Electronic Supplementary Material (ESI) for Analytical Methods.
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1. Electronic Supplementary information for

Frequency characterization of flow magnitude and phase in resonant microfluidic circuits

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Section 1: Complete bead position fitting result for the series resonant device

Section 2: Figure illustrating the phase difference between bead position and velocity.

Section 3: Voltage to bead streaks conversion function

Section 1: Complete bead position fitting result for the series resonant device

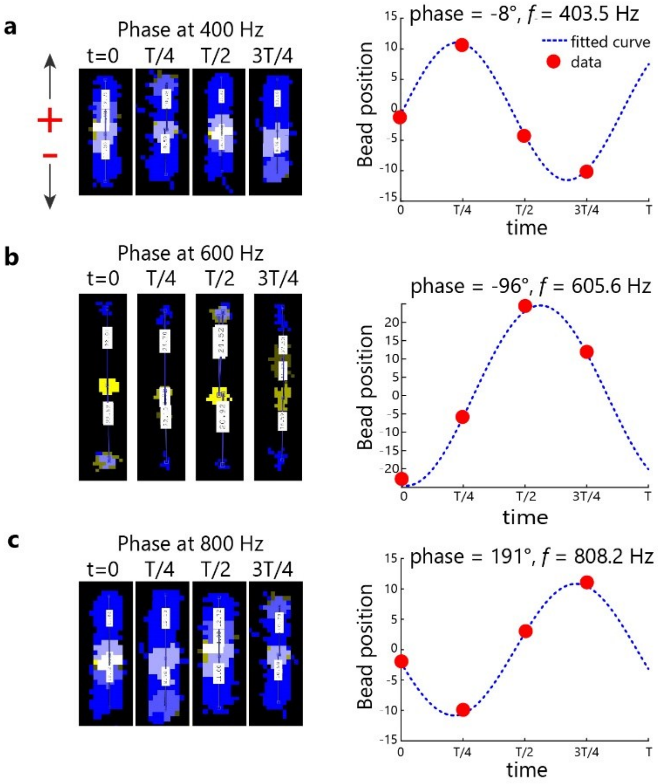

Figure S1: Phase calculation using bead position at the four offsets. **a.** shows the bead traverse at 400 Hz. The bead position (red) relative to the 'rest' position (yellow) at the four offsets is plotted on the right. Positive and negative directions are indicated. Note that while it appears that at t=0 the bead was moving up (positive) - towards the top extreme, in the actual fit, the position was slightly negative because we corrected for any drift in the center rest position across the four measurements. The dashed curve show the result of the sum of sines fit. The fitted phase and frequency of the fundamental frequency from sum of sines are shown. The fitted phase was adjusted (+90°) for the position-to-velocity conversion (see section 2). The normalized magnitude of the harmonics were orders of magnitude (10^{-5}) smaller compared to the fundamental (*f*). **b.** and **c.** show the bead positions and the fit for 600 Hz and 800 Hz respectively.

Section 2: Figure illustrating the phase difference between bead position (x) and velocity (v)

