Electronic Supplementary Material (ESI) for Soft Matter.

Supplementary Information and Figures: Simultaneous measurement of the Young's Modulus and the Poisson Ratio of thin elastic layers

Supplementary Figure S1: Correction Factor in the Model of Dimitriadis et al.

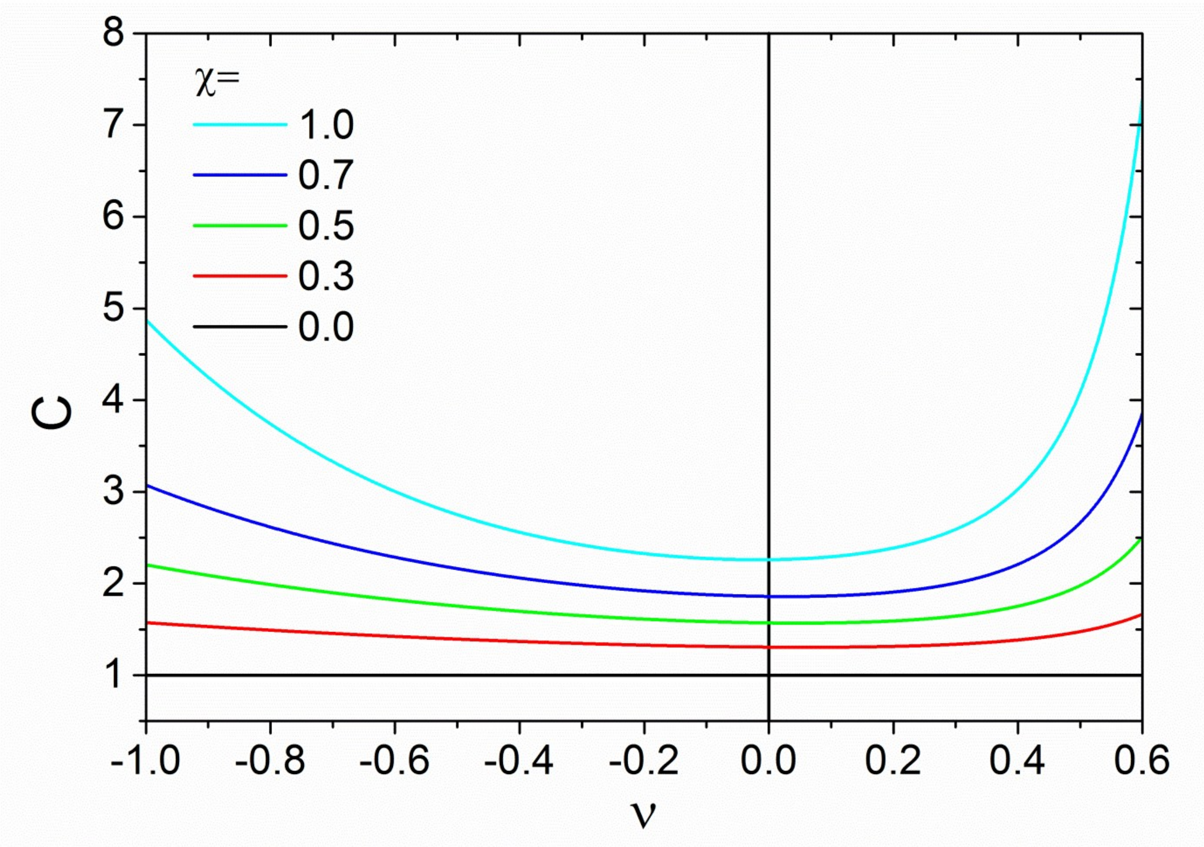

Figure S1: Thickness correction factor C in the model of Dimitriadis et al. (equation 3) as a function of the Poisson ratio for a soft layer bound to a stiff substrate.

