A Re-Evaluation of Transparent Conductor Requirements for Thin-Film Solar Cells

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Supplementary Information

Approximating the efficiency factor

In section 3 of the main text we have given approximate forms of the efficiency factor which are amenable to being used as figures of merit for transparent conductors. Equation (5) was derived first by setting the derivative of (2a) with respect to W equal to zero, giving

$$2\frac{J_{mp}^{0}}{V_{mp}^{0}}R_{s}W^{3} - \frac{J_{mp}^{0}}{V_{mp}^{0}}sR_{s}W^{2} - 3s = 0$$
(S1)

and then noting that as the shading loss ${}^{S}/W$ should be no larger than a few percent for the optimal W, the second term can be safely neglected by comparison with the first, giving the expression for W^* quoted in (5). For equation (6) an identical procedure applies, first taking the limit of (2b) in which R_M vanishes, and then differentiating to obtain

$$2\frac{J_{mp}^{0}}{V_{mp}^{0}}R_{s}p^{3} - \frac{J_{mp}^{0}}{V_{mp}^{0}}wR_{s}p^{2} - 12s = 0$$
(S2)

In which again the second term can be neglected, being of order W/p.

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Figure S1: (Left) A comparison between the metallized efficiency factor as calculated using equation (2b) with the optimizations performed numerically, and the figure of merit defined by equation (6). (Right) A comparison between the bare figure of merit (equation (5)) and the full figure of merit, in which the effect approximation is invisible. Two values of the interconnect width s are shown. In both cases we take T = 100%, making these plots of the maximum efficiency factor E_{max} .

In figure S1 we illustrate the close agreement between the expression (7) and the full efficiency factor as defined by equation (2b) when the optimization is performed numerically. Two different wire heights are chosen for the efficiency factor to illustrate that the figure of merit models the limit $h \rightarrow \infty$, or equivalently $R_M \rightarrow 0$. Similarly we show the even closer agreement between the figure of merit for a bare electrode equation (6) and *E* as defined by equation (2a). In this case the discrepancy is actually invisible in the R_s range shown.

Dead-space Losses

In the text it was claimed that metallizing a transparent electrode may reduce the impact of interconnect dead-space on the efficiency of a thin-film module. This statement is quantified in Figure S2 where we plot the interconnection loss versus sheet resistance by calculating the optimum cell width W using equation (2a). For sheet resistances in the vicinity of $10\Omega/sq$ metallization allows for cell widths which are 2-3 times larger than for bare electrodes, causing a corresponding reduction in the interconnection loss. At larger sheet resistances the effect is even more pronounced.



Figure S2: (Left) A plot of optimal cell widths versus wire spacing for different electrode types. (Right) Corresponding interconnection losses expressed as a fraction of cell efficiency.

Efficiency Factors for Tandem Cells

The efficiency factors defined by (2) were introduced for the purpose of modelling the top electrode on a stand-alone cell, but a similar strategy can be pursued to model the performance of TCs in tandem cells with only slightly different equations. If the tandem efficiency is written as a sum of contributions from the top and bottom cells $\eta_{tandem} = \eta_{top} + \eta_{bottom}$ then the efficiency factor takes a corresponding two-part form. For the rear TC of a tandem top-cell, the electrode's sheet resistance only impacts on the top cell's voltage, whilst the transparency only reduces the bottom cell's current, thus (neglecting interconnection losses for simplicity):

$$E(R,T) = T\left(1 - \frac{w}{p^*}\right)(1 - \phi) + (1 - \frac{1}{3}\frac{J_{mp}^0}{V_{mp}^0})(1 - [W^2R_M + p^{*2}R_s])\phi$$
(S3)

where $\phi = \frac{\eta_{top}}{\eta_{top}} + \eta_{top}$, which can be used as before to determine the optimal (R_s, T) combinations and wire spacing, with the minor caveat that T is no longer a common factor so that E_{max} cannot be defined as previously. In Fig. S3 we use this equation to again show the dramatic effect of wire width on the performance of high R_s electrodes by plotting the tandem efficiency factor above for technologies spanning a wide range of sheet resistances, using parameters appropriate to a perovskite-on-silicon combination.



Figure S3: Plots of the tandem efficiency factor for the rear-electrode of a tandem topcell, defined by equation (S3) and calculated using experimental (R_s , T) data from the literature. Here $\phi = \frac{2}{3}$, $\frac{J_{mp}^0}{V_{mp}^0} = 19\Omega^{-1}$ cm⁻² corresponding approximately to a perovskite-silicon device, and W = 1cm.

Contact resistance in metallized electrodes

In presenting equation (1b) for the voltage loss in a metallized electrode we have neglected the contribution of contact resistance between the TC and metal wires. Assuming that the contact is ohmic, these losses can be modelled with an additional term added to the voltage loss (1a):

$$\frac{\Delta V_c}{V_{mp}^0} = \frac{1}{3} \frac{J_{mp}^0}{V_{mp}^0} \frac{\rho_c}{w} p$$

in which ρ_c is the specific contact resistance. Using the approximately optimal wire spacing defined by equation (S2) gives the condition

$$\rho_c \ll w \frac{J_{mp}^0}{V_{mp}^0} \left(\frac{J_{mp}^0 R_s}{6V_{mp}^0} \right)^{1/3}$$

under which the contribution of contact resistance is negligible. Using typical numbers found in the text gives an upper bound of of $\rho_c \approx 1 \cdot 10^{-2} \Omega \text{cm}^{-2}$, which is well satisfied for typical metallic contacts to conductive oxides and graphene (1), but may require attention in other cases such as carbon nanotubes (2).

Bibliography

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