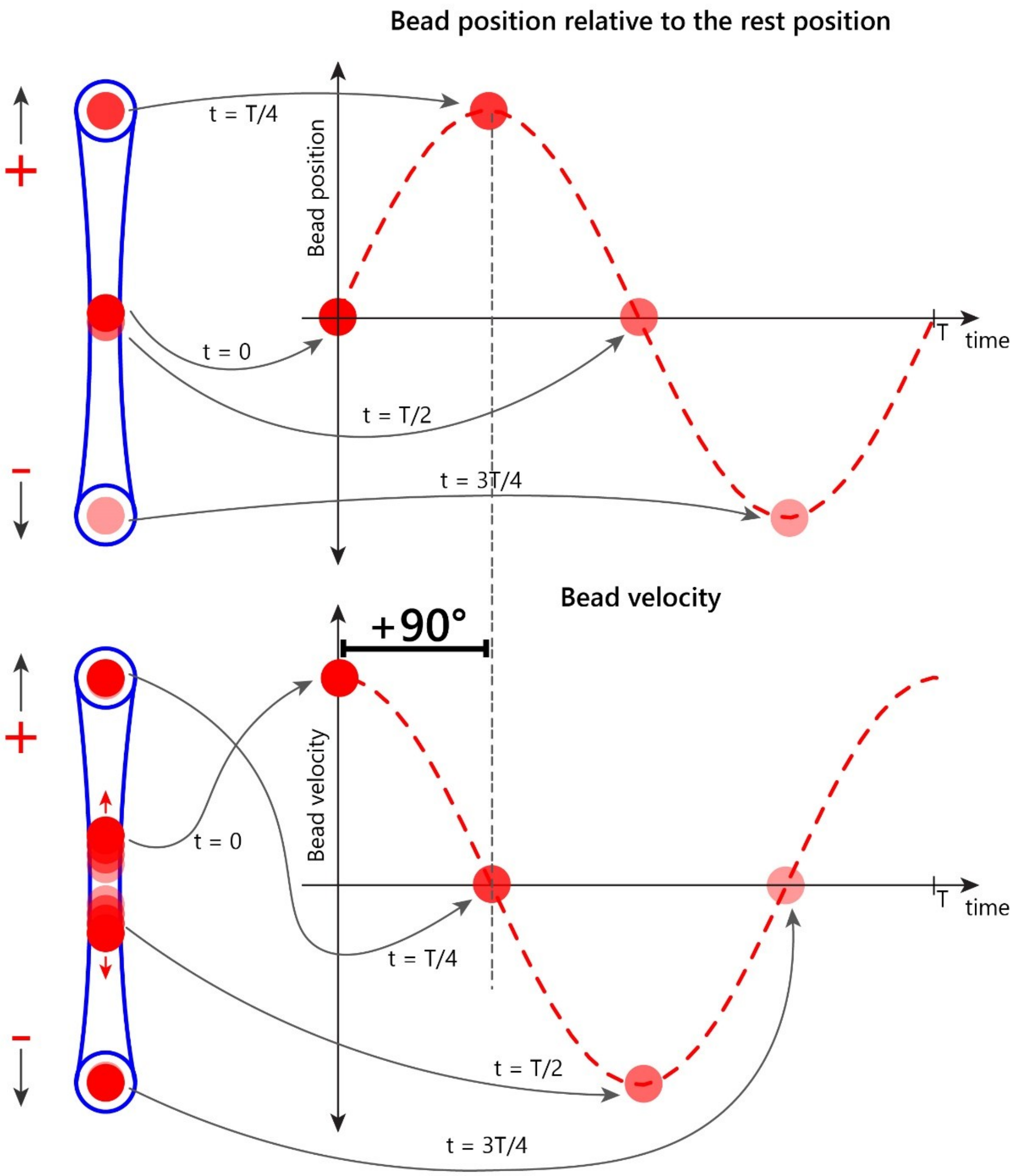

Figure S2: Top panel shows bead position with respect to the center rest position at different time points. In this example, the bead starts from the center rest position (minimum position) and moves to the top extreme (maximum position) by T/4. By T/2, the bead is back to the center position but moving towards the bottom extreme, which it reaches by 3T/4. The fitted curve for bead position reveals a sine wave with about 0° phase. The bottom panel shows the bead velocity for the same example. At t=0, when the bead is at the center rest position and moving to the top extreme, its velocity is maximum. When the bead reaches the extreme position, it comes to a stop, much like a pendulum. Hence its velocity is minimum. The plot of velocity versus time shows that the velocity (v) is shifted by +90^{o} relative to bead position (x). Since our experiments measured bead position, phase shift determined from bead position was shifted by +90^{o} to allow direct comparison to phase shift of the AC flowrate provided as the output from the circuit model.

Section 3: Voltage to bead streaks conversion function

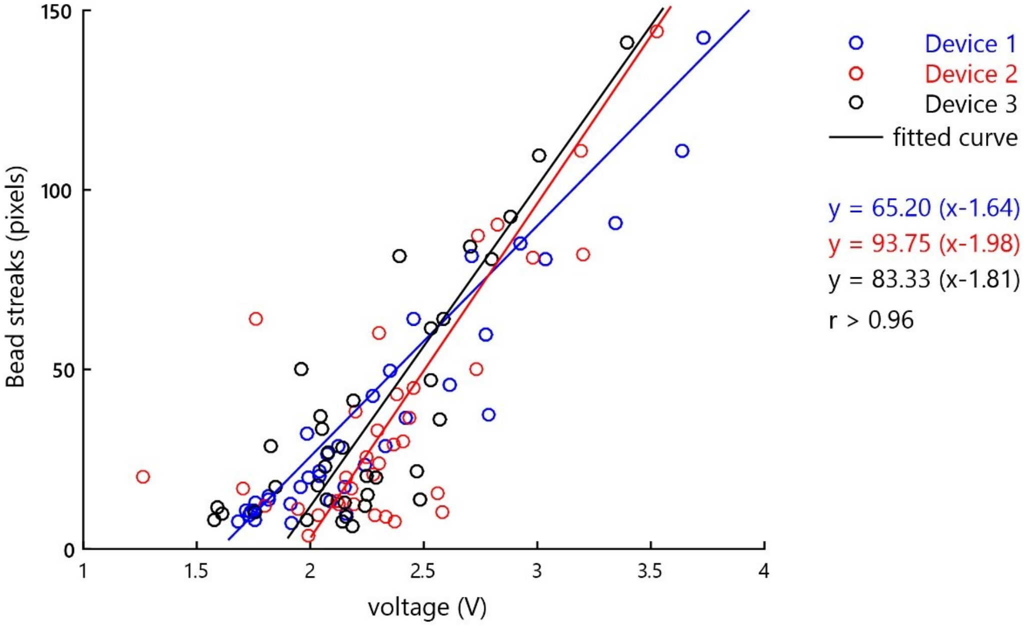

Figure S3: The linear function for converting sensor voltage to equivalent bead streak was calculated using a set of three calibration devices (series resonant device, f_s = 250 Hz, channel length = 16 mm, capacitor radius = 3.5 mm). Flow magnitude was measured using beads in the channel and through the capacitor. Since flowrate calculated by both methods should be equal in magnitude and phase, the bead streak length (pixels) measured using the images from the channel and the raw sensor voltage (V) from the diaphragm-lens sensor for the same frequency correlate linearly as shown in the plot. The fitted parameters (slope and intercept) from the three devices were averaged and the resulting function was used to convert future sensor measurements to equivalent bead streaks, which were used to calculate the flowrate using the same techniques as in the channel