Supplementary Information

1. Fitting of the tight-binding-ladder model.

Suppose that the hopping integrals along the left and right edges are t_1 and t_2, respectively. The on-site energies of two edges are ε_l and ε_r, respectively. Then, the band structures of the two edges are $E_l(k)=\varepsilon_l-2t_1 coskb$ and $E_r(k)=\varepsilon_r-2t_2 coskb$, where b is the lattice constant along the Y direction. Since the left edge is spin-polarized, the on-site coulomb repulsion energy U between the spin-up and spin-down electrons should be considered. For the spin-up and down electrons, the M_σ takes -1 and +1 respectively. If the hopping integral between two edges is t_0, the Hamiltonian is as follows:

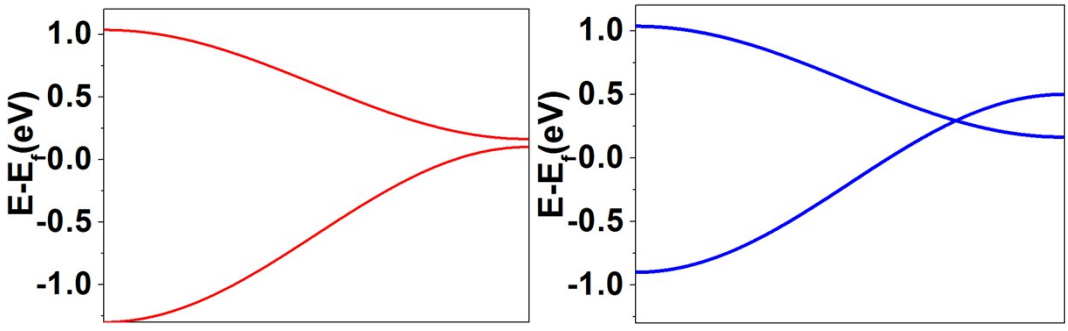
$$H=\begin{bmatrix} \varepsilon_l-2t_1\cos kb+M_\sigma U & t_0 \\ t_0 & \varepsilon_r-2t_2\cos kb \end{bmatrix}$$

Thus, the band structure near Fermi level takes the form

$$E(k)=\frac{E_l(k)+E_r(k)+M_\sigma U}{2}\pm\frac{1}{2}\sqrt{[E_l(k)-E_r(k)+M_\sigma U]^2+4t_0^{\,2}}$$

Fig. 1. The band structure is simulated by the parameters: ε_l= -0.4, ε_r= 0.6, U= 0.2; t_1=0.35, t_2=-0.218, t_0=0.0. This figure can be used to fit to Figure 3 of manuscript. (a) spin up band; (b) spin down band. The simulated figures can help to understand the band gap opening of spin up band and band closure of spin down band.

(a) (b)



2. Fitting of the relativistic linear dispersion in band structure.

The linear dispersion of the band structure in the vicinity of the crossing point can be qualitatively described by the following Hamiltonian

$$H=\begin{bmatrix} \varepsilon_1 & -t_1[1+\exp(i\vec{k}\cdot\vec{b})] & 0 & 0 \\ -t_1[1+\exp(-i\vec{k}\cdot\vec{b})] & \varepsilon_2 & -t_2 & 0 \\ 0 & -t_2 & \varepsilon_3 & -t_3[1+\exp(-i\vec{k}\cdot\vec{b})] \\ 0 & 0 & -t_3[1+\exp(i\vec{k}\cdot\vec{b})] & \varepsilon_4 \end{bmatrix}$$

Where ε_i is the on-site energy of atom i (i=1, 2, 3, 4) and $-t_i$ ($t_i > 0$) is the hopping parameter between atom i and atom i+1 (i=1, 2, 3). The qualitative relationship between on-site energies of these four atoms contributed to the linear dispersion state was found as $\varepsilon_2 > \varepsilon_3 > \varepsilon_4 > \varepsilon_1$. The strength relationship of hopping parameters was found as $t_1 > t_2 > t_3$. We note the real case is that more atoms and electrons involve in the formation of this relativistic state. Hence it is more complex. Our model here is only to provide a qualitative physical scenario.

Fig. 2. The following linear dispersion near the crossing point is simulated by the parameters: t1=0.5, t2=0.45, t3=0.43; ε1= 0.45, ε2= 1.55, ε3= 1.15, ε4= 0.85. This figure can be used to fit to the linear dispersion in Figure 4 of manuscript.