Supplementary Information 2: Reconstruction of the Elastic Modulus E and the Poisson ratio ν

The elastic modulus E and the poisson ratio ν can be reconstructed by determining the pair of E and ν which most accurately reproduces the measured datasets $\{\delta_i\}$ and $\{h_i\}$ for all radii $\{R_j\}$. We define the quality of a pair of E and ν by calculating the squared deviation from the model. We calculate an estimate for the elastic modulus using equation 3 assuming Poisson values between 0 and 0.6 with a step size of 0.001 for all measured data points E:

$$E = < E(\{\delta_i, h_i, R_j\}, \rho_s, \rho_{PBS}; \nu) > \qquad \text{(S1)}$$

We then determine the estimated indentation depths $\delta_{i,est}$ for all data points for each set of E and ν by solving equation 3 with the Euler method for δ. We observed that equation 3 can have 2 positive solutions for δ of which the lower one is always realized in the experiment. We ensure that the Euler method converges at the lower solution by setting the start value to 1/10th of the measured δ_i.

We then choose the optimal pair of E and ν by minimizing the standard deviation of δ:

$$\sum_{i,j} (\delta_i - \delta_{i,est})^2 \rightarrow min \qquad \text{(S2)}$$

In our experiments, δ, h, and R were the main contributors causing uncertainties in the reconstruction. We use a statistical approach to estimate the precision of the reconstruction by varying the input dataset $\{\delta_i, h_i, R_j\}$. We vary every data point δ_i, h_i and R_j by generating normally distributed values around the measured data points and a standard deviation of their respective errors. We then determine an optimal pair of E and ν by minimizing the standard deviation of the indentations as described above. By repeating the data generation and reconstruction for 10,000 iterations, we calculated distribution functions for both E and ν.

For the three AA gels with Poisson ratios close to 0.5, these distribution functions were symmetric and normally distributed (See figure S2 A, B, and C). We therefore fitted a normal distribution function and report the fitted standard deviation as the error of both parameters respectively. In the case of the NIPA-gel which has a Poisson ratio of about 0.33, we observe that the distribution is asymmetric and wider towards lower Poisson ratios. We therefore fitted an asymmetric normal distribution with two different standard deviations σ_l and σ_u on both sides of the maximum and report these standard deviations as the errors of the reconstruction:

$$p(x) = \begin{cases} Ae^{-\frac{(x-\mu)^2}{2\sigma_l^2}}, & x < 0 \\ Ae^{-\frac{(x-\mu)^2}{2\sigma_u^2}}, & x \geq 0 \end{cases} \qquad \text{(S1)}$$

Additionally, about 10 percent of the reconstructions yielded $0 < \nu < 0.05$ in the NIPA-measurement (see figure S2 D). We could reproduce this peak by simulating comparable indentation data for two spheres with radii of $200\mu m$ and $500\mu m$ for a gel with $E = 15kPa$ and $\nu = 0.3$ by solving equation 3 for δ (figure S3 A). We added normally distributed noise to δ, h and R of a magnitude similar to our experimental noise and applied our reconstruction algorithm. The results of one dataset which shows a distinctive peak at a Poisson ratio of $0 < \nu < 0.05$ are shown in figure S3 B and C. The distributions are similar to our measured PNIPA data set, showing also a distinct peak near $\nu = 0$ (blue) far away from the value of $\nu = 0.3$ used during data generation. Therefore, we attribute this peak to finite measurement errors and suggest, that it is an inherent property of equation 3. For this reason, we did not consider reconstructions yielding $\nu < 0.05$ when fitting equation S3 to the distributions. Using this method, the reconstruction of 50 simulated datasets yielded $E = (15.4 \pm 1.3) kPa$ and $\nu = 0.29 \pm 0.05$, which is in excellent agreement with the original material parameters used to generate the data sets. Therefore, our method is capable of reconstructing the material parameters.

