (S9), we get

$$\frac{p}{p_o} = 1 + \frac{8\eta\left(\bar{u}_z - V_{Pole}\right)}{R^2 p_o} z \tag{S11}$$

Eqs. (S10) and **Eqs.** (11) are simplified as **Eqs.** (3) and **Eqs.** (4).