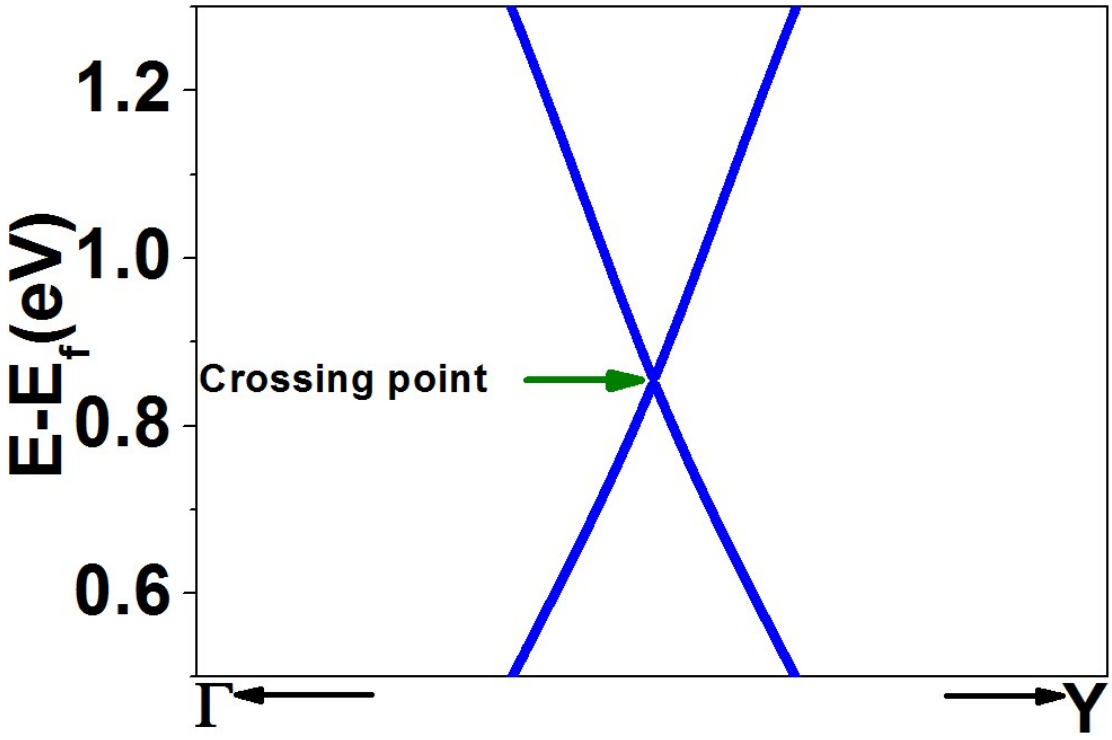
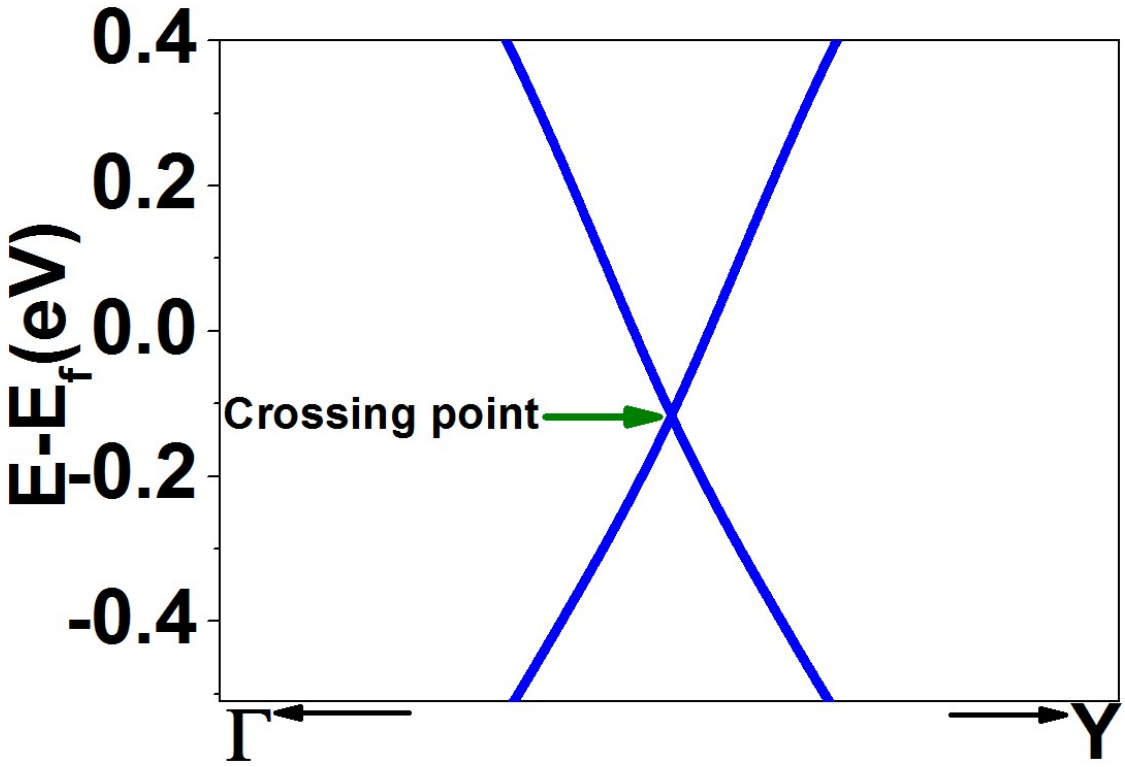


Fig. 3. The following linear dispersion near the crossing point is simulated by the parameters: t1=0.5, t2=0.45, t3=0.43; ε1= -0.50, ε2= 0.60, ε3= 0.20, ε4= -0.10. This figure can be used to fit to the linear dispersion in Figure 6.


0.4
0.2
0.0
-0.2
-0.4
E-E$_f$(eV)
Crossing point
Γ
Y