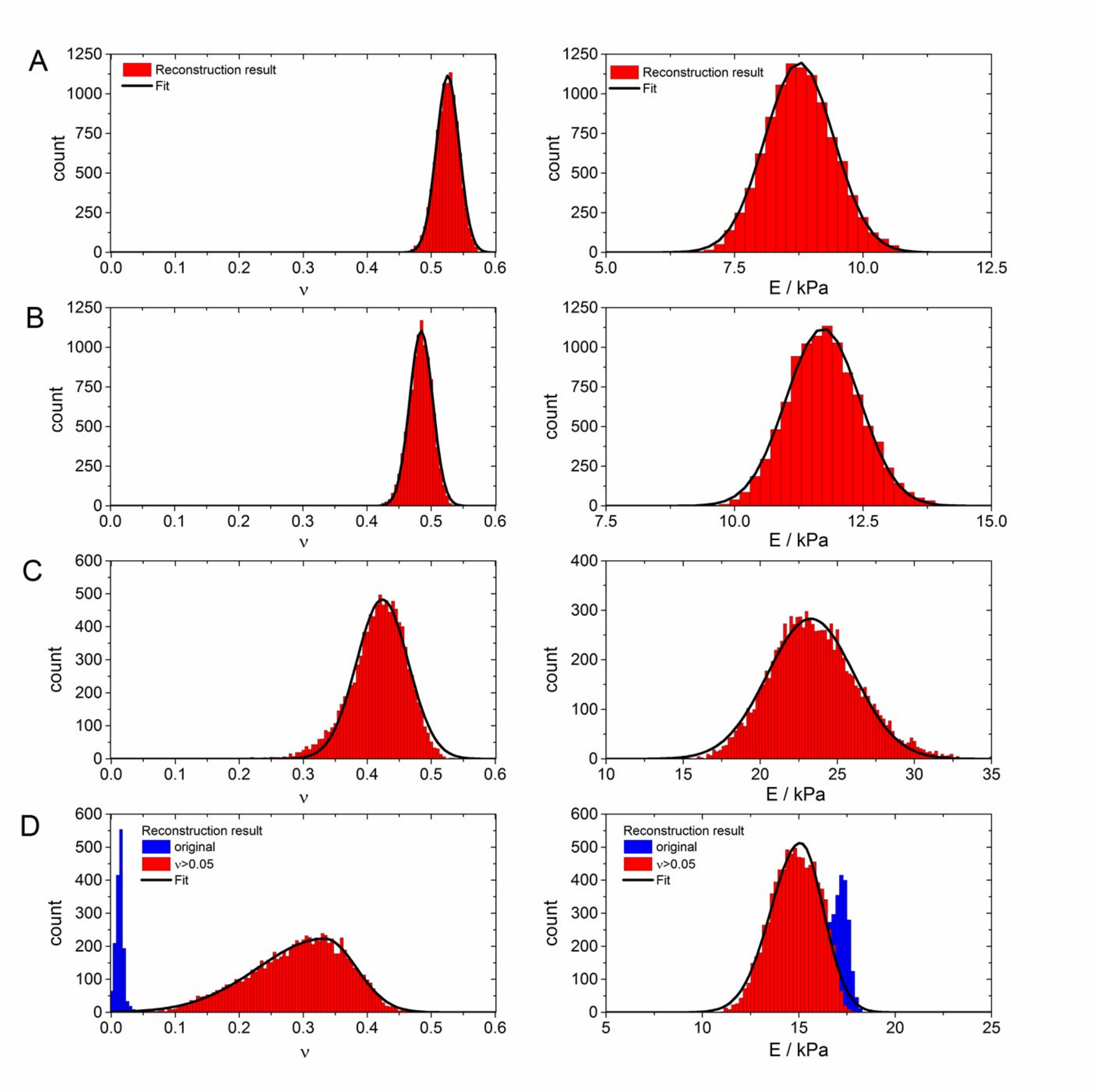

Figure S2: Reconstructed distribution functions for E and ν, obtained by the steel sphere method. The distribution functions for (A) 10% AA and 0.03% BIS, (B) 10% AA and 0.06% BIS, and (C) 10% AA and 0.10% BIS are symmetric and were fitted with a normal distribution function. For the 10% PNIPA gel (D), the distribution is wider towards lower Poisson ratios. We therefore fitted an asymmetric normal distribution with two different standard deviations above and below the maximum.

Furthermore, the distribution has a peak near $\nu = 0$. We excluded all reconstructions which yielded $\nu < 0.05$ (marked in blue) from the fitting.

