Supporting Information

Experimental details

To synthesis the Ag₉GaSe₆ compound, high purity raw elements, Ag (shots, 99.999%, Alfa Aesar), Ga (shots, 99.999%, Alfa Aesar), and Se (pieces, 99.999%, Alfa Aesar) were weighted out in stoichiometric proportions and then sealed in silica tubes under vacuum. The tubes were heated to 1373 K and held at this temperature for 12 hours before quenching into cold water. Then, the quenched ingots were annealed at 873 K for 5 days. Finally, the products were ground into fine powders and sintered by Spark Plasma Sintering (Sumitomo SPS-2040) at 813–833 K under a pressure of 40 MPa for 10 minutes. High densities (> 99% of the theoretical density) were obtained for all samples.

Characterization methods

The phase purity and crystal structure were examined by the powder X-ray diffraction (PXRD) with Cu K α radiation at 300 K. The electrical conductivity and Seebeck coefficient were measured by using ZEM-3 (ULVAC) from 300 to 800 K. The thermal conductivity was calculated from $\kappa = DC_p\rho$, where the thermal diffusivity (*D*) was obtained by using a laser flash method (Netzsch LFA 457), the specific heat (C_p) was measured by differential scanning calorimetric (Netzsch DSC 404F3), and the density (ρ) was measured by using the Archimedes method. Hall coefficient (R_H) was measured in a Physical Property Measurement System (Quantum Design) by sweeping the magnetic field up to 3 T in both positive and negative directions. Hall carrier concentration (p_H) and Hall mobility (μ_H) were estimated by $p_H = 1/eR_H$ and $\mu_H = \sigma R_H$, respectively. Measurements of the transverse and longitudinal sound velocities were performed on a sample with a diameter of 10 mm and a thickness of 2 mm using an Advanced Ultrasonic Measurement System (TECLAB).

Calculation details¹⁻⁵

Average sound velocity (v_a) is calculated from the sound velocity

$$\frac{1}{v_a} = \left[\frac{1}{3}\left(\frac{1}{v_l^3} + \frac{2}{v_t^3}\right)\right]^{1/3}, \qquad (1)$$

where v_l is the longitudinal sound velocity and v_l is the transverse sound velocity.

Young's modulus (E) is calculated by

$$E = \frac{\rho v_t^2 (3v_l^2 - 4v_t^2)}{(v_l^2 - v_t^2)}, \qquad (2)$$

where ρ is the sample density.

Poisson ratio (v_p) is calculated by

$$v_p = \frac{1 - 2(v_t/v_l)^2}{2 - 2(v_t/v_l)^2}.$$
 (3)

Shear modulus (G) is calculated by

$$G = \frac{E}{2(1+\nu_p)} \tag{4}$$

The Gruneisen parameter (γ) is calculated by

$$\gamma = \frac{3}{2} \left(\frac{1 + \nu_p}{2 - 3\nu_p} \right)$$
 (5)

Debye temperature (θ_D) is calculated by

$$\theta_D = \frac{h}{k_B} (\frac{3N}{4\pi V})^{1/3} \nu_a , \qquad (6)$$

where *h* is Planck's constant, k_B is the Boltzmann constant, *N* is the number of atoms in the primitive unit cell (N = 64 for Ag₉GaSe₆) and *V* is the unit cell volume. Phonon mean free path (*l*) is calculated by

$$\kappa_L = \frac{1}{3} C_V v_a l , \qquad (7)$$

where C_V is the heat capacity at constant volume.

Table S1 Sound velocities (longitudinal sound velocity v_l , transverse sound velocity v_l , average sound velocity v_a) and lattice thermal conductivity κ_L of Ag₉GaSe₆. The elastic properties (Young's modulus *E*, Shear modulus *G*, Poisson ratio v_p), the Grüneisen parameter γ , Debye temperature θ_D and phonon mean free path *l* are

Т	ĸ	VI	<i>V</i> _t	Va	Ε	G	v_p	γ	θ_D	l
(K)	(Wm ⁻¹ K ⁻¹)	(ms ⁻¹)	(ms ⁻¹)	(ms ⁻¹)	(GPa)	(GPa)			(K)	(nm)
300	0.23	2865	1130	1281	26.25	9.321	0.408	2.72	137.1	0.28

derived based on Eq. (1)-(7) based on the measured sound velocity.



Fig. S1 (a) Temperature dependence and (b) Hall carrier concentration (*n*) dependence of *PFs* for Ag₉GaSe_{6-x}. The solid curve in Fig. S1b shows the calculated curve based on the single parabolic band (SPB) model with $m^* = 0.11m_e$ dominated by acoustic phonon scattering.

References

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