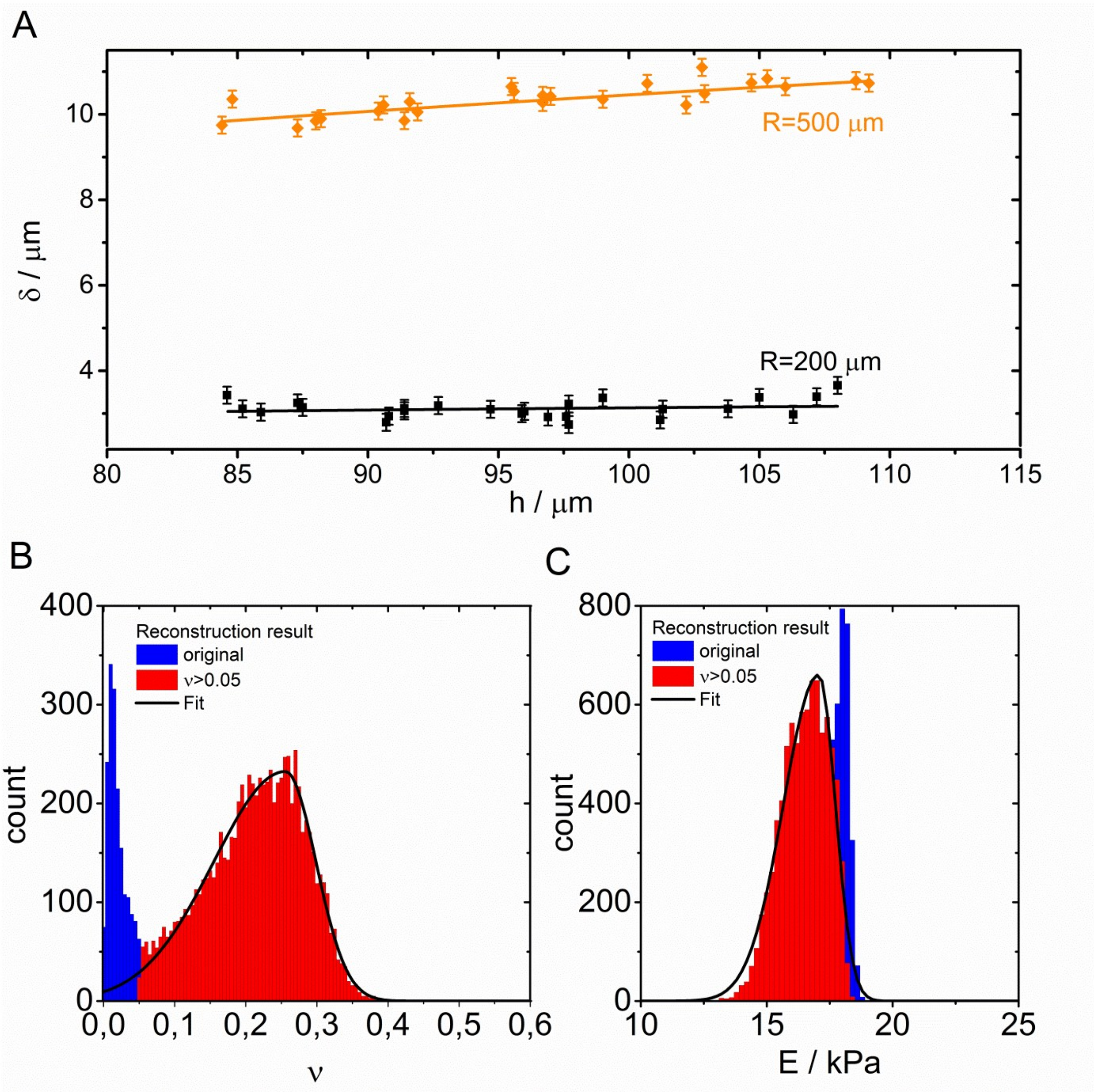

Figure S3: Reconstruction of a simulated experiment. (A) Simulated data for a gel with $E = 15kPa$ and $\nu = 0.3$. Error bars display the standard deviation of the normal distribution used to apply noise. The results of the reconstruction algorithm are displayed in the subfigures